ROBUST SPECKLE METROLOGY
Techniques for Stress Analysis and NDT

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Preface

The invention of the laser in the early 1960s allowed for light sources with a high coherence degree, which generated many novel research lines in order to make use of them. People working with these light sources noticed that a high-contrast and fine-scale granular pattern was produced when a rough surface was illuminated with laser light. This effect was called a “speckle effect,” characterized by a random distribution of scattered light. After recognizing that each speckle has a definite phase, several techniques were developed to measure deformations, displacements, stresses, vibrations, and inner defects.

Several multiauthor books have been published beyond the first one published in 1978 (Speckle Metrology, edited by R. K. Erf)—including Digital Speckle Pattern Interferometry and Related Techniques, edited by P. K. Rastogi, and Advances in Speckle Metrology and Related Techniques, edited by G. H. Kaufmann—show new branches in speckle metrology, new proposed schemes and improvements in processing techniques, and optical approaches that have occurred over the last 20 years.

The main goal of nondestructive testing (NDT) is to detect and characterize anomalies that can adversely affect the performance of the component under test without impairing its intended service.

Optical techniques can be considered as alternative approaches to traditional NDT methods. They are very attractive for NDT due to their noncontacting nature and their high relative speed of inspection. The application of digital techniques allows for automatic processing. Consequently, a fast inspection procedure enables the evaluation of large areas (e.g., aircraft wings and ship structures) or a large number of parts (e.g., automotive components). Speckle techniques have the advantages cited for optical methods. Additionally, they are appropriate for the evaluation of real components without further preparation of the surface or time-intensive analysis.

This book provides tips, ideas, and examples for the successful application of optical techniques (more specifically based on the speckle phenomenon) outside the laboratory room. Readers can see that the topics presented in the following nine chapters have been selected to benefit graduate students, engineers, and scientists who are interested in the in-field application of
speckle techniques to solve specific problems related to optical metrology, experimental mechanics, and NDT.

Chapter 1 discusses aspects to consider when designing mechanical parts and structures for safe and reliable products because several applications are usually related with human life and ecology. This chapter also shows that the working conditions influence the performance and mechanical integrity of the part. This influence can sometimes cause an accident due to a lack of corrective actions. For this reason, the chapter highlights the use of NDT to foresee possible accidents and focuses on optical techniques, especially speckle methods.

Chapter 2 addresses the theoretical aspects of the origin and formation of the speckle phenomenon. The most important principles for speckle interferometry are then developed, showing how the phase of the speckle distribution carries essential information for measuring displacements fields, object shapes, etc. For this reason, several tools to quantify the phase of the speckle distribution are presented, as well as the phase-unwrapping principles that are used to deal with $2\pi$ jumps obtained after the use of phase-shifting techniques.

Chapter 3 presents traditional digital-speckle-pattern-interferometry (DSPI) optical configurations used to measure displacement fields and their derivatives. Measurements are divided into (a) out-of-plane and (b) in-plane displacements. For the former, the working principle is presented, as well as a possible laboratory optical setup. For the latter, traditional interferometers with in-plane sensitivity are presented; radial, in-plane interferometer setups capable of measuring polar coordinates are also presented. Finally, principles for shearography are shown.

Chapter 4 gives a more-detailed description of the requirements for robust optical setups. The chapter offers tools, tips, and reference parameters to guide the development and design of interferometers based on the speckle phenomenon for use outside of the laboratory. Additionally, various environment agents are described, showing the effect that they have on the measuring performance of the optical system.

Chapter 5 discusses the application of DSPI to measure mechanical stresses as an auxiliary tool for structural integrity assessment. After a short introduction, the principles for traditional strain-gage sensors are presented. Some interferometric solutions are shown in order to measure 3D displacements (along three sensitivity directions) and displacements in polar coordinates. For the latter, several tips are listed for the measurement of large strain fields without loss of correlation. Finally, an application example shows the effectiveness of the proposed solution.

Many service failures of structural or mechanical components are caused by a combination of residual stress fields in the material and mechanical stresses produced by applied loads. For this reason, Chapter 6 provides experimental solutions to compute residual stresses. The traditional method
combines strain gages with the hole-drilling technique. In this case, a small hole is introduced into the material, allowing for local stress relief that enables stress measurements. The chapter also explores a combination of the hole-drilling technique and DSPI. A practical application outside the laboratory is described, showing the high potential of the technique as an integrity-evaluation tool.

