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Numerical Investigation of the Effect of Number of Zones On Fresnel Zone Plates Imaging Performance Under Different Coherence Conditions

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ABSTRACT

In recent years, integral imaging has received increasing attention as a promising 3D imaging technique with one single exposure. In medical diagnostics, the X-ray integral imaging system potentially has a much shorter exposure time than the conventional computed tomography, reducing the radiation damage to the patient. By replacing the micro-lens array with a micro-Fresnel Zone Plate (FZP) array, the classical visible integral imaging system can be transposed to an X-ray system. However, limited by the micro-scale dimensions of FZP in the array and current manufacturing techniques, the number of zones of FZP is required to be small. This may have an important impact on the FZP imaging performance. Based on the scalar diffraction theory, this paper introduces a simulation method and numerically investigated the effect of the number of zones on the FZP imaging performance under different coherence conditions.

Keywords: Fresnel zone plate; modeling; wave propagator; coherence; X-ray integral imaging

1. INTRODUCTION

Nowadays, computed tomography (CT) is one of the most common 3D imaging techniques in clinical diagnosis, providing highly accurate information of internal structure and guiding medical treatment. The principle of CT is rendering 3D image of the sample from multiple projections recorded under different angles of view. It is apparent that large projection number will bring a better reconstruction quality.¹ Unfortunately, it will meanwhile deliver a high radiation dose for the patients and increase the probability of cancer.^{2,3} X-ray integral imaging can provide an alternative to CT in medical imaging as it has the prospect of reconstructing 3D images from one single capture, consequently leading to a lower dose and a shorter exposure time.

Integral imaging approach, first proposed by Lippmann in 1908,⁴ consists in capturing simultaneously the spatial and angular information of the light ray intersecting on the photosensor, the so-called 4D light field. The captured 4D light field allows us to refocus the image on different planes and render a 3D model of the sample with one exposure.^{5,6} For visible light, various integral imaging acquisition systems have been proposed, such as camera arrays,⁷ micro-lens arrays,⁸ and amplitude masks.⁹ The design of placing a micro-lens array in the front of conventional camera's photosensor is the most widespread, in view of the diversity and availability of micro-lens array. However, to convert the integral imaging system from visible light to X-ray, we must search alternative optical components for the refractive lenses. Because the refractive index is very close to unity for all materials at X-ray wavelengths, leading to a extremely long focal distances at kilometer scale for a single lens and a low numerical aperture.¹⁰

The Fresnel Zone Plate (FZP), relying on the diffraction to realize X-ray focusing, is widely used and has been demonstrated to achieve a high spatial resolution.¹¹ The FZP array is thus an available alternative to the refractive lens array in X-ray imaging.¹²⁻¹⁴ FZP geometry dominates its imaging performance. For a desired

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focus, the larger the number of zones is, the higher resolution FZP can attain. Here, the resolution refers to the width of the focused point. Therefore, its number of zones is one of the most important structural parameters. Yet, due to the micro-scale dimensions of FZP in the array of an integral imaging system and current manufacturing techniques, the number of zones of FZP is greatly limited.

Therefore, this paper attempts to numerically investigate the impact of FZP number of zones on its imaging performance under different coherence conditions, especially the case of very small number of zones. Accordingly, a simulation method based the scalar diffraction theory is introduced in this paper. Several simulation results of FZP imaging are presented and discussed, then compared to ideal refractive lens.

2. METHOD

A Fresnel zone plate is a diffractive optic, its alternating transparent and opaque zones make the incident light constructively interfere at the desired focus. The FZP's focusing properties can be analyzed by the scalar diffraction theory. Figure 1 represents a generalized schematic of a FZP optical system.

Figure 1. Generalized schematic of Fresnel Zone Plate (FZP) optical system: the point source P on the plane (ξ, η) emitting a spherical wave is intercepted and diffracted by the FZP on the plane (x, y) . The image of the source is formed on the plane (u, v) . z_1 and z_2 represent respectively the source-FZP distance and the FZP-image distance.

