Comparative accuracy analysis for two types of scanning IR Earth horizon sensors

Oleg Vetrov, Alexander Dmitriev, Michail Pirogov
ABSTRACT

A comparative accuracy analysis is performed for two types of scanning IR Earth horizon sensors developed during a few last years. Resulting measurement error contribution of certain error sources, including instability of hardware parameters, as well as influence of seasons latitude variations of the Earth IR radiance, is estimated based on computer simulation. The conclusion is motivated concerning application of the obtained results for reducing measurement errors of some IR Earth horizon sensors currently exploiting on board of the satellites.

1. INTRODUCTION

The main principles of IR Earth horizon sensors design, including scanning and static devices, are well known since 70’s of last century [1]. In spite of development of the modern static devices [2] during a few resent years the overwhelming majority of the Earth infrared horizon sensors, currently exploiting on board of the satellites, are the devices of scanning type. Moreover, the new implementations of scanning IR Earth horizon sensors are developed during a few last years: with unidirectional conical scanning [3] and with bidirectional (oscillatory) scanning [4]. Therefore comparative analysis of accuracy for such the devices is a topical task. In particular, the analysis results may be applied to processing the data from some IR Earth horizon sensors currently exploiting on board of the satellites.

2. METHODOLOGICAL APPROACH TO SCANNING SENSORS’ ACCURACY ANALYSIS

According to the principle of operation the scanning IR Earth horizon sensors are described by their geometric parameters, which state the position of scanning trajectory (or trajectories). For the most simple the Earth spherical model IR horizon, observed from a satellite orbit, is a circle with an angular radius of

\[ \text{Rea} = \arcsin\left(\frac{(\text{Ha}+\text{Rel})}{(\text{Ho}+\text{Rel})}\right) \]

where

- \( \text{Ha} \) – effective height of atmosphere;
- \( \text{Rel} = 6371 \text{ km} \) – linear radius of the Earth sphere;
- \( \text{Ho} \) – height of orbit.

The sensor measures by any way an angular position for points of the trajectories intersection with a horizon circle determined by the effective atmosphere height. Having three or more points we may calculate, in particular, a position of the mentioned circle centre, i.e. determine a direction of the geocentric vertical. Corresponding formulae for the sensor with oscillatory scanning presented in [4], and for circular conical scanning presented, for example, in [3]. These formulae include, in particular, a half angle of the scanning cone or an equivalent parameter – an angular radius of the scanning trajectory. If the mentioned parameters differ from the stated values, the formulae allow to estimate corresponding contribution of these differences into the total error of the measured pitch and yaw.

There are also the additive components in the formulae [3] intended “to take into account the electronics transfer function, the earth radiance profile and the earth oblateness by manufacturer as calibration data”. However, the problem of the Earth oblateness belongs to exclusive geometrical part of an analysis and has no need in new approaches. Contrary, the problems of taking into account the electronics specific transfer functions and effective radiance profiles of the Earth atmosphere need in special approaches.

We propose to solve them by means of mathematical simulation with corresponding software implementation. Development and application of such models for specific the Earth horizon sensors allow analyse in details measurement accuracy and obtain methodical facilities for the accuracy improvement.

3. MATHEMATICAL SIMULATION OF THE EARTH EFFECTIVE RADIANCE AND ELECTRONICS TRANSFER FUNCTION

Information of the sensor’s instantaneous field of view (FOV) position relative to the Earth horizon contains in a value of the Earth radiance energy passed into the mentioned FOV. The optical system, radiance detector and electronic channel transform an input radiance energy into the output electric signal being used for measurements. For an arbitrary signal form on the transfer channel input the form of the output signal is determined by means of the Duhamel integral.
\[ Y(t) = \int_{0}^{t} X'(\tau) \cdot h(t - \tau) \cdot d\tau, \text{ where} \] (2)

\[ X' \] – derivative of an input signal;
\[ h \] – transfer function as a response on unit function;
\[ \tau \] – moments of the unit function steps.

Thus, an input signal of an arbitrary form is substituted by the sum of delayed in time unitary influences in a form of a signal stepwise jumps. The output signal is the integral sum of elementary responses of the transfer channel to the unitary input influences.

