FREQUENCY-DOMAIN ANALYSIS WITH DFTs
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Preface

Fourier transforms provide a mechanism for translating suitable mathematical functions between the time domain and the frequency domain. Many excellent references describe the theoretical basis for using Fourier transforms to analyze the frequency content of mathematical functions. Fourier transforms have also been adapted for applied scenarios, such as estimating the frequency content of discretely sampled signals. Procedures for discrete frequency-domain analysis using Fourier methods are, however, laden with subtle foibles. Many of the quirks that are unique to the analysis of discretely sampled signals elude the insight of the typical, rigorous Fourier development that is so elegantly valuable for analyzing mathematical functions. In my experience, many mathematically oriented references leave the details of practical implementation for readers to discover on their own. The main objective of this book is to provide a practical guide for the implementation of Fourier transform methods to perform frequency-domain analysis of discretely sampled time-series signals.

The topic of frequency-domain analysis is initially motivated by presenting a basic example: examining the frequency content of a synthetic line voltage that is created by summing the sinusoidal signals and a noise component. The composite sum is displayed as a time-series signal, where the sinusoidal components are not necessarily easy to identify in the time domain. A discrete Fourier transform (DFT) peak-amplitude spectral density profile (periodogram) of the composite signal is presented that shows how the sinusoidal contributions are explicitly revealed in the frequency domain.

Creating and interpreting the DFT periodogram requires specific knowledge of Fourier transform theory, combined with techniques that are tailored for discretely sampled signals. Salient mathematical
concepts that are pertinent to Fourier analysis are explored, with the aim of providing enough theory to fully understand and interpret the Fourier frequency spectra of mathematical functions. The theory is followed by precise explanations of essential practical concepts that are required for analyzing discretely sampled time-series signals using Fourier transform methods. A discussion of common DFT pre-processing techniques, such as de-trending, data padding, data windowing, and remedies for certain discretization issues, is presented. MATLAB® source code is provided that implements all of the ideas discussed in the text. (MATLAB® is a registered trademark of The Mathworks, Inc.) Some validation and verification activities for the source code are explored. The source code is then put to good use in several applied examples that illustrate some of the potent capabilities of DFT frequency-domain analysis. The code can be downloaded at http://spie.org/Samples/Pressbook_Supplemental/PM282_sup.zip

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