Consider a point source $P(\xi, \eta)$ emitting spherical wave diffracted by a FZP on the plan (x, y) and its image formed on the plane (u, v) . The distances between the point source to the FZP and the FZP to the image are respectively noted as z_1 and z_2 . According to Huygens-Fresnel principle and the linearity of the optical system,¹⁵ the impulse response $h(\xi, \eta, u, v)$ of the FZP can be expressed in the form of a Fourier Transform

$$h(0, 0, u, v) = \frac{z_2}{z_1} \exp\left[\frac{ik}{2z_2}(u^2 + v^2)\right] \times \mathcal{F}\left\{t(\lambda z_2 x', \lambda z_2 y') \exp\left\{\frac{ik}{2}\left(\frac{1}{z_1} + \frac{1}{z_2}\right)[(\lambda z_2 x')^2 + (\lambda z_2 y')^2]\right\}\right\} \quad (1)$$

where $t(x, y)$ is the transmission function of the FZP, with $x' = \frac{x}{\lambda z_2}$, $y' = \frac{y}{\lambda z_2}$.

The image $Img(u, v)$ can be obtained by the convolution of the image predicted by geometrical optics $img(u, v)$ with the impulse response $h(\xi, \eta, u, v)$.¹⁶ It is worth noting that the application of the point spread function (PSF) on an image depends on the degree of coherence of the source. Here, we consider two extreme cases, fully coherent and fully incoherent. The image $Img(u, v)$ can be written as

$$Img(u, v) = \begin{cases} |h(\xi, \eta, u, v) \otimes img(u, v)|^2 & \text{fully coherent} \\ |h(\xi, \eta, u, v)|^2 \otimes |img(u, v)|^2 & \text{fully incoherent} \end{cases} \quad (2)$$

According to the convolution theorem, Equation (2) can be further converted to

$$\text{Img}(u, v) = \begin{cases} \mathcal{F}^{-1} \left\{ |\mathcal{F}\{h(\xi, \eta, u, v)\}| \times \mathcal{F}\{|\text{img}(u, v)|^2\} \right\} & \text{fully coherent} \\ \mathcal{F}^{-1} \left\{ \mathcal{F}\{|h(\xi, \eta, u, v)|^2\} \times \mathcal{F}\{|\text{img}(u, v)|^2\} \right\} & \text{fully incoherent} \end{cases} \quad (3)$$

This conversion reduces the complexity of the computation and allows greatly increasing the computation speed thanks to the use of FFT.

This method is flexible for the definition of FZP and the adjustment of imaging system structure, allowing to study the effects of different parameters on the FZP imaging performance. More detailed derivation and validation tests about this method are provided in reference.¹⁷

3. RESULTS

To our best knowledge, the quality of the image formed by a FZP versus the number of zones has never been studied. In this section, by the proposed method, the images of the FZP with different N were performed and compared to the images formed by the refractive lens, respectively, under fully incoherent and fully coherent sources. The effect of the number of zones on FZP imaging performance is then discussed.

3.1 Fully incoherent case

A series of numerical images of a resized 1951 USAF resolution test chart formed by FZP with increasing N are displayed in Figures 2a–e and the part of interest on the test chart is zoomed in Figure 2f. In the following text, the blue-, orange-, and green-colored bars in Figure 2f are named as elements 2.2, 2.3, and 2.4, respectively, with the width of $1.36 \mu\text{m}$, $1.18 \mu\text{m}$, and $0.99 \mu\text{m}$ on the image plane. Figures 2a–e are carried out with a fully incoherent source of 11 keV . The object to FZP distance is $3f$, leading to the creation of an image at $1.5f$. It is apparent from Figures 2a–e that as N increases, the first order image becomes sharper. As the radius of FZP is meanwhile broadened along N , the field of view is enlarged.

Figure 2. (a–e) False color numerical images of the USAF 1951 test target imaged by FZP with different number of zones under fully incoherent illumination. The object–FZP distance is $3f$ and the FZP–image distance is $1.5f$. Each image dimension is $200 \times 200 \mu\text{m}^2$. (f) Zoomed part of interest on the test chart: for the following part, the blue-, orange- and green-colored elements of bars in group 2 are respectively noted as elements 2.2, 2.3, and 2.4.

Measured the contrast of Figures 2 a–e, we remark the maximum contrast value is as low as 0.2 (see Figure 7 in References [17]). In order to study its origin, we modeled the image formation with a theoretical perfect refractive lens. Supposed the FZPs in Figure 2 can be replaced by the refractive lenses with the same numerical aperture (NA), the transmission of refractive lens $t_{Lens}(x, y)$ is defined as

$$t_{lens}(x, y) = \begin{cases} \exp[-\frac{ik}{2f}(x^2 + y^2)], & \sqrt{x^2 + y^2} < R \\ 0, & else \end{cases} \quad (4)$$

where R and f are the radius and focal length of lens, respectively.