The dynamic features of a radiance detector and the circuits of the electronic channel are simulated by corresponding set of differentiating and integrating circuits. In the simulation software implementation we propose chose a proper small time increment and, without sacrifice accuracy, substitute derivatives by the linear expressions.

Output signal of a differentiating circuit on the given calculation step is:

\[ \text{SDO}_i = \text{SDO}_{i-1} \cdot (\frac{\text{SD}_i - \text{SD}_{i-1}}{\text{JI}}) \cdot \frac{\tau_{\text{diff}} \cdot \text{dt}}{\text{JI}} + \frac{\text{SD}_i - \text{SD}_{i-1}}{\text{JI}} \cdot \text{dt}, \text{ where} \] (3)

\[ \text{SDO}_{i-1} \] – value of the output signal on the previous calculation step;
\[ \text{SD}_i \] – value of the input signal on the given calculation step;
\[ \text{SD}_{i-1} \] – value of the input signal on the previous calculation step;
\[ \tau_{\text{diff}} \] – differentiating time constant;
\[ \text{dt} \] – time step of calculations.

Output signal of an integrating circuit on the given calculation step is:

\[ \text{SIO}_i = \text{SIO}_{i-1} + (\frac{\text{SI}_i - \text{SI}_{i-1}}{\text{JI}}) \cdot \text{dt} \cdot \tau_{\text{int}}, \text{ where} \] (4)

\[ \text{SIO}_{i-1} \] – value of the output signal on the previous calculation step;
\[ \text{SI}_i \] – value of the input signal on the given calculation step;
\[ \text{SI}_{i-1} \] – value of the input signal on the previous calculation step;
\[ \tau_{\text{int}} \] – integrating time constant;
\[ \text{dt} \] – time step of calculations.

More specific is determination of a form for the sensor channel input signal, i.e. time form of an effective radiance, perceived by the sensor detector. For example, is known [5] usage of calculated data of the Earth atmosphere IR radiance profile (in the 15 mkm CO₂ absorption band), expressed as a function of tangent height relative to the earth surface. The profiles are calculated for the stated latitudes and season. An normalised profile, averaged according to the calculated data for six latitudes of the North hemisphere in July, is shown, as an example, in Fig. 1 (Radiance Profile).

The shown profile form corresponds to the FOV, which tends to zero. However, the value of radiance, perceived in the real sensor FOV, depends, in particular, on the relationship of the FOV angular dimensions and angular dimensions of the Earth figure part being observed.

To determine a form of input signal, generated by the stated arbitrary profile of the atmosphere radiance, we propose an approach, which generalised application of the Duhamel integral.

Let the real radiance profile be substituted by a sum of unitary concentric circular layers, which have radii corresponding to the crossing points of the layers with the radiance profile (ref. Fig. 1). The signals from every unitary layer are calculated by unified method for the current position of the sensor FOV relative to the Earth horizon. Maximal signal from one unitary layer corresponds to coverage entire FOV by the layer. Depending on the profile form the signals from some unitary layers may be negative. Total signal, corresponding to the given mutual position of the sensor FOV and the Earth horizon profile, is determined as an algebraic sum of calculated signals from the unitary layers. Normalisation is performed by means of division of the total signal by the sum of maximum signals.

Let us apply the proposed approach to the Earth horizon sensor [4], which has two circular FOV. A signal from the radiance profile unitary layer is proportional to the intersection area for two circles on the celestial sphere: the FOV circle and the unitary layer circle.

If an angular distance \( d \) between the centres of the mentioned circles is greater than the sum of their angular radii \( R_e \) and \( R_f \), then the intersection area is equal to zero.
Sc = 0 when \( d > Re + Rf \).  

