

# **ANALOG AND DIGITAL HOLOGRAPHY WITH MATLAB<sup>®</sup>**



# **ANALOG AND DIGITAL HOLOGRAPHY WITH MATLAB®**

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# Table of Contents

<i>Preface</i>	xi
<i>List of Acronyms and Abbreviations</i>	xiii
<b>1 Introduction and Preliminaries</b>	<b>1</b>
1.1 History of Holography	1
1.1.1 Introduction	1
1.1.2 Types of holograms	2
1.1.3 Holographic recording media	7
1.2 Scalar Theory of Diffraction	14
1.2.1 Maxwell's equations	14
1.2.2 Spatial frequency transfer function and Fresnel diffraction	17
1.2.3 Fraunhofer diffraction	18
1.2.4 Fourier transform property of an ideal lens	22
1.2.5 Gaussian beam optics	25
1.2.6 $q$ -transformation of Gaussian beams	27
1.2.7 Focusing a Gaussian beam	29
1.3 Example 1: MATLAB Code for Calculating Diffraction with the Fast Fourier Transform	31
1.4 Example 2: MATLAB Code for Calculating Forward and Backward Gaussian Beam Propagation	34
1.5 Example 3: MATLAB Code for Gaussian Beam Propagation through a Lens	34
1.6 Generalized Diffraction Example via the Fresnel Transform	37
References	44
<b>2 Analog Holography, Holographic Interferometry, and Phase-Shifting Holographic Interferometry</b>	<b>47</b>
2.1 Fourier Optics Theory	47
2.2 Analog Holography Theory and Setups	49
2.3 Analog Holographic Interferometry Theory and Setups	52
2.4 Phase Unwrapping in 1D and 2D	55
2.5 Application of Phase Unwrapping in Holographic Interferometry	63

2.6	Phase-Shifting Holography through Dynamic Holography and Self-Diffraction	74
	References	80
<b>3</b>	<b>Fringe Deciphering Techniques Applied to Analog Holographic Interferometry</b>	<b>83</b>
3.1	Introduction	83
3.2	Interferogram Processing Using Frequency Techniques	84
3.2.1	Demodulating simulated fringes due to a tilt	84
3.2.2	Demodulating fringes embedded with a carrier	84
3.3	Interferogram Processing Using Fringe Orientation and Fringe Direction	90
3.3.1	Definition of fringe orientation and fringe direction	91
3.3.2	Orientation computation methods	93
3.3.2.1	Gradient-based method	93
3.3.2.2	Plane-fit method	94
3.3.2.3	Spin-filter method	97
3.3.2.4	Fourier transform method	100
3.3.2.5	Accumulate-differences method	106
3.3.2.6	Comparison of the different methods	107
3.3.3	Phase unwrapping and fringe direction computation using regularized phase tracking	107
3.4	Phase Demodulation Using the Hilbert Transform Technique	112
3.5	Fringe Skeletonization and Normalization	120
3.6	Contrast Enhancement of Fringe Patterns	127
3.7	Phase Unwrapping: Interferogram Analysis	130
3.7.1	Path-dependent techniques	131
3.7.2	Path-independent techniques	132
	References	132
<b>4</b>	<b>Digital Holography and Digital Holographic Microscopy</b>	<b>137</b>
4.1	Basics of Digital Holography	137
4.2	Digital Holography Reconstruction Algorithms	138
4.2.1	Numerical reconstruction by the discrete Fresnel transformation	139
4.2.2	Numerical reconstruction by the convolution approach	141
4.2.3	Numerical reconstruction by the angular spectrum approach	141
4.3	DC Suppression during Reconstruction	142
4.4	Digital Holography Example	145
4.5	Digital Holograms of Large Objects	152
4.6	Digital Holographic Microscopy	153
4.7	Digital Holographic Microscopy Example	157
4.8	Optimization of the Fresnel Transform	157
4.9	General Functions for Digital Holography Using MATLAB	164
	References	180