Chapter 7 begins with a list of the traditional nondestructive techniques used in defect detection. The chapter highlights shearography as a NDT tool with important applications in the automotive, aeronautical, and petroleum and gas industries. Several optical configurations suitable for in-field applications are presented. One of the most important components in a shearographic device is the loading/excitation setup. For this reason, several possible methods are described. Finally, applications in some industries, mechanical parts, and structures are shown. Available commercial systems highlight the fast growth of shearography as a NDT technique. Some significant commercial devices are illustrated in this chapter.

Previous chapters address principles, optical setups, and application examples for interferometric techniques based on the speckle phenomenon. Another optical speckle technique that has grown quickly over the last two decades is digital image correlation (DIC), which is considered a noninterferometric technique. A short review of the available literature about this technique is presented in Chapter 8, which is oriented to NDT applications.

Finally, Chapter 9 briefly discusses all of the presented techniques to help readers select the best optical setup for their needs, or, beyond that, develop new solutions (for those cases where there are none) to measure a specific measurand.

We would like to thank the following people: Prof. Guillermo Kaufmann and SPIE Press Manager Tim Lamkins for their encouragement before writing this book; Prof. Gary Schajer for his kind help and valuable collaboration with some figures obtained by residual stress measurements with the hole-drilling techniques; Prof. Gustavo Galizzi for his help during the elaboration of some simulated figures used in the phase-unwrapping section; Dr. Gordon Craggs for several fruitful discussions about Chapters 2 and 3 and for his help with some phrasing; the peer reviewers for their important comments and corrections; and Scott McNeill and the SPIE editorial department for their help and support.

Last, but not least, we are grateful to our families for their support and patience during our time “inside the book.” In particular, we would like to give thanks to God for the opportunity to write this book.

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August 2014

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<thead>
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<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
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<td>$A$</td>
<td>Cross-sectional area of a uniform conductor</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Camera calibration matrix</td>
</tr>
<tr>
<td>$a_{ij}$, $b_{ij}$</td>
<td>Matrices of calibration coefficients</td>
</tr>
<tr>
<td>AFM</td>
<td>Atomic force microscopy</td>
</tr>
<tr>
<td>AOM</td>
<td>Acousto-optical modulator</td>
</tr>
<tr>
<td>AOV</td>
<td>Angle of view of the camera</td>
</tr>
<tr>
<td>ASTM</td>
<td>American Society for Testing and Materials</td>
</tr>
<tr>
<td>atan2</td>
<td>Full four-quadrant arctangent function</td>
</tr>
<tr>
<td>$b$</td>
<td>Diameter of the aperture</td>
</tr>
<tr>
<td>BS</td>
<td>Beamsplitter</td>
</tr>
<tr>
<td>$c$</td>
<td>Matrix operator that acts over the curvature of the solution for nonuniform residual stresses</td>
</tr>
<tr>
<td>CASI</td>
<td>Computer-aided speckle interferometry</td>
</tr>
<tr>
<td>$C_{Ax}$</td>
<td>External axial load</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge-coupled device</td>
</tr>
<tr>
<td>CFRP</td>
<td>Carbon-fiber-reinforced polymer</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary metal–oxide semiconductor</td>
</tr>
<tr>
<td>CT</td>
<td>Computed tomography</td>
</tr>
<tr>
<td>$d$</td>
<td>Displacement vector</td>
</tr>
<tr>
<td>$d = [u, v, w]$</td>
<td>Displacement vector at point $P$</td>
</tr>
<tr>
<td>$d_f$</td>
<td>Diameter of the conductor after application of the strain</td>
</tr>
<tr>
<td>DIC</td>
<td>Digital image correlation</td>
</tr>
<tr>
<td>$d_o$</td>
<td>Diameter of the conductor before the application of the strain</td>
</tr>
<tr>
<td>DOE</td>
<td>Diffractive optical element</td>
</tr>
<tr>
<td>$d_{opt}$</td>
<td>Optimal displacement vector</td>
</tr>
<tr>
<td>$d(P')$</td>
<td>Displacement vector at point $P'$</td>
</tr>
<tr>
<td>DPSS</td>
<td>Diode-pumped solid state</td>
</tr>
<tr>
<td>DSCM</td>
<td>Digital speckle correlation method</td>
</tr>
<tr>
<td>$d_{sp}$</td>
<td>Averaged speckle size</td>
</tr>
<tr>
<td>DSPi</td>
<td>Digital speckle pattern interferometry</td>
</tr>
<tr>
<td>$(d_{sp})_{obj}$</td>
<td>Size of the speckle on the illuminated object</td>
</tr>
<tr>
<td>$dx$, $dy$</td>
<td>Distances between adjacent pixel in the $x$ and $y$ directions</td>
</tr>
</tbody>
</table>
\( d_{\tau} \)  
Time interval between successive registrations