If an angular distance between the centres of the mentioned circles is less than the difference of their angular radii, then the intersection area is equal to the area of the FOV smaller circle with the radius \( Rf \).

\[
Sc = S0 = 2\pi(1 - \cos(Rf)) \quad \text{when} \quad d < Re - Rf. \tag{6}
\]

If an angular distance \( d \) between the centres of the mentioned circles is greater than the leg of the orthogonal spherical triangle with the hypotenuse, which is equal to the angular radius \( Re \) of the greater circle, and the second leg is equal to the angular radius \( Rf \) of the smaller circle (ref. Fig. 2), then

\[
Sc = S1 + S0 - S2 - 2S3, \quad \text{where} \quad S1 = 2\pi(1 - \cos(Re)) - \text{area of the intersection sector, which belongs to the greater circle; } \]
\[
S2 = 2\pi(1 - \cos(Rf)) - \text{area of the intersection sector, which belongs to the smaller circle; } \]
\[
S3 = \alpha + \beta + \gamma - \pi - \text{area of the spherical triangle with the sides of } Re, \ Rf \ \text{and } d; \ \alpha - \text{angle between the sides } Re \ \text{and } d \ \text{of the mentioned triangular sphere; } \]
\[
\beta - \text{angle between the sides } Rf \ \text{and } d \ \text{of the mentioned spherical triangle; } \gamma - \text{angle between the sides } Re \ \text{and } Rf \ \text{of the mentioned spherical triangle.} \tag{7}
\]

Else (ref. Fig. 3)

\[
Sc = (Re - Rf) < d < \arccos(\cos(Re)/\cos(Rf)) \]

when \( (Re-Rf) < d < \arccos(\cos(Re)/\cos(Rf)) \)

\[
Sc = S1 + S0 - S2 - 2S3 \tag{8}
\]

Equations 5 – 8 give a solution for the most simple case of more complicated problem concerning areas of the spherical polygons [6]. Signal from the stated radiance profile of the Earth horizon, normalised in the range from zero to unit, is equal to

\[
St = \frac{1}{n^*} \sum_{i=1}^{n} \frac{Sc_i}{S0}, \quad \text{where} \tag{9}
\]

\( n^* \) – number of unitary layers, which generate positive signals \( Sc_i \),
\( n \) – total number of unitary layers, which generate signals \( Sc_i \).

### 4. USING SIMULATION FOR COMPARISON OF TWO SCANNING METHODS

For clarity of two methods comparison let us simulate operation of the Earth horizon sensors [3] and [4] in the same conditions and with equivalent characteristics stating the following nominal parameters:

- Height of the circular orbit \( Ho = 500 \text{ km} \);
- Scanning frequency \( Fc = 4 \text{ Hz} \);
- Encoder quantification \( Da = 2 \text{ arc.min} \);
- Angular radius of scanning \( Rs = 45 \text{ degr.} \);
- Angular radius of the FOV \( Rf=1,5 \text{ degr.} \);
- Detector diff. time constant \( \tau_{ddif} = 10 \text{ ms} \);
- Detector int. time constant \( \tau_{dint} = 0.1 \text{ ms} \);
- Electronics diff. time constant \( \tau_{edif} = 10 \text{ ms} \);
- Electronics int. time constant \( \tau_{eint} = 0.5 \text{ ms} \);
- Relative threshold level \( St/Smax = 0.5 \).

The last parameter corresponds to the “50% radiance normalised locator” algorithm from [5].

For the first stage of simulation the following conditions were stated additionally:

- zero displacement from the Earth centre in pitch and yaw;
- uniform radiance of the Earth disc without taking into account the Earth atmosphere;
- sinusoidal oscillation of the scanning axis for sensor [4] with the amplitude of 80 degrees.

The results of simulation for sensor [4] are illustrated in Fig. 4, where the input radiance and the output signal, which correspond to Space-to-Earth transition, presented as functions of the axis rotation angle of the scanning mechanism.
The angular position of true horizon and angular position measurement result, corresponding to a half of difference between maximal and minimal levels of the electronic channel output signals, are also shown in the Fig. 4. Difference between true and measured positions of the Earth horizon is determined by the input radiance rate of increasing and the electronic channel dynamic characteristics. For the nominal conditions time lag may be taken into account as a calibration parameter. However, this parameter is not a constant but depends on actual measurement conditions and instability of the nominal parameters. Besides an angular position of the scanning mechanism axis and the parameters, mentioned as simulation data in the beginning of the given section, the actual measurement conditions are characterised by current season and latitude of the horizon area being crossed by the scanning trajectory.

Fig. 5 illustrates the results of Space-to-Earth transition simulation for the sensor [4] in the form of two functions:
- Input radiance (Earth) from the Earth disk without taking into account the atmosphere;
- Input radiance (Atmosphere) from atmospheric profile for July according to [5] (ref., also, Fig. 1).

