Evaluation of communication performance for adaptive optics corrected geo-to-ground laser links

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EVALUATION OF COMMUNICATION PERFORMANCE FOR ADAPTIVE OPTICS CORRECTED GEO-TO-GROUND LASER LINKS
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I. INTRODUCTION
By using existing single mode components developed for fiber technologies (optical detectors and amplifiers, MUX/DEMUX...), the very high throughput of future satellite-to-ground optical communication links might be achievable at a reasonable cost. However, atmospheric turbulence induced perturbations compromise the injection efficiency of the wave into single mode components. The resulting channel impairments can be significantly reduced by the use of appropriate adaptive optics (AO) compensation at the expense of potentially complex and expensive systems if very high stability of the injection is required. Feasibility demonstrations of AO correction were recently reported in the US [1] and France [2] with Low Earth Orbit (LEO) satellites, and in Germany with GEO satellites [3]. To reduce the loss in useful information and to relax by the same way the specifications and cost of AO systems, numerical techniques such as error-correcting codes have to be considered as well. These codes introduce a degree of data redundancy, which allows a transmitted codeword to be correctly interpreted despite the loss of a significant fraction of the individual codeword elements. However, they are usually adapted to combat randomly distributed errors rather than bursty errors. Therefore, over the free-space optical channel, a fading event that is much longer than the codeword duration will usually result in an unrecoverable loss of information. To overcome this non-uniform distribution of errors that occur in slow fading channels, symbol interleaving is mandatory. Such numerical interleavers after disassembling each codeword will space the individual symbols with a temporal interval comparable to a characteristic duration of the channel fades. At the receiver, the symbols are reassembled in the proper order but the errors are redistributed such that there is a higher probability that each codeword is correctly decoded and reconstructed. In addition to the drawback induced by the latency introduced by the symbol spreading, this interleaving process could lead to unmanageable buffer size. Extensive work on numerical approaches to mitigate channel impairments were conducted in the US [4], Germany [5] and Japan [6]. Whereas the AO correction will impact the temporal characteristics of the channel and can therefore, in turn, diminish the drawbacks related to the interleaving process, none of the reported work considered the influence of the AO residuals on the characteristics of the channel. Consequently, the need for a joint optimization of the performances of the AO system and the interleaving process, with respect to power constraints and coding rate, appears necessary. In this paper, appropriate telecom metrics such as the channel capacity and outage probabilities are computed using a simplified AO corrected channel model to assess the influence of AO residuals on the required interleaver size. The specific case of a GEO to ground link with on-off keying (OOK) modulation is addressed. The outline of this paper is organized as follows. In Section II the fading model is presented. The assumptions and underlying hypotheses made for modeling the coupled optical power fluctuations after partial AO correction and the electrical signal after photo-detection are first synthetized. Then the expression of the channel capacity based on the computation of the mutual information (MI) between the received and sent symbols, as well as the way of obtaining the link outage probabilities given a prescribed code rate are detailed. Optimal interleaving using the mean durations of fading events is explained in the same section. In Section III, after introducing relevant telecommunications metrics, the performances of a GEO-to-ground link are analyzed for two cases of AO correction.

II. FADING MODEL
A. Propagation channel: atmospheric turbulence and partial AO
The use of AO has become of significant interest in the community, as it appears now as a mature technology capable of effectively mitigating injection losses in order to meet the requirements characterizing communication applications. Being extensively used by astronomers since the early 90’s [7], it benefits from reliable performance estimation and end-to-end simulation tools. However, in the framework of satellite-to-ground links some simplifications in terms of the modeling of the system can be made. For example, because of the high optical power requested for high data rates (between 40 and 140 photons per bit [8]) and the typical timescale of evolution of the atmospheric turbulence (milliseconds to fractions of milliseconds), it is reasonable to consider that noise influence on wavefront sensing is not a major contributor to the AO error budget. Hence it will be neglected in what follows. In the case of a Shack-Hartmann wave-front sensor this assumption directly
implies that the influence of scintillation on wave-front sensing is neglected as the main effect due to scintillation at the scale of a sub-aperture will be the fluctuations of the signal to noise ratio. In that case, performance will mostly be limited by the number of available actuators of the deformable mirror, the finite number of points for the measurement of the phase perturbations and the control loop frequency. Power fluctuations after such an AO correction can be generated using a simulation tool that jointly takes into account aperture averaged scintillation and coupling losses into a SMF as well as their temporal correlation. This numerical model, relying on a Monte Carlo approach assumes statistical independence between scintillation and phase effects and, makes the hypothesis that the spatial variations of the amplitude on the coupling efficiency are neglected. It is described in more details in [9]. The advantage of this simplified adaptive optics simulation tool (SAOST) over end-to-end simulations resides in its simplicity of use and lower calculation resources and time needed. Compared to similar approaches developed previously[10] this model takes into account aperture averaged scintillation as in the case of very good correction, aperture averaged scintillation can be the major contributor to the injected power fluctuations.

