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Fractal and Wavelet Image Compression Techniques

Stephen Welstead

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Printed in the United States of America.
To the memory of my parents
Introduction to the Series

Since its inception in 1989, the Tutorial Texts (TT) series has grown to more than 80 titles covering many diverse fields of science and engineering. The initial idea for the series was to make material presented in SPIE short courses available to those who could not attend and to provide a reference text for those who could. Thus, many of the texts in this series are generated by augmenting course notes with descriptive text that further illuminates the subject. In this way, the TT becomes an excellent stand-alone reference that finds a much wider audience than only short course attendees.

Tutorial Texts have grown in popularity and in the scope of material covered since 1989. They no longer necessarily stem from short courses; rather, they are often generated by experts in the field. They are popular because they provide a ready reference to those wishing to learn about emerging technologies or the latest information within their field. The topics within the series have grown from the initial areas of geometrical optics, optical detectors, and image processing to include the emerging fields of nanotechnology, biomedical optics, fiber optics, and laser technologies. Authors contributing to the TT series are instructed to provide introductory material so that those new to the field may use the book as a starting point to get a basic grasp of the material. It is hoped that some readers may develop sufficient interest to take a short course by the author or pursue further research in more advanced books to delve deeper into the subject.

The books in this series are distinguished from other technical monographs and textbooks in the way in which the material is presented. In keeping with the tutorial nature of the series, there is an emphasis on the use of graphical and illustrative material to better elucidate basic and advanced concepts. There is also heavy use of tabular reference data and numerous examples to further explain the concepts presented. The publishing time for the books is kept to a minimum so that the books will be as timely and up-to-date as possible. Furthermore, these introductory books are competitively priced compared to more traditional books on the same subject.

When a proposal for a text is received, each proposal is evaluated to determine the relevance of the proposed topic. This initial reviewing process has been very helpful to authors in identifying, early in the writing process, the need for additional material or other changes in approach that would serve to strengthen the text. Once a manuscript is completed, it is peer reviewed to ensure that chapters communicate accurately the essential ingredients of the science and technologies under discussion.

It is my goal to maintain the style and quality of books in the series and to further expand the topic areas to include new emerging fields as they become of interest to our reading audience.

James A. Harrington
Rutgers University
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Preface

This book is a tutorial text that examines the techniques behind fractal and wavelet approaches to image compression. The field of image compression has experienced an explosion of interest recently because of the growth of the Internet and other multimedia applications. While standard image and data compression methods exist and are in extensive use today, the demand for ever increasing storage requirements and transmission speed have spurred continued research for improved methods. Fractals and wavelets provide two different avenues for such research. For scientists, engineers, students and researchers interested in learning more about fractal and wavelet image compression, this book provides both an introduction to the subject matter and implementation details sufficient for beginning their own investigations into these exciting new technologies.

Prior knowledge of image compression, fractal geometry or wavelet concepts is not necessary to benefit from this book. The level of mathematical presentation is accessible to advanced undergraduate or beginning graduate students in technical fields. Mathematical concepts that would be helpful to know include the idea of convergence of a sequence, multiple integrals, linear independence and basis vectors. Experienced image processing practitioners will probably be disappointed at the minimal amount of coverage devoted to traditional techniques such as the discrete cosine transform and entropy coding. These topics are covered in depth in other books. For example, entropy coding, which can be applied to the output of any compression algorithm, including fractal and wavelet approaches, is not included in the system applications developed here. The present book focuses on the mathematical aspects of fractal and wavelet image compression.

In addition to learning the theory behind fractal and wavelet image compression, readers of this book will have access to software that will enable them to explore these ideas on their own. The software accompanying this book can be found on the web at [http://www.spie.org/bookstore/tt40/](http://www.spie.org/bookstore/tt40/). Details on how to use the software, and how it is constructed, are covered in the book's Appendixes A, B, and C. Three complete Windows-compatible software systems are included with the accompanying software. The IFS System allows readers to create their own fractal images using iterated function systems. The IMG System compresses images using fractal techniques, displays the decoded images, and computes the error between the original and decoded images through image subtraction. The WAV System performs similar functions on images using wavelet techniques and, in addition, displays the wavelet transform of an image. Each system uses a standard Windows interface and includes options for saving and retrieving information from files. The programs run on 32-bit Windows systems, including Windows NT, 95 and 98. Finally, to enable readers to explore beyond the boundaries of the included software, complete C/C++ source code is provided.