<b>5</b>	<b>Digital Holographic Interferometry and Phase-Shifting Digital Holography</b>	<b>183</b>
5.1	Digital Holographic Interferometry: Basic Principles	183
5.2	Two-Illumination-Point Technique	187
5.3	3D Stress and Strain Sensors from Three Digital Hologram Recordings	189
5.4	Phase-Shifting Digital Holography	195
5.5	Techniques to Perform Phase-Shifting Digital Holography	199
5.6	One-Shot Phase-Shifting Digital Holography Using Wave Plates	203
5.7	General Functions for Digital Holographic Interferometry and Phase-Shifting Digital Holography Using MATLAB	207
	References	210
<b>6</b>	<b>Digital Holographic Tomography</b>	<b>211</b>
6.1	Introduction	211
6.2	Single-Shot Optical Tomography Using the Multiplicative Technique (SHOT-MT)	212
6.3	Single-Shot Optical Tomography Using the Radon Transform Technique	220
6.4	Recording Considerations for Holographic Tomography	229
6.4.1	Multiple-angle, single-exposure methods	229
6.4.2	Multiple-angle, multiple-exposure methods	232
6.4.3	Microscopic tomography methods	237
6.4.4	Angular sampling considerations	242
6.5	Examples of Digital Holographic Tomography Using MATLAB	255
	References	256
<b>7</b>	<b>Multiwavelength Digital Holography</b>	<b>257</b>
7.1	Holographic Contouring	257
7.2	Principle of Multiwavelength Digital Holography	261
7.3	Hierarchical Phase Unwrapping	263
7.4	Multiwavelength Digital Holography	264
7.5	Multiwavelength Digital Holography with Spatial Heterodyning	267
7.6	Multiwavelength Digital Holographic Microscopy	280
7.7	Multiwavelength Digital Holographic Microscopy with Spatial Heterodyning	280
7.8	Holographic Volume-Displacement Calculations via Multiwavelength Digital Holography	291
7.9	Multiwavelength Digital Holography: Image-Type Setup and Results	297
	References	304
<b>8</b>	<b>Computer-Generated Holography</b>	<b>307</b>
8.1	A Brief History	307
8.2	Fourier Transform Holograms: Detour Method	308
8.3	Phase-Only CG Hologram	311
8.4	Gerchberg–Saxton Algorithm for Recording a CG Hologram	314
8.5	Point-Source Holograms and the Wavefront Recording Plane Method	318

8.6	Recent Developments in CGH	324
8.6.1	Fourier ping-pong algorithm	325
8.6.2	Interference-based algorithms	326
8.6.3	Diffraction-specific algorithm	326
8.6.4	Binarization algorithms	329
8.7	CGH-based Display Systems	329
8.7.1	Advantages	329
8.7.2	Challenges	331
8.7.3	Computational loads	333
	References	333
<b>9</b>	<b>Compressive Sensing and Compressive Holography</b>	<b>337</b>
9.1	Compressive Sensing: Background	337
9.2	Compressive Holography	341
9.3	Experimental Setups and MATLAB Examples	347
	References	355
<b>10</b>	<b>Contemporary Topics in Holography</b>	<b>357</b>
10.1	Transport-of-Intensity Imaging	357
10.2	Nonlinear Holography	372
10.3	Coherence Holography	375
10.4	Polarization Imaging Using Digital Holography	384
	References	408
<b>11</b>	<b>Progress in Stereoscopic, Head-Mounted, Multiview, Depth-Fused, Volumetric, and Holographic 3D Displays</b>	<b>411</b>
11.1	Introduction to 3D Displays	411
11.1.1	Characteristics of an optimal 3D display	412
11.1.2	Display-technology depth cues related to the human visual system	412
11.2	Stereoscopic 3D Displays	416
11.2.1	Spectral-based stereoscopic display (anaglyph)	417
11.2.2	Polarization-based stereoscopic display	417
11.2.3	Alternate-frame stereoscopic display	420
11.3	Head-Mounted Displays (HMDs)	421
11.4	Autostereoscopic 3D Displays	424
11.4.1	Multiview 3D display technology	424
11.4.1.1	Introduction to different multiview systems	424
11.4.1.2	Occlusion-based system	427
11.4.1.3	Refraction-based system	430
11.4.1.4	Reflection-based system	435
11.4.1.5	Diffraction-based system	435
11.4.1.6	Projection-based system	437
11.4.1.7	Super multiview (SMV) 3D display	440