B. Noise and block fading channel model

We study a communication system using intensity modulation/direct detection (IM/DD), making the assumption that the background illumination level is negligible. Moreover, we emphasize here on an OOK modulation format. Usually, in such systems the detection is either PIN-based or APD-based. In the former case, the incoming signal is optically pre-amplified using for example a single mode EDFA in order to overcome the photodiode noise floor and improve the detection sensitivity. However, after photo-detection (often described as a square-law detection) the process of optical pre-amplification produces ASE noise that generates additional beat noises (spontaneous-spontaneous and signal-spontaneous) in the electrical domain of the receiver. Therefore the overall electrical noise is fundamentally non-Gaussian. Several analytic approximations of the statistics of the electrical signal corrupted by ASE beat noises solely exist. The first of which corresponds to a chi-square distribution as described in [11]. In addition, the photo-detector thermal noise cannot be in reality totally neglected and has to be taken into account as well as an additive Gaussian noise. Thus, as a simplification it is common to pretend that all random fluctuations of the detector’s current obey Gaussian statistics as described in [12], which is in fact only valid in an asymptotic way. However, when only an “order of magnitude” in terms of the capacity or error-rate of the transmission is investigated, such a simplification can be convenient. In the case of an APD-based detector, the receiver will be shot-noise limited. It is well known that shot-noise arises from random fluctuations of the current flowing through the photo-detector and is modeled by a Poisson process. However, it has been shown in [13] and [14] that this distribution can be approximated by a Gaussian distribution. Therefore, for both architectures, at a “quasi-abstract” level, one can make the assumption that the detection noise is an AWGN as long as it is assumed that only estimates of the subsequent error-rate and channel capacity are investigated. To do so, as optical communications are characterized by very high data rates (Gbps to Tbps), one can make the sound assumption that the symbol duration or the bit time will be much less than the typical coherence time of the atmospheric channel and thus of the optical power fluctuations at the input of the SMF (typically of the order of the millisecond). In other words, one faces a slow-fading channel. This type of channel can be accurately modeled as a block-fading channel [15]. Under this simplification, one can consider that the channel is fixed for the duration of the coherence interval of the optical power fluctuations, taking a value drawn from their distribution.

C. Capacity estimation

A soft receiver is assumed and, for a fixed value of the power fluctuation at the exit of the SMF noted \( A_{\text{SMF}} \), the channel is therefore a binary input-continuous output channel characterized by additive Gaussian noise \( N \). The received electrical signal can be represented as:

\[
Y = R. (A_{\text{SMF}} \cdot X) + N, \quad (1)
\]

where \( R \) is a constant representing in a broad sense the photo-electric conversion efficiency, \( X \) represents the transmitted signal that is taken as symbols drawn from an OOK constellation such that \( X \in \{0, P_{T_{R}}\} \) and \( P_{T_{R}} \) is the average transmitted optical power. As previously explained, here the noise is assumed independent of the signal and represented by a zero-mean Gaussian distribution of variance \( \sigma_{n}^{2} \). We define the received electrical signal-to-noise ratio (SNR) as in [16]:

\[
\text{SNR}(A_{\text{SMF}}) = \frac{P_{R} R^{2} A_{\text{SMF}}^{2}}{\sigma_{n}^{2}}. \quad (2)
\]

The channel capacity \( C \) is the maximum achievable data rate that can be reliably communicated between the transmitter and the receiver and can be computed from the mutual information (MI) according to [17]:

\[
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\]

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where \( f_{Y|X}(Y|X) \) follows a Gaussian distribution of mean \( R(X.A_{SMF}) \) and variance \( \sigma^2 \). The probability of sending either a 0-bit (corresponding to \( P_{Tr} = 0 \text{ W} \)) is \( \Pr(X = 0) \) and is equal to the probability of sending a 1-bit, that is \( \Pr(X = 1) = \Pr(X = P_{Tr}) = 0.5 \). This expression is commonly assumed for memoryless channel with perfect channel state information (CSI) at receiver side [18]. Its use might be questionable for free space optical channel [19]. However it is justified here as interleavers longer than the coherence time of the channel are considered. Concerning CSI, perfect knowledge of the statistics of \( A_{SMF} \) at receiver is assumed.