Additionally, the initial radiance profile (Profile) calculated using 1 as function of the rotation angle of the scanning mechanism axis is shown, also, on Fig. 5. According to Fig. 5 data, in the stated measurement condition presence of the atmosphere essentially shifts the observed position of the Earth horizon. However, the radiance characteristic slope and general form of the signal front differ significantly from initial atmospheric profile. This difference increases just more after signal transformation in the device electronic channel, but slightly depends on the scanning method itself.

Fig. 6 illustrates the results of Space-to-Earth and Earth-to-Space transitions simulation for the sensor [3].

In Fig. 6 zero rotation angle of the scanning mechanism axis corresponds to coincidence of the sensor FOV centre with true position of the Earth horizon. The Earth-to-Space transition is shown along with a conventional substitution of the scanning polarity for clear comparison of the signals.

5. COMPARATIVE ANALYSIS OF SIMULATION RESULTS

Initial data to determine an angular position of the Earth centre is the measurement results for the angles corresponding to the Earth disc edges. For the sensor [3] the scanning trajectory intersects one edge of the Earth disc at a transition Earth-to-Space, and intersects the
other edge at a transition Space-to-Earth. For the sensor [4] both the mentioned transitions occur at both the Earth disc edges. With the stated for simulation parameters of the sensors and the measurement conditions functions of output signals versus rotation angle of the scanning axis are, in practise the same for the same transitions. In the Table 1 the simulation results are presented for measurement of the Earth disc edges angular position at transitions Earth-to-Space and Space-to-Earth. The values of the axis rotation angle are shown in degrees for the following cases:
- intersection the scanning trajectory with the Earth disc geometric edge (true horizon);
- relative level 0.5 of maximum signal (ref. algorithm from [5]) at the nominal conditions;
- the same at deviations of the nominal parameters.
The chosen values of the nominal parameters relative deviations (+1% and −1%) allow estimate linearity of the errors’ functions and perform interpolation or extrapolation in some vicinity of the nominal values’ deviation. In the Tables 1 and 2 the results are shown for variations of the following parameters:
- the scanning trajectory angular radius (1);
- the FOV angular radius (2);
- the channel differentiating time constant (3);
- the channel integrating time constant (4);
- the radiance profile height (5);
- the radiance profile curvature (6).

The found polarity of components 1, 4, 5 seems to be quite obvious. However, polarity of components 2, 3, 6 as well as all the components values are not trivial.

As it’s shown for example in [3], an angular position of the Earth centre in the specified device frame is calculated according to the measured values of “chords” and also according to the angular positions of the chords’ middles. The chords correspond to arcs of intersection of the scanning trajectories with the Earth horizon circle.

Let us name the errors of determining the angular position of the chords’ middles – the first kind errors. They directly characterise the sensor methodical bias along the angular co-ordinate corresponding to rotation around the axis of scanning mechanism. The errors of determining the angular values of chords we name the second kind errors. They are proportional to the sensor methodical bias along the other angular co-ordinate, which is orthogonal to the scanning axis.

There are corresponding calculation formulae in [3], in accordance with them the biases of the chords middles positions determination are equal to a half of algebraic sum for biases of the Earth disc edges determination at transitions Earth-to-Space and Space-to-Earth.

In Table 3 the errors of the first kind are presented for deviations of the sensor [3] parameters from the nominal values. The bias components are shown in absolute values (arc.min) taking into account their sign.