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11.4.1.8	Head-tracking autostereoscopic 3D display	441
11.4.1.9	Directional-backlight autostereoscopic 3D display	442
11.4.2	Depth-fused 3D display technology	445
11.4.3	Volumetric 3D display technology	446
11.4.3.1	Passive static-screen display	447
11.4.3.2	Active static-screen display	450
11.4.3.3	Passive swept-screen display	452
11.4.3.4	Active swept-screen display	457
11.4.4	Holographic 3D display technology	458
11.4.4.1	Spatial light modulators (SLMs)	459
11.4.4.2	MIT holographic 3D displays: holovideo	463
11.4.4.3	SeeReal 3D displays	466
11.4.4.4	Zebra holographic 3D displays	466
11.4.4.5	QinetiQ holographic 3D displays	468
11.4.4.6	IMEC holographic 3D display	469
11.4.4.7	Holographic Reconstruction (HORN)	469
11.4.4.8	Image hologram	470
11.4.4.9	Coherent stereogram	471
11.4.4.10	NICT 3D holographic system	473
11.4.4.11	University of Arizona's updatable holographic display	473
11.5	Comparison of the Different 3D Display Techniques	476
11.6	Commonly Misunderstood Nonholographic, Non-3D Displays	479
11.6.1	Pepper's ghost illusion	479
11.6.2	Heliodisplay	481
	References	483
	<i>Appendix: Additional MATLAB Functions</i>	495
	<i>Index</i>	501



# Preface

Although the concept of holography has been known for decades, the field has seen significant development due to the availability of moderately priced lasers in the market for holographic applications. Also due to the advances in computer technology and computational processes, gathering and processing the experimental data has become much more tangible. Holography is a useful technique because it is the only truly three-dimensional imaging method available. It is used in a plethora of fields, such as 3D nonintrusive testing of cracks and fatigue in equipment, high-axial and lateral-resolution 3D topography of surfaces, 3D particle image velocimetry, 3D stress and deformation measurement, 3D microscopy of transparent phase objects for biomedical imaging, and holographic displays for the entertainment industry, just to name a few. For these reasons, digital and analog holography, along with their many variations (i.e., holographic interferometry, holographic microscopy, holographic tomography, multiwavelength digital holography, phase-shifting holography, compressive holography, coherence holography, computer-generated holography, etc.) have become the methods of choice for various metrological applications in 3D imaging.

This book begins with a brief introduction of the history of holography, types of holograms, and materials used for hologram recording, followed by a discussion of the basic principles of analog and digital holography and an in-depth explanation of some of the most famous fringe-deciphering techniques for holographic interferometry. Besides the traditional topics already mentioned, other related topics are discussed—dynamic holography, non-Bragg orders, and compressive holographic tomography—as well as a nonholographic technique for 3D visualization, i.e., transportation of intensity imaging. Furthermore, the latest topics in the field of holography are discussed for the first time here: compressive holography, coherence holography, nonlinear holography, and polarization holography. The last chapter is dedicated to the progress in holographic and nonholographic 3D display technologies.

Multiple holographic techniques are presented, and readers may master their basic concepts through in-depth theory and applications. This book is a comprehensive study in the sense that traditional and up-to-date topics

concerning holographic imaging and displays are presented. The focus is not so much the theory of these 3D imaging techniques, which exists in many references and will be briefly mentioned in this book, but rather the programming side, namely, the exact code that is needed to perform complex mathematical and physical operations. The code associated with each section help the reader grasp the mathematical concepts better through changing and adapting the parameters. Programming these complex equations is tedious and not straightforward; supplying the code with the text makes it easier for students and experienced researchers to concentrate on performing the experiment and simply changing the parameters in the code to get their results.

