WINDOWED FRINGE PATTERN ANALYSIS
WINDOWED FRINGE PATTERN ANALYSIS

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To Xiaocong and Zihan
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Preface

Fringe patterns can be formed coherently using various interferometers and incoherently using the moiré technique. They can also be designed in fringe projection profilometry. All of these techniques are useful for full-field, noncontact, and high-sensitivity measurement. The primary goal of fringe pattern analysis is to extract the hidden phase distributions that generally relate to the physical quantities being measured. This book addresses the challenges and solutions involved in this process. Both theoretical analysis and algorithm development are covered to facilitate the work of both researchers and engineers. The information herein may also serve as a specialized subject for students of optical and computer engineering. Readers are encouraged to provide the author with feedback for improvement.

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Qian Kemao
Nanyang Technological University
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Glossary of Terms and Acronyms

\(a, a_x, a_y\) Background intensity of a fringe pattern and its derivatives
A3 Accurate, automatic, and accelerated
ACED Adapted coherence enhancing diffusion
AIA Advanced iterative algorithm
AQF Adapted quadratic filter
arctan Arctangent function
arg max \(p\) that maximizes
arg min \(p\) that minimizes
atan2 Four-quadrant arctangent function
AWFF2 Adaptive windowed Fourier filtering
\(b, b_x, b_y\) Fringe amplitude and its derivatives
bQFGRPT Amplitude included, quadratic-phase-matched and frequency-guided RPT
CCD Charge-coupled device
CED Coherence enhancing diffusion
CO Congruence operation
CO-LSF Congruence operation and least squares fitting
CPF Cubic phase function
CPU Central processing unit
CUDA Compute unified device architecture
\(c_{xx}, c_{xy}, c_{yy}\) Local curvatures
D1 Ill-posedness problem
D2 Sign ambiguity problem
D3 Order ambiguity problem
D4 Noise problem
D5 Discontinuity problem
DIC Digital image correlation
EMD Empirical mode decomposition
EPF Exponential phase field
ESPI | Electric speckle pattern interferometry
---|---
f | Fringe intensity
Ff | Fourier spectrum of f
FFRPT | Fringe-follower RPT
FFSD | Fast FSD
FFT | Fast Fourier transform
FG | Frequency guidance or frequency guided
FGRPT | Frequency-guided regularized phase tracking
FGWFR2 | Frequency-guided WFR2
fn | Fringe pattern with zero background and unit amplitude
Fn | Fourier spectrum of n
\|f\|_p | L⁰ or Lᵖ norm of f
FPDem | Fringe pattern demodulation
FPDen | Fringe pattern denoising
FPP | Fringe projection profilometry
fps | Frames per second
FSD | Frequency-guided sequential demodulation
FSD-LM | FSD with LM optimization
FT | Fourier transform
fv | Fringe pattern with zero background
\(g(x,y)\) | 2D window function
GPGPU | General-purpose graphic processing unit
GPU | Graphic processing unit
GRPT | Generalized RPT
\(g_x(x), g_y(y)\) | Window functions
\(g_{\xi_x}(x), g_{\xi_y}(y)\) | Windowed Fourier kernels
\(g_{\xi_x, \xi_y}(x,y)\) | 2D windowed Fourier kernel
I2L2 | Interlaced indexed linked list
IFSAR | Interferometric synthetic aperture radar
IILL | Interlaced indexed linked list
ILL | Indexed linked list
Im | Imaginary part of a complex number
IMF | Intrinsic mode function
j | Imaginary number (\(\sqrt{-1}\))
k | Integer
LAQF | Local adaptable quadrature filter
LDV | Laser Doppler vibrometry
LFR | Least-frequent rereferencing
LL | Linked list
LM | Levenberg–Marquardt
LS1U | Least squares for 1 unknown
LS2U | Least squares for 2 unknowns
LS3U | Least squares for 3 unknowns
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<th>Term</th>
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<td>LSF</td>
<td>Least squares fitting</td>
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<td>LSxU</td>
<td>Least squares for $x$ unknowns</td>
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<td>MAPE</td>
<td>Maximum absolute phase error</td>
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<td>MFR</td>
<td>Most-frequent rereferencing</td>
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<td>ML</td>
<td>Maximum likelihood</td>
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<td>$n$</td>
<td>Noise</td>
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<td>PZT</td>
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<td>QG</td>
<td>Quality guidance or quality guided</td>
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<td>Quadrature transform</td>
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<td>Re</td>
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<td>(x, y)</td>
<td>Spatial coordinates</td>
</tr>
<tr>
<td>(x&lt;sub&gt;i&lt;/sub&gt;, y&lt;sub&gt;i&lt;/sub&gt;)</td>
<td>Spatial coordinates of pixel i</td>
</tr>
<tr>
<td>Z</td>
<td>Integer numbers</td>
</tr>
<tr>
<td>α, α&lt;sub&gt;x&lt;/sub&gt;, α&lt;sub&gt;y&lt;/sub&gt;</td>
<td>Respective intermediate values for a, a&lt;sub&gt;x&lt;/sub&gt;, a&lt;sub&gt;y&lt;/sub&gt; during optimization</td>
</tr>
<tr>
<td>α&lt;sub&gt;xx&lt;/sub&gt;, α&lt;sub&gt;xy&lt;/sub&gt;, α&lt;sub&gt;yy&lt;/sub&gt;</td>
<td>Respective intermediate values for c&lt;sub&gt;xx&lt;/sub&gt;, c&lt;sub&gt;xy&lt;/sub&gt;, c&lt;sub&gt;yy&lt;/sub&gt; during optimization</td>
</tr>
<tr>
<td>β, β&lt;sub&gt;x&lt;/sub&gt;, β&lt;sub&gt;y&lt;/sub&gt;</td>
<td>Respective intermediate values for b, b&lt;sub&gt;x&lt;/sub&gt;, b&lt;sub&gt;y&lt;/sub&gt; during optimization</td>
</tr>
<tr>
<td>δ(·)</td>
<td>Delta function or error of estimation</td>
</tr>
<tr>
<td>Δφ</td>
<td>Phase change or shift</td>
</tr>
<tr>
<td>θ</td>
<td>Fringe orientation</td>
</tr>
<tr>
<td>δ</td>
<td>Fringe direction</td>
</tr>
<tr>
<td>λ, λ&lt;sub&gt;1&lt;/sub&gt;, λ&lt;sub&gt;2&lt;/sub&gt;</td>
<td>Weights</td>
</tr>
<tr>
<td>μ(·)</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>ξ&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Frequency coordinates corresponding to t</td>
</tr>
<tr>
<td>ξ&lt;sub&gt;ti&lt;/sub&gt;</td>
<td>Frequency sampling interval of ξ&lt;sub&gt;t&lt;/sub&gt;</td>
</tr>
<tr>
<td>[ξ&lt;sub&gt;tl&lt;/sub&gt;, ξ&lt;sub&gt;th&lt;/sub&gt;]</td>
<td>Frequency band of ξ&lt;sub&gt;t&lt;/sub&gt;</td>
</tr>
<tr>
<td>ξ&lt;sub&gt;x&lt;/sub&gt;</td>
<td>Frequency coordinate corresponding to x, also intermediate value for ω&lt;sub&gt;x&lt;/sub&gt; during optimization</td>
</tr>
<tr>
<td>ξ&lt;sub&gt;xi&lt;/sub&gt;</td>
<td>Frequency sampling interval of ξ&lt;sub&gt;x&lt;/sub&gt;</td>
</tr>
<tr>
<td>[ξ&lt;sub&gt;xl&lt;/sub&gt;, ξ&lt;sub&gt;xh&lt;/sub&gt;]</td>
<td>Frequency band of ξ&lt;sub&gt;x&lt;/sub&gt;</td>
</tr>
<tr>
<td>ξ&lt;sub&gt;y&lt;/sub&gt;</td>
<td>Frequency coordinate corresponding to y, also intermediate value for ω&lt;sub&gt;y&lt;/sub&gt; during optimization</td>
</tr>
<tr>
<td>ξ&lt;sub&gt;yi&lt;/sub&gt;</td>
<td>Frequency sampling interval of ξ&lt;sub&gt;y&lt;/sub&gt;</td>
</tr>
</tbody>
</table>
Glossary of Terms and Acronyms