D. Outage probability

We assume as well that the transmitter sends information at a fixed coding rate (the proportion of the data-stream that is useful i.e. non-redundant) of \( R_0 \) bits/channel. Shannon’s noisy-channel coding theorem [17] states that the capacity of the channel must be greater than the rate of any code in order for the code to achieve an arbitrarily small error probability. Note that this theorem guarantees successful decoding only for ideal “capacity-achieving” codes. Because the slow-fading channel is random and fixed for a long period of time, there is a non-zero probability that the capacity \( C \) falls under the code rate \( R_0 \). When such an event occurs the code in use will not allow for an “error-free” decoding. These events are called link outages [15]. They correspond to:

\[
P_{Out}(R_0) = \Pr(C(SNR(A_{SMF})) < R_0),
\]  

(4)

That is the cumulative distribution function (CDF) of the channel capacity. Given the form of the latter (4), one can notice that there is a direct bijection between the channel capacity fluctuations \( C \) and the fluctuations of the coupled optical power \( A_{SMF} \) (or the fluctuations of the electrical signal-to-noise ratio \( SNR(A_{SMF}) \) equivalently). Therefore (4) can be expressed in terms of the cumulative distribution function (CDF) of \( SNR(A_{SMF}) \) [20] as:

\[
P_{Out}(R_0) = \Pr(SNR(A_{SMF}) < C^{-1}(R_0)).
\]  

(5)

In the work presented here, the CDF of \( SNR(A_{SMF}) \) is computed numerically from correlated time series provided by the SAOST tool (see section II. A.).

E. Interleavers dimensioning using mean duration of fades

Assuming that the transmitter sends information at a fixed prescribed code rate \( R_0 \) and that the receiver knows the statistical and temporal characteristics of the optical power fluctuations at the input of the SMF after AO correction one would now like to assess the benefits of using numerical interleavers. The implementation of a practical numerical interleaver will aim to span across \( L \) intervals of duration equal to at least the coherence time of the power fluctuations in order to ensure that each one of the output codewords (i.e. after deinterleaving) will be comprised of \( L \) groups of symbols that have experienced uncorrelated fluctuations. Therefore \( L \) represents the diversity of the interleaved channel in our model. Dimensioning the depth of interleavers using the coherence time of the propagation channel does not allow for assessing the effectiveness of the interleaver with respect to the achievable coding rates of the transmission. Therefore, we propose here to set the depths of our interleavers to \( L \) times a duration that depends on the transmitter code rate \( R_0 \) as well as on the averaged transmitted optical power \( P_{Tr} \). This parameter is defined as the mean duration of fading events, that is the mean duration of the fall-offs of the electrical signal-to-noise ratio \( SNR(A_{SMF}) \) under the threshold \( SNR_0 = C^{-1}(R_0) \). This duration, noted \( \mu_{FT}(C^{-1}(R_0)) \) is an important parameter allowing for an optimization of the interleavers in combating errors’ bursts in long fading channels [21] [22]. It is inferred numerically in this paper.

III. INVESTIGATION OF TELECOM PERFORMANCE METRICS IN A GEO DOWNLINK CASE

A. Simulation parameters

The simulation parameters are summarized in the present subsection. They refer to the turbulence conditions as well as the geometry of the link and, the different AO system parameters considered. The vertical turbulence profile is derived from a HV profile and is taken from ITU recommendation ITU-R P.1621-1. \( C_n^2 \) vertical profile is computed according to:

\[
C_n^2(h) = 8.148 \times 10^{-56} \cdot V_{RMS} \cdot h^{16/1000} + 2.7 \times 10^{-16} \cdot e^{-h/15000} + C_0 \cdot e^{-h/1000} \cdot m^{-2/3},
\]  