Because MATLAB<sup>®</sup> has become the programming language of choice for engineering and physics students, we decided to use this fantastic tool for our code examples. A few authors suggest the use of MATLAB for optics-oriented books, but none is adequate for use in practical situations. There are many books about analog and digital holography, but this book is more practical in terms of MATLAB code and examples because it includes all of the different techniques and codes in a single volume. A supplemental CD-ROM is included, which has a detailed version of the code and functions, as well as typical test images, so that readers do not need to perform the experiment to use the code in the book.

Special thanks to Dr. Partha Banerjee, Dr. Joe Haus, and Dr. Andrew Sarangan, Dr. John Loomis, and Dr. Russel Hardie from the University of Dayton. Also, special thanks to those who contributed to some of the original code in this book, namely, Mr. Thanh Nguyen (CUA), Dr. D. J. Brady (Duke University), Dr. J. Antonio Quiroga (The University of Madrid), Drs. J. Bioucas-Dias and G. Valadão (the Instituto Superior Técnico, Lisboa, Portugal), Dr. Munther Gdeisat [the General Engineering Research Institute (GERI) at Liverpool John Moores University], Dr. Miguel Arevallilo Herraiez (the Mediterranean University of Science and Technology, Valencia, Spain), Dr. Justin Romberg (Georgia Tech.), Mr. Peyman Soltani (University of Zanjan, Iran), Dr. Jeny Rajan (the National Institute of Technology, Karnataka Surathkal, Mangalore, India), Dr. Wei Wang (the Heriot–Watt University, Edinburgh), Dr. Laura Waller (UC Berkeley), and Drs. Lei Tian and George Barbastathis (the Department of Mechanical Engineering at MIT). We offer special thanks to our parents, for without them this work would not have been possible.

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# List of Acronyms and Abbreviations

2D-SRNCP	Sorting by reliability, following a non-continuous path
AgBr	Silver bromide
AgCl	Silver chloride
AgH	Silver halide
AgI	Silver iodide
AH	Analog holography
AHI	Analog holographic interferometry
AO	Acousto-optic
AOM	Acousto-optic modulator
API	Application program interface
AR	Active retarder
AS	Angular spectrum
BaTiO <sub>3</sub>	Barium titanate
BC	Branch cuts
BLU	Backlight unit
BMA	Block-matching algorithm
BS	Beamsplitter
BSO	Bismuth silicon oxide
CAD	Computer-aided design
CCD	Charge-coupled device
CGH	Computer-generated holography
CH	Coherence holography
CL	Collimating lens
CO <sub>2</sub>	Carbon dioxide
CPAS	Compensated phase-added stereogram
CRS	Cathode ray sphere
CRT	Coherent raytrace
CRTs	Cathode ray tubes
CS	Compressive sensing
CsH	Compressive holography
DBS	Direct binary search

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DC	direct current (zero order)
DCT/IDCT	Discrete cosine transform/inverse discrete cosine transform
DFD	Depth-fused display
DFT	Discrete Fourier transform
DH	Digital holography
DHI	Digital holographic interferometry
DHI-SM	Digital holographic interferometry with spatial multiplexing
DHM	Digital holographic microscopy
DHT	Digital holographic tomography
DL	Delay line
DLP	Digital light processing
DMD	Digital micromirror device
DND	Diffraction nanodevice
DOE	Diffraction optical element
DPSI	Digital phase shifting interferometry
DS	Diffraction specific
DSCP	Diffraction-specific coherent panoramagram
EASLM	Electrically addressable spatial light modulator
EBL	E-beam lithography
ED	Error diffusion
ESPI	Electronic speckle pattern interferometry
fft	Fast Fourier transform
FK	Fresnel–Kirchhoff
FOV	Field of view
FP	Full parallax
FPA	Focal plane array
FPGA	Field programmable gate array
FZP	Fresnel zone plate
GLV	Grating light valve
GPU	Graphical processing unit
GS	Gerchberg and Saxton
GVF	Gradient vector field
GWS	Guided wave scanner
H&D	Hurter–Driffield
HC	Holocamera
HDTV	High-definition TV
HMD	Head-mounted display
HOE	Holographic optical element
HORN	Holographic reconstruction
HPO	Horizontal parallax only
HTh	Hard thresholding
HWP	Half-wave plate
IR	Infrared
ITO	Indium tin oxide