<table>
<thead>
<tr>
<th>Parameter \ Transition</th>
<th>Earth-to-Space</th>
<th>Space-to-Earth</th>
</tr>
</thead>
<tbody>
<tr>
<td>True horizon</td>
<td>58.29810</td>
<td>301.7019</td>
</tr>
<tr>
<td>Nominal conditions</td>
<td>59.78196</td>
<td>300.9667</td>
</tr>
<tr>
<td>Scanning radius +1%</td>
<td>60.04668</td>
<td>300.7064</td>
</tr>
<tr>
<td>Scanning radius -1%</td>
<td>59.77809</td>
<td>300.9650</td>
</tr>
<tr>
<td>Field of view radius +1%</td>
<td>59.78357</td>
<td>300.9705</td>
</tr>
<tr>
<td>Field of view radius -1%</td>
<td>59.78357</td>
<td>300.9683</td>
</tr>
<tr>
<td>Channel differentiating time constant +1%</td>
<td>59.78357</td>
<td>300.9705</td>
</tr>
<tr>
<td>Channel differentiating time constant -1%</td>
<td>59.78031</td>
<td>300.9662</td>
</tr>
<tr>
<td>Channel integrating time constant +1%</td>
<td>59.78680</td>
<td>300.9715</td>
</tr>
<tr>
<td>Channel integrating time constant -1%</td>
<td>59.77711</td>
<td>300.9646</td>
</tr>
<tr>
<td>Radiance profile height +1%</td>
<td>59.79600</td>
<td>300.9598</td>
</tr>
<tr>
<td>Radiance profile height -1%</td>
<td>59.76798</td>
<td>300.9809</td>
</tr>
<tr>
<td>Radiance profile curvature +1%</td>
<td>59.78461</td>
<td>300.9657</td>
</tr>
<tr>
<td>Radiance profile curvature -1%</td>
<td>59.77932</td>
<td>300.9693</td>
</tr>
</tbody>
</table>

The radiance profile curvature deviation is conventionally simulated by the following. Two heights for the stated boundary relative levels of radiance (for example, 0.1 and 0.9) are fixed. A height of any arbitrary relative level of radiance, within the limits of the mentioned boundary heights, is changed by the specified percent from difference between an initial height of the given level and the height of the nearest boundary level.

According to the Tables 1 data the biases (in arc.min) are presented in the Table 2 for varying the stated parameters, the biases being measured between the given and nominal positions of the Earth disc edges.

<table>
<thead>
<tr>
<th></th>
<th>Earth-to-Space</th>
<th>Space-to-Earth</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1%</td>
<td>-16.52730</td>
<td>15.80653</td>
</tr>
<tr>
<td>+1%</td>
<td>0.22671</td>
<td>0.229650</td>
</tr>
<tr>
<td>-1%</td>
<td>-0.09950</td>
<td>-0.031140</td>
</tr>
<tr>
<td>+1%</td>
<td>-0.29152</td>
<td>0.290407</td>
</tr>
<tr>
<td>-1%</td>
<td>-0.83917</td>
<td>0.854769</td>
</tr>
<tr>
<td>+1%</td>
<td>-0.15882</td>
<td>0.157421</td>
</tr>
</tbody>
</table>

The radiance profile height deviation is simulated by the specified percent deviation of each absolute height, corresponding to any arbitrary relative level of radiance.

The values of the axis rotation angle are shown in degrees for the following cases:
- intersection the scanning trajectory with the Earth disc geometric edge (true horizon);
- relative level 0.5 of maximum signal (ref. algorithm from [5]) at the nominal conditions;
- the same at deviations of the nominal parameters.

The chosen values of the nominal parameters relative deviations (+1% and −1%) allow estimate linearity of the errors’ functions and perform interpolation or extrapolation in some vicinity of the nominal values’ deviation. In the Tables 1 and 2 the results are shown for variations of the following parameters:
- the scanning trajectory angular radius (1);
- the FOV angular radius (2);
- the channel differentiating time constant (3);
- the channel integrating time constant (4);
- the radiance profile height (5);
- the radiance profile curvature (6).
Calculation of the chords middles positions for the sensor [4] is performed using the difference between two measured positions of two the Earth disc edges at two transitions Space-to-Earth. Therefore at the stated conditions of simulation for the sensor [4] type scanning method errors of the first kind are equal to zero.

Calculation of the chords values for the sensor [3] is performed using the difference of the measured positions of the Earth disc edges. Correspondingly, bias of chords values determination is equal to difference of errors for different transitions Space-to-Earth and Earth-to-Space.

Calculation of the chords values for the sensor [4] is performed using the sum of the Earth disc edges positions measured at the transitions Space-to-Earth. Correspondingly, bias of chords values determination is twice as big as the error at Space-to-Earth transition.

In Table 4 the second kind errors are presented for both types of scanning.