(6)
where the altitude average wind velocity parameter is set to \( V_{RMS} = 21 \text{ m.s}^{-1} \) and the \( C_n^2 \) at ground level is set to \( C_n^2 = 5.4 \times 10^{-14} \text{ m}^{-2/3} \). This ensures a Fried parameter of \( r_0 = 3 \text{ cm} \) at 550 nm at zenith and a log-amplitude variance \( \sigma_d^2 = 0.06 \) for the same wavelength. We consider a Bufton wind profile which high altitude layer has been modified to take into account an altitude origin at sea level [23]:

\[
V(h) = V_0 + V_he^{(h - 12448)/4800^2},
\]

where \( V_0 = 5 \text{ m.s}^{-1} \) and \( V_h = 25 \text{ m.s}^{-1} \). The case of a GEO downlink is presented here. The wavelength considered is 1.55 \( \mu \text{m} \) (telecom wavelength). The distance between the ground station and the satellite is set to 36000 km. Elevation is 40 deg. For this elevation the Fried parameter is \( r_0 = 8 \text{ cm} \) and Rytov log-amplitude variance is \( \sigma_d^2 = 0.04 \). The diameter of the receiver is set to 50 cm and two AO systems are considered. A “high-performance” (HP) case, corresponding to 9 corrected radial orders with a control loop sampling frequency of 800 Hz and a two frames delay. This leads to an average coupled power of \( A_{SMF} = -2.7 \text{ dB} \). The coherence time of the fluctuations is roughly 8 ms. A “low-performance” (LP) system is considered as well, with 5 radial corrected orders at 400 Hz, leading to a power penalty of \( A_{SMF} = -5.2 \text{ dB} \). In that case the coherence time of the fluctuations is roughly 5 ms. The correction cases are taken from [9]. They both correspond to state of the art performances with commercial off the shelf components.

### B. Telecommunication performance metrics

Two performance metrics that allow for a simple and straightforward characterization of the link are introduced in this section. The reasoning is to use the non-fading channel as a reference (neither power fluctuations nor average power losses due to injection efficiency). The losses introduced by the optical fluctuations are then characterized in terms of average power differences with respect to this unfaded baseline. The first metric consists of the difference between the power required to sustain a given capacity in the case of an unfaded AWGN channel and the power required to sustain fading capacity. The latter is often called ergodic capacity. It is in fact the capacity averaged over all the fading, thus simulating the case of an “infinite interleaver”:

\[
C_{Ergo} = \int_0^\infty C(SNR(A_{SMF}))f(SNR(A_{SMF}))dSNR,
\]

where \( f(SNR(A_{SMF})) \) is the probability density function (PDF) of the electrical SNR conditioned by the fluctuations of \( A_{SMF} \). This loss is non-recoverable by means of numerical mitigation techniques. However, using an AO system compensates it. The second metric of interest is called the finite interleaver loss. For a prescribed code rate \( R_c \), it is the difference between the power corresponding to a given probability of outage and the power corresponding to the fading capacity. In the following subsection, these metrics are used to compare two performance levels of AO correction in the case of a GEO-to-ground link.

### C. Results: An example of link budgeting for two AO performance levels

Figure 1 shows the evolution of the ergodic capacity \( C_{Ergo} \) against the unfaded required average SNR for both the cases of the “High performance” (HP) and “Low performance” (LP) AO systems. The ergodic capacity of the AWGN channel is shown as well. For the remainder of this paper, we set the prescribed, time invariant, code rate \( R_c \) to 0.85. The required SNRs to sustain the ergodic capacity corresponding to this code rate are reported in the cases of the pure AWGN channel, the HP AO system, and the LP AO system on Fig. 1.
Using (5), the outage probabilities for both cases of performance without interleaving as well as after interleaving with \( L = 5, 20 \) and 80 are reported on Fig. 2. The interleaving depths are set over the mean fading times that are reported on Fig. 3 for both AO systems. In the case of the LP AO system, in order to reach an outage probability of 0.001 the required unfaded SNRs are 17.37 dB when \( L = 80 \), 17.93 dB when \( L = 20 \), 18.23 dB when \( L = 5 \) and, 20.92 dB when no interleaving is used. Thus, the corresponding finite interleaver losses are 0.17 dB, 0.73 dB, 1.02 dB and, 3.72 dB when no interleaving is used. The decrease in finite interleaver losses induced by the interleaving process has to be pondered by the required buffer sizes. Indeed, from Fig. 3, one sees that the average fading times are of roughly 10 ms, 8 ms and 7 ms at the corresponding required unfaded SNRs and interleaving diversities L. Therefore, at 10 Gbps, in order to compensate almost completely the finite interleaver losses, 8160 MB buffers are needed whereas to limit to 1 dB the finite interleaver loss, 335 MB buffers only are required. In the case of the HP AO system a similar analysis leads to buffer sizes of 6900 MB for compensating almost completely the finite interleaver loss and 230 MB for accepting roughly 1 dB of finite interleaver loss. The HP case enables an almost 30% decrease in interleaver size compared to the LP case. The interleaver power and size requirements to sustain an outage probability of 0.001 for both cases of AO corrections are summarized in Table 1. The buffer sizes in MB are given in the case of a 10 Gbps data rate.