\( E \)  
Modulus of elasticity

ESPI  
Electronic speckle pattern interferometry

\( f \)  
Focal length of the optical system

\( F \)  
Reference image

\( \bar{F} \)  
Medium value for the subset

\( F_{b} \)  
Numerical aperture of the optical system

\( F_{1} \ldots F_{n} \)  
Set of external loads

\( G \)  
Image after displacement

\( \bar{G} \)  
Medium value for the subset

\( G_{e} \)  
Geometric factor associated with the directions of illumination and observation

GLARE  
Glass-reinforced aluminum laminate

GPGPU  
General-purpose computing on graphics processing unit

\( G_{r} \)  
Modulus of elasticity in shear (also known as the modulus of rigidity)

\( H_{0}(r_{s}) \)  
Zero-order Fourier coefficient

\( H_{1}(r_{s}) \)  
First-order Fourier series coefficient

\( H_{2}(r_{s}) \)  
Second-order Fourier coefficient

\( H_{nS}(r_{s}) \)  
Total magnitude of the \( n^{th} \) harmonic

\( H_{nS}(r_{s}), H_{nc}(r_{s}) \)  
Sine and cosine components, respectively, of the \( n^{th} \) Fourier series coefficient

\( \hat{i}, \hat{k} \)  
Unitary vectors for the \( x \) and \( z \) directions, respectively

\( I_{0} \)  
Averaged (or background) intensity

\( I_{0f} \)  
Averaged correlation intensity

\( I_{1}, I_{2} \)  
Intensities of the interfering beams

\( I_{12} \)  
Subtraction of the intensities of the interfering beams

\( I_{c} \)  
Cosine intensity obtained from the wrapped difference phase map \( \Delta \phi_{w}(m,n,t_{1},t_{2}) \)

IEEE  
Institute of Electrical and Electronics Engineers

Im  
Imaginary part

\( I_{M} \)  
Modulation intensity

\( I_{per} \)  
Moment of inertia of the section

IR  
Infrared

\( I_{s} \)  
Sine intensity obtained from the wrapped-difference phase map \( \Delta \phi_{w}(m,n,t_{1},t_{2}) \)

\( k \)  
Sensitivity vector

\( K_{0}, K_{x}, K_{y} \)  
Constant fitting values of the bending plane

\( K_{0R}, K_{1C}, K_{1S}, K_{2C}, K_{2S}, K_{0} \)  
Least-square fitting coefficients