A group of images formed by the refractive lens is simulated under the same structure of Figure 2, displayed in Figure 3. For the ease of comparison with Figure 2, the NA of refractive lens is written as a function of the number of zones. Compared with Figure 2, the intensity of the refractive lens images is obviously more uniform, and there is no longer a halo around the image. The contrast value measured from refractive lens images can reach a maximum of 0.94, much higher than that of FZP images (see Figure 9 in References [17]). This is because the first order image of FZP overlaps with the other orders, resulting in a brightened and complex background.

Figure 3. False color numerical images of the USAF 1951 test target imaged by refractive lens under fully incoherent illumination. The NA of the refractive lens equals to the NA of the FZP when the number of zones of FZP is 5, 20, 40, 60, and 100. Each image dimension is $200 \times 200 \mu\text{m}^2$.

3.2 Fully coherent case

Keeping the same configuration as mentioned in the fully incoherent case, we discuss FZP imaging performance under fully coherent illumination in this section. The simulated images of FZP versus N are shown in Figure 4. As N increases, the first order image of FZP becomes more resolved, while the images of the other orders are much more apparent in the fully coherent case than in the fully incoherent images. To visualize more directly the FZP images difference of images under different coherence conditions, Figure 5 displays side by side the fully incoherent and fully coherent FZP images for $N = 100$. Unlike the other orders' images with recognizable details in the fully coherent case, we can only see a diffuse halo overlapping the first order image in the fully incoherent case. Indeed, the overlap of different order images substantially degrades the first order image quality compared to the fully incoherent case.

Figure 4. False color numerical images of the USAF 1951 test target imaged by FZP with different number of zones under fully coherent illumination. The object—FZP distance is $3f$ and the FZP—image distance is $1.5f$, as same as for Figure 2. Each image dimension is $160 \times 160 \mu\text{m}^2$.

Figure 5. Comparison between fully incoherent and fully coherent images formed by FZP. The other orders images become more visible in the fully coherent case than the fully incoherent case, greatly degrading the first order image quality. N is equal to 100. Each image dimension is $160 \times 160 \mu\text{m}^2$.

To bring out the characteristics of a FZP image, we can compare the Figure 4 to Figure 6. The set of images in Figure 6 are simulated with the refractive lenses of the same NA as the FZPs in Figure 4. Without the influence of different orders images, the refractive lens images are clearer and more visible than FZP ones. On the other hand, we can see the diffraction patterns of each element of the test chart appear under fully coherent illumination in Figure 6, which shows the impact of coherence in imaging. On closer observation, we can also find the diffraction pattern changes with different N , which is related to the divergence of the source in our model. The images can be regarded as a result of collective interference between each point of the test chart. The variation of the diffraction pattern then leads to the oscillation of the image intensity along N . In the fully coherent case, strong interference results in more complex images. It is complicated to quantify the evolution of the image by the contrast as used in the fully incoherent case.

Figure 6. False color numerical images of the USAF 1951 test target imaged by refractive lens with different number of zones under fully coherent illumination. The NA of the refractive lens equals to the NA of the FZP in Figure 4 when the number of zones of FZP is 5, 20, 40, 60, and 100. Each image dimension is $160 \times 160 \mu\text{m}^2$.

4. CONCLUSIONS

In this paper, a simulation method based on scalar diffraction has been introduced. The presented method is able to simulate the images formed by FZP with different parameters and under different coherence conditions. The effect of the number of zones on image formation has been discussed under fully incoherent and fully coherent conditions, especially the case of very small number of zones. The results have shown that surprisingly image can be formed with a number of zones as low as 5. However, due to the overlap of different orders of FZP, the images have relatively low contrast, and this influence of the other orders becomes more important in the fully coherent case than in the fully incoherent case. The reported results in this paper bring helpful information for the design of FZP for an integral imaging system in X-ray. Future study can be extended to the axial effect of the number of zones, the formation of images in the case of partially coherent beam and the application on the full set-up of integral imaging system.

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