KNbO <sub>3</sub>	Potassium niobate
LCD	Liquid crystal display
LCOS	Liquid crystal on silicon
LCP	Left circularly polarized light
LED	Light-emitting diode
LEM	Lumped-element model
LID	Laser induced damage
LiNbO <sub>3</sub> , LN	Lithium niobate
LP	Linear polarizer
LUT	Look-up table
MAC	Multiplication and accumulation
MEMS	Microelectromechanical systems
MO	Microscope Objective
MWDH	Multi-wavelength digital holography
MWDH-SH	Multi-wavelength digital holography with spatial heterodyning
MWDHM	Multi-wavelength digital holographic microscopy
MWDHM-SH	Multi-wavelength digital holographic microscopy with spatial heterodyning
Nd:YAG	Neodymium-doped yttrium aluminium garnet
NLS	Nonlinear Schrödinger
NMRS	Numerical mean square error
OALCD	Optically addressed LCD
OL/ETL	Offset diverging lens and electrically tunable lens
OPEN-GL	Open Graphics Library
OR	Orthoscopic
PAS	Phase-added stereogram
PBS	Polarized beamsplitter
PC	Phase conjugate
PDLC	Polymer-dispersed liquid crystal
PE	Phase enhanced
PMMA	Poly(methyl methacrylate)
POCS	Projection onto constrained sets
PP	Ping-pong
PR	Photorefractive
PR	Patterned retarder
PS	Pseudoscopy
PSDH	Phase-shifting digital holography
PSDH-WP	Phase-shifting digital holography using a wave plate
PSF	Point spread function
PSH	Phase-shifting holography
PTP	Photothermoplastic
PUMA	Phase unwrapping via maximum flow/min-cut
PZT	Piezoelectric transducer

QWT	Quarter-wave plate
RCP	Right circularly polarized light
RGB	Red, green, and blue
RHS	Right hand side
RIP	Restricted isometry property
RMSE	Root-mean-square error
ROACH	Referenceless on-axis complex hologram
RPT	Regularized phase tracking
RS	Rayleigh–Sommerfeld
SADBS	Simulated annealing direct binary search
SAW	Surface acoustic wave
SBN	Strontium–barium niobate
SCF	Spatial coherence function
SF	Spatial filter
SHOT	Single-beam holographic tomography
SHOT-MT	Single-beam holographic tomography using a multiplicative technique
SHOT-RTT	Single-beam holographic tomography using a Radon transform technique
SIP	Shutter in panel
SLM	Spatial light modulator
SMV	Super multiview
SNR	Signal-to-noise ratio
SOP	State of polarization
SVEA	Slowly varying envelope approximation
TCH	Tomographic compressive holography
TCH-MT	Tomographic compressive holography using a multiplicative technique
TeO <sub>2</sub>	Tellurium dioxide
TIE	Transport of intensity equation
TIR	Total internal reflection
TMP	Tree-matching pursuit
TV	Total variation
TWIST	Two-step iterative shrinkage thresholding
USAF	United States Air Force
UV	Ultraviolet
VA	Visual acuity
VPO	Vertical parallax only
VR	Virtual reality
VW	Viewing window
WRPM	Wavefront recording plane method



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