\( K_{11} \) to \( K_{66} \)  
Coefficients of elasticity of the material

\( k_{c} \)  
Multiplicative constant for a speckle distribution

\( k_{i} \)  
Wave-propagation vectors corresponding to the illumination direction
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>k_o</td>
<td>Wave-propagation vectors corresponding to the observation direction</td>
</tr>
<tr>
<td>L</td>
<td>Length of a uniform conductor</td>
</tr>
<tr>
<td>l_0 \times l_0</td>
<td>Cross-section of the illuminated area</td>
</tr>
<tr>
<td>l_f</td>
<td>Final length of the bar</td>
</tr>
<tr>
<td>l_o</td>
<td>Initial length of the bar</td>
</tr>
<tr>
<td>LUS</td>
<td>Laser ultrasound</td>
</tr>
<tr>
<td>M</td>
<td>Bending moment applied to the beam</td>
</tr>
<tr>
<td>M_1</td>
<td>45-deg mirror in the radial interferometer</td>
</tr>
<tr>
<td>M_2</td>
<td>Mobile mirror in the radial interferometer</td>
</tr>
<tr>
<td>M_3</td>
<td>Fixed mirror in the radial interferometer</td>
</tr>
<tr>
<td>M_g</td>
<td>Magnification of the optical system</td>
</tr>
<tr>
<td>M(T)</td>
<td>Middle-crack tension</td>
</tr>
<tr>
<td>(n, m, t)</td>
<td>Nondimensional coordinates of the discrete image</td>
</tr>
<tr>
<td>n_A, n_B</td>
<td>Illumination unitary vectors</td>
</tr>
<tr>
<td>NDT</td>
<td>Nondestructive testing</td>
</tr>
<tr>
<td>NINT</td>
<td>Rounding to the nearest integer</td>
</tr>
<tr>
<td>N_n \times N_m</td>
<td>Sensor pixel number (horizontal and vertical)</td>
</tr>
<tr>
<td>\mathbf{n_o}, \mathbf{n_i}</td>
<td>Unitary vectors</td>
</tr>
<tr>
<td>NSSD</td>
<td>Normalized sum of square difference</td>
</tr>
<tr>
<td>N_t</td>
<td>Number of successive acquired images</td>
</tr>
<tr>
<td>OMS</td>
<td>Optical measurement system</td>
</tr>
<tr>
<td>OPA</td>
<td>Operational amplifier</td>
</tr>
<tr>
<td>OPD</td>
<td>Optical path difference</td>
</tr>
<tr>
<td>p</td>
<td>Parameter vector of the shape function</td>
</tr>
<tr>
<td>p, q, t</td>
<td>Combination variables between the strains ( \varepsilon_1, \varepsilon_2, \text{ and } \varepsilon_3 )</td>
</tr>
<tr>
<td>P</td>
<td>Equal-biaxial stress</td>
</tr>
<tr>
<td>P_1, P_2</td>
<td>Points at the illuminated surface</td>
</tr>
<tr>
<td>\mathbf{P_1}, \mathbf{P_2}</td>
<td>Unwrapping paths</td>
</tr>
<tr>
<td>p_r</td>
<td>Period of the grating structure</td>
</tr>
<tr>
<td>PZT</td>
<td>Piezoelectric translator</td>
</tr>
<tr>
<td>Q</td>
<td>45-deg shear stress</td>
</tr>
<tr>
<td>Q(x, y)</td>
<td>Points at the imaging plane</td>
</tr>
<tr>
<td>q_{mult}</td>
<td>Multiplying factor</td>
</tr>
<tr>
<td>r</td>
<td>Position vector</td>
</tr>
<tr>
<td>r, \theta</td>
<td>Polar coordinates</td>
</tr>
<tr>
<td>R</td>
<td>Resistance of a uniform conductor</td>
</tr>
<tr>
<td>r_0</td>
<td>Radius of the hole</td>
</tr>
<tr>
<td>R_e</td>
<td>Real part</td>
</tr>
<tr>
<td>r_{ex}, r_{in}</td>
<td>External and internal radius, respectively, of the pipe</td>
</tr>
<tr>
<td>r_i</td>
<td>Curvature center of the incident wavefront</td>
</tr>
<tr>
<td>\mathbf{R_i}</td>
<td>Rotation matrix</td>
</tr>
<tr>
<td>r_o</td>
<td>Position vector of the observation point</td>
</tr>
<tr>
<td>ROI</td>
<td>Region of interest</td>
</tr>
</tbody>
</table>
List of Symbols and Notation

- $r_s$: Sampling radius
- $s$: Scale factor
- $S$: Scattering surface
- $S_A$: Sensitivity of the metallic alloy used as a conductor
- SAE: Society of Automotive Engineers
- SAW: Submerged arc welding
- SEM: Scanning electron microscopy
- SG: Strain gage
- $\text{sgn} I$: $\text{sgn}[I(1) - I(2)]$ for the Carrè algorithm
- $\text{sg}$: Sign function
- SLM: Spatial light modulator
- SNR: Signal-to-noise ratio
- $s_{OPD}$: Optical path difference
- SSD: Sum of squared deviations
- $T$: Shear stress
- $t_1, t_2$: Time for the first and second interferogram acquisition
- $T_i$: Translation vector
- $u$: Component of the displacement field along the $x$ direction
- $(u_1, v_1, 1)$, $(u_2, v_2, 1)$: Image coordinates for both cameras
- UOE: U-shape, O-shape, and expansion (pipe-fabrication process)
- $u_r$: Radial component of the in-plane displacement
- USB: Universal serial bus
- $u_t$: Amount of uniform rigid-body translation
- $V$: Fringe visibility or contrast
- VDIC: Volumetric digital image correlation
- $V_f$: Correlation fringe visibility or contrast
- $V_r$: Radial phase variation around a pixel
- $w$: Component of the displacement of the object surface along the $z$ direction
- $w(x)$: Weighting function
- $x(x, y)$: Coordinates on the observation plane
- $(x, y, z)$: Cartesian coordinates
- $(x_{c1}, y_{c1}, z_{c1}, 1)$, $(x_{c2}, y_{c2}, z_{c2}, 1)$: Homogeneous coordinates corresponding to point Q in camera 1 and camera 2, respectively
- $(x_w, y_w, z_w, 1)$: Homogeneous coordinates in the world coordinate system
- $y$: Distance from the neutral line
- $z$: Distance between the screen where the scattered light is gathered and the object
- $z_f$: Distance between the aperture and the imaging plane
- ZN SSD: Zero-normalized sum of squared differences
- $z_o$: Distance from the object to the aperture
- ZSSD: Zero-mean sum of squared differences
List of Symbols and Notation