In comparison of the absolute values of separate error components it should be taken into account that actually relative instabilities of the examined parameters aren’t be equal to the values stated for simulation and aren’t be equal to each other. However, data of Tables 3 and 4 may be used to estimate the required accuracy of the initial data for calculating corrections on the corresponding parameters variations.

In the Table 5, according to the Tables 3 and 4 data, the relationships are shown between the relative values of the first kind errors for sensor [3] (A), as well as the first kind errors and the second kind errors for sensor [4] (B and C), in the following form. The maximum error value for the examined case (A, B or C) is taken as 100%. The values of other errors for the examined case expressed in percents of maximum error.

According to the Tables 3 – 5 data ratio of the errors, due to variation of each examined parameter by ±1%, is:
- approximately 70 times for A case (ref. line 6 Radiance profile curvature);
- more than 250 times for B case (ref. line 4 Channel integrating time constant);
- less than 3 times for C case (ref. line 6).

Ratio of maximum errors, due to alternate variation of all the examined parameters, is
- approximately 7 times for A case (ref. line 6, +1%);
- more than 400 times for B case (ref. line 3 Channel differentiating time constant, -1%);
- less than 200 times for C case (ref. line 3, +1%).

Difference of the obtained results is originated in principal difference of the scanning methods [3] and [4].

Circular conical scanning in one direction produces transition Earth-to-Space at one edge of the Earth disc and transition Space-to-Earth at the other edge of the Earth disc. At the first edge a delay in sensing the Earth horizon due to the atmospheric radiance is added to a delay produced in the electronic channel.
Contrary, at the second edge overtaking due to the atmospheric radiance partly compensates the electronics delay. For example, according to the data illustrated in Fig. 6 the angular position of the half relative level is minus 1.01 degr. for transition Space-to-Earth and is plus 1.61 degr. for transition Earth-to-Space relative to the position corresponding to true horizon.

For oscillatory reciprocating scanning both edges of the Earth disc may be sensed using the same transitions, for example, Space-to-Earth. Additionally, an opportunity is reserved to use, if needed, other combinations of the transitions.

As whole the simulation results prove that oscillatory reciprocating scanning, implemented in the sensor [4], provide achievement of higher accuracy comparing with unidirectional circular conical scanning. Taking into account that time intervals between the moments of the Earth disc edges intersection have an order of tenths second, we assume that the main source of difference between the horizon transitions for the sensor [4] is seasonal & latitude features of the Earth atmospheric radiance at the Earth disc edges. However, it should be noted the following. For both the sensor types possible instability of the scanning angular radius depends on the structure and design of optical system and scanning mechanism. In particular, the instability may be caused by the thermal deviations for the refraction index of germanium optics commonly used in the IR devices. The FOV angular radius also may vary due to thermal instability, for example, of the sensor lens focal length. The major source of instability for the time constants of differentiating and integrating circuits is also changes of electronic components’ temperature.

At the same time the main external source of thermal changes during in-flight exploitation of the Earth horizon IR sensors on satellites is seasonal variations of solar illumination. These variations may cause seasonal variations of the sensor temperature, for example, on satellites, which have orbital orientation on the Sun synchronous orbit.

The simulation results demonstrate that the measurement bias components have very diversified values, polarity due to the sources, which generate errors. At the first turn it concerns the sensors with the unidirectional scanning. Therefore development of the methods improving the Earth horizon IR sensors accuracy requires comprehensive analysis and separation specific main sources of bias, including special in-flight operations.

Application of the described simulation facilities and their expansion for processing the results of the mentioned in-flight operations is a subject of a separate discussion.

6. CONCLUSION

The presented methodology of mathematical simulation and analysing bias errors for the Earth horizon scanning sensors is successfully used in the development practice for the devices with circular FOV. A complete set of the simulation software is implemented at the enterprise NPP “Geofizika-Cosmos” being used for investigating the sensors operation in wide range of flight altitudes, including geo-stationary orbit. Other FOV configurations may be analysed, for example, square form, elliptical etc. Large volume of data concerning the Earth atmosphere radiance for different latitudes and seasons, as well as facilities for experimental confirmation of the mathematical models using simulators of the Earth IR radiance, provide implementation of precision devices and developing motivated methods of in-flight accuracy improvement for specific sensors.

7. REFERENCES