The source code for the accompanying software is written in a combination of C and C++. It is not necessary to know either of these languages to benefit from the ideas of this book or to run the programs included with the software. There are a few code examples listed with the text. For the most part, the computational code is written in C. When there is an obvious benefit to exploiting the object-oriented characteristics of C++, then that language is used. In either case, the computational code is kept separate from
the user-interface and display code modules that access Windows. Thus, the
computational source code, with perhaps minor modifications, should be portable to
other platforms, such as UNIX. The user-interface code, where there is an obvious
benefit to using object-oriented properties such as inheritance, is written in C++. The
source code includes its own C++ application framework for developing simple
Windows applications. It does not depend on Microsoft’s Foundation Classes (MFC) or
other third-party frameworks. The code here was developed using Borland’s C++ for
Windows, version 4.5. It has also been compiled with Symantec C++ 7.2 and Microsoft
Visual C++ 4.0. It should be possible to re-compile the code with any C++ compiler that
accesses the Windows Application Programming Interface (API) and supports
development of 32-bit Windows applications from source code files.

Outline of Topics

The book begins with an overview of the image compression problem, including a brief
discussion of general topics such as information and entropy, arithmetic coding, and a
look at current compression approaches such as JPEG. These general topics are
introduced in order to place fractal and wavelet image compression techniques in the
context of the overall theory of image compression. The remainder of the book is
devoted to fractal and wavelet topics and will not focus on general compression topics,
such as entropy coding, which are covered in other texts.

Fractal image compression is motivated by initially looking at iterated function systems
(IFS). The mathematics of IFS theory, including the contraction mapping theorem,
Barnsley’s collage theorem, and affine transformations, is covered here. These topics are
important to understanding why fractal image compression works. Computer examples
show how to use IFS techniques to synthesize fractal images resembling natural objects.

Partitioned iterated function systems extend the ideas of IFS theory to more general real-
world images and enable fractal encoding and compression of those images. Once the
theory behind fractal encoding has been established, the book considers practical
implementation issues such as how to set up a system of domain and range subimages
and the transformations between these subimages. Computer examples illustrate concepts
such as quadtree partitioning of range cells and the convergence of image sequences to
an attractor image.

Long encoding times have hindered the acceptance of fractal techniques for image
compression. Two approaches for speeding up the encoding process have received recent
attention in the literature. Feature extraction reduces the number of computations needed
for domain-range comparisons. Classification of domains reduces search times for
finding a good domain-range match. This book examines techniques for feature
extraction and the use of neural networks for domain classification. Examples show that
these techniques reduce encoding times from hours to seconds and make PC
implementation viable.

The book then introduces wavelets as an alternative approach to image compression.
Basic Haar wavelets illustrate the idea of wavelet decomposition as a process of
averaging and detail extraction at different resolution levels. The book presents a
unifying approach to the seemingly disparate multiple entry points into wavelet analysis.
Image resolution leads the reader from the ideas of averaging and detail extraction on discrete sequences to scaling functions and wavelet functions. The fact that these functions form basis sets in certain vector spaces leads to the idea of multiresolution analysis. The wavelet transform can be derived from any of these entry points. Averaging and detail extraction can be represented as matrix operators, which leads to a particularly simple formulation of the wavelet transform. The essential features of these operators can be extended to more general highpass and lowpass filtering operators. This analysis leads to more complex wavelet systems, such as the Daubechies wavelets, which provide high compression of commonly occurring signal and image components. With the wavelet framework established, the book examines wavelet image compression techniques, beginning with simple wavelet coefficient quantization schemes and moving on to more complex schemes such as wavelet zerotree encoding. Code samples will illustrate key implementation steps, and computer examples will show how the techniques work. The book also discusses recent research in hybrid techniques which apply the ideas of fractal encoding to data in the wavelet transform domain. Computer examples compare the performance of fractal, wavelet, and hybrid image compression techniques.

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Stephen T. Welstead
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