\( \alpha \) Angle that defines the translation direction

\( \alpha_P, \alpha_Q, \alpha_T \) Factors to control of the amount of regularization

\( \alpha_i \) Relative phase shift between acquired interferograms

\( \gamma \) Illumination angle

\( \Upsilon \) Wrapping operator

\( \gamma_1, \gamma_2 \) Illumination angles for an in-plane interferometer

\( \gamma_{ij} \) Shear strains being \( i = x, y, \) or \( z \) and \( j = x, y, \) or \( z \)

\( \Delta \) Change in the parameter

\( \Delta d \) Displacement difference

\( \Delta l \) Change of the length

\( \delta_{NL} \) Angle of the neutral line

\( \Delta OPC \) Optical path change generated by the PZT displacement

\( \Delta PZT \) Displacement of the piezoelectric transducer

\( \Delta r \) Displacements of the scattering surface

\( \Delta S_{OPD} \) Optical path difference

\( \delta_x \) Lateral shift in shearography

\( \delta_{s_{MAX}} \) Angular position of the maximum stress axis

\( \Delta \phi \) Variation of the phase of the speckle in the object beam produced by the displacements of the diffuser

\( \Delta \phi \) Variation in the phase difference

\( \Delta \phi(m,n) \) Wrapped phase to be determined

\( \Delta \phi_{m,n} \) Measured wrapped phase

\( \varepsilon_1, \varepsilon_2 \) Principal strains

\( \varepsilon_1, \varepsilon_2, \varepsilon_3 \) Measured strains by gages 1, 2, and 3 (see Fig. 6.2)

\( \varepsilon_i \) Normal strains, being \( i = x, y, \) or \( z \)

\( \zeta(x, p) \) Subset function

\( \eta \) Principal angle (for mechanical stresses)

\( \lambda \) Wavelength of the illumination source

\( \nu \) Poisson ratio

\( \xi \) Diffraction angle for the \( m \)-order

\( \xi(x,y) \) Surface height at \( (x, y) \)

\( \rho \) Specific resistance of a uniform conductor

\( \sigma \) Normal stress

\( \sigma_1, \sigma_2 \) Principal stresses

\( \sigma_B \) Bending stress

\( \sigma_C \) Stress along the circumferential direction

\( \sigma_L \) Stress along the longitudinal direction

\( \sigma_L(x,y) \) Longitudinal stress component measured in each angular point

\( \tau \) Shearing stress

\( \tau \) Time on the observation plane

\( \phi \) Phase of the speckle

\( \phi_1, \phi_2 \) Phases of the interfering wavefronts
\( \phi^E_i \)  
Phase value for the \( i^{th} \) pixel on the external circle

\( \phi^I_i \)  
Phase of \( i^{th} \) pixel on the internal circle

\( \varphi_s \)  
Random component of the phase of the speckle

\( \phi_w \)  
Wrapped phase

\( \chi \)  
Angle of the conical mirror

\( \chi(p) \)  
Cost function

\( \psi \)  
Deterministic component of the phase of the speckle

\( \psi_i \)  
Initial optical phase

\( \psi_o \)  
Object phase

\( \Omega(r) \)  
Complex amplitude at each point in a speckle pattern

\( \omega_i \)  
Complex amplitude of the incident light in \((x,y)\)