**Table 1** Interleaver power and buffer size budget for the HP and LP AO corrections

<table>
<thead>
<tr>
<th>( L )</th>
<th>80</th>
<th>20</th>
<th>5</th>
<th>No interleaving</th>
</tr>
</thead>
<tbody>
<tr>
<td>( &lt;A_{SMF}^{}&gt; )</td>
<td>( 15 )</td>
<td>( 15.45 )</td>
<td>( 15.72 )</td>
<td>( 16.12 ) (1.42)</td>
</tr>
<tr>
<td>( &lt;A_{SMF}^{}&gt; )</td>
<td>( 17.37 )</td>
<td>( 17.93 )</td>
<td>( 18.23 )</td>
<td>( 20.92 ) (3.72)</td>
</tr>
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<td>( 20.92 ) (3.72)</td>
</tr>
<tr>
<td>( &lt;A_{SMF}^{}&gt; )</td>
<td>( 17.37 )</td>
<td>( 17.93 )</td>
<td>( 18.23 )</td>
<td>( 20.92 ) (3.72)</td>
</tr>
<tr>
<td>Mean fading time [ms]</td>
<td>( 8.6 )</td>
<td>( 5.6 )</td>
<td>( 4.7 )</td>
<td>( 2.3 )</td>
</tr>
<tr>
<td>Buffer Size [MB] ([ms])</td>
<td>( 6880 (688) )</td>
<td>( 1120 (112) )</td>
<td>( 230 (23) )</td>
<td>( 335 (33.5) )</td>
</tr>
</tbody>
</table>
A better AO system will obviously lead to a lower average power penalty and therefore a lower capacity loss. However the preceding analysis seems to show that numerical interleavers could drastically relax the specifications of AO systems. In order to clarify this aspect, the outage probabilities after compensating the average power penalties proper to each of the AO system studied are presented on Fig. 4. By doing so, only the distinct impacts of each of the AO corrections regarding the mitigation of the channel fluctuations are highlighted. The losses in terms of average coupled power are ignored or, in other words, it is assumed that we compensate them by increasing the optical power at the transmission. On Fig. 5 the evolution of the corresponding average fading times are reported.
In the case where no interleaving is used, Fig. 4 underlines that even by compensating the average power penalties, the HP system allows for a gain in the required SNR for sustaining a transmission with an outage probability of 0.001 of roughly 2.5 dB. When interleavers with L=5, L=20 or L=80 are used, the required SNRs for the same outage probability for both AO systems are almost identical. Moreover the depth of the interleavers in both cases are very close: at 12 dB the mean fading time for both system is approximately 15 ms; at 13 dB the mean fading times for the HP system is 5 ms and 7.5 ms for the LP system. These results highlight the fact that, in mitigating the temporal fluctuations of the channel, numerical interleavers even with reasonable sizes could be as power efficient than high performance AO correction.

IV. CONCLUSION

Temporal statistics of AO corrected injection losses have been taken into account to assess interleaving influence on outage probability. Two AO correction cases have been considered: a high performance one and a low performance one. Without interleaving significant residual power fluctuations make a 2.5 dB supplementary power necessary to obtain comparable outage probability for the same code rate. Interleaving over only 5 times the mean fading duration permit to achieve comparable performances if the average power penalty is
compensated. However, since the HP case induces lower mean fading times, the required buffer size are 30% smaller than for the LP case. Considering the manageable complexity of the HP correction case (the specifications of the correction are compatible with commercially available components) the work presented here confirms the relevancy of these specifications while taking into account the potential gain brought by the interleaving process. These conclusions merit further investigations, as some of the hypotheses assumed in this model seem notably simplistic (e.g. noise statistics after photo-detection; neglecting of the log-amplitude spatial variations of SAOST times series). They will be consolidated by end-to-end simulations.

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