\( \xi_{y_i} \)  
Frequency sampling interval of \( \xi_{y_i} \)

\([\xi_{y_i}, \xi_{y_i+1}]\)  
Frequency band of \( \xi_{y_i} \)

\( \sigma_{(\cdot)} \)  
Standard deviation

\( \sigma_x, \sigma_y \)  
Window size

\( \tau \)  
Temporal coordinate

\( \varphi \)  
Phase distribution

\( \varphi_w \)  
Wrapped phase

\( \psi \)  
Intermediate value for \( \varphi \) during optimization

\((\omega_{cx}, \omega_{cy})\)  
Carrier frequency

\((\omega_x, \omega_y)\)  
Local frequency

\( \angle (\cdot) \)  
Angle of a complex number

\( | \cdot | \)  
Amplitude of a complex number

\( (\cdot)^\dagger \)  
Conjugate of a complex number

\( (\cdot)^T \)  
Transpose of a matrix or vector

\( (\cdot)^a \)  
With sign ambiguity

\( (\cdot)^\wedge \)  
An estimated value

\( (\cdot)^\sim \)  
A filtered value

\( (\cdot)^\sim\)  
An intermediate value during optimization

\( \infty \)  
Infinity

\( \otimes \)  
Convolution

\( \nabla \)  
Gradient operator

\( \nabla \cdot \)  
Divergence operator

\( \triangleq \)  
“Define as”

\( \subset \)  
A subset of

\( \in \)  
An element of