Thin-Film Design
Modulated Thickness and Other Stopband Design Methods
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Introduction to the Series

Since its conception in 1989, the Tutorial Texts series has grown to more than 60 titles covering many diverse fields of science and engineering. When the series was started, the goal of the series was to provide a way to make the material presented in SPIE short courses available to those who could not attend, and to provide a reference text for those who could. Many of the texts in this series are generated from notes that were presented during these short courses. But as stand-alone documents, short course notes do not generally serve the student or reader well. Short course notes typically are developed on the assumption that supporting material will be presented verbally to complement the notes, which are generally written in summary form to highlight key technical topics and therefore are not intended as stand-alone documents. Additionally, the figures, tables, and other graphically formatted information accompanying the notes require the further explanation given during the instructor’s lecture. Thus, by adding the appropriate detail presented during the lecture, the course material can be read and used independently in a tutorial fashion.

What separates the books in this series from other technical monographs and textbooks is the way in which the material is presented. To keep in line with the tutorial nature of the series, many of the topics presented in these texts are followed by detailed examples that further explain the concepts presented. Many pictures and illustrations are included with each text and, where appropriate, tabular reference data are also included.

The topics within the series have grown from the initial areas of geometrical optics, optical detectors, and image processing to include the emerging fields of nanotechnology, biomedical optics, and micromachining. When a proposal for a text is received, each proposal is evaluated to determine the relevance of the proposed topic. This initial reviewing process has been very helpful to authors in identifying, early in the writing process, the need for additional material or other changes in approach that would serve to strengthen the text. Once a manuscript is completed, it is peer reviewed to ensure that chapters communicate accurately the essential ingredients of the processes and technologies under discussion.

It is my goal to maintain the style and quality of books in the series, and to further expand the topic areas to include new emerging fields as they become of interest to our reading audience.

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This text is written for thin-film designers and students with advanced knowledge of multilayer, optical thin-film coatings. It focuses specifically on coatings that have high reflectance performance requirements in more than one spectral wavelength band or region. Many advanced optical systems that employ optical thin-film coatings rely on the performance attributes of multiple spectral bands. Several new analytical design methods that produce multiple stopbands as well as passbands are presented in this text. These analytical design methods produce discrete thin-film designs that will, in most cases, achieve specifications without any refinement of the design. If needed, the spectral performance of these analytical designs could be improved via common computer refinement algorithms. The theory of each design method in this text is presented along with design examples. Relatively basic exercises are provided for students as well as challenging ones for researchers.

This text does not attempt to cover the vast amount of material already published on thin films. The reader is expected to have a general knowledge of classical thin-film physics and designs. Several other texts cover the gamut of classical thin-film theory and designs, and additional information is readily available in several conference proceedings and many journal articles. Detailed derivations of Fresnel equations, complex refractive index, effective index, p- and s-polarization, phase, classical thin-film coating designs, etc., were intentionally omitted from this text. However, a brief introductory chapter on thin-film theory and design is included for completeness.

The coating designs that readers produce using the methods described in this text can be readily manufactured using common coating materials and process methods. In general, the layer thicknesses of these designs vary from a quarter-wave stack to a modulated profile. For some designs, process adjustments may be required for thin layers. However, thin-film manufacturing processes are not covered in this text since other texts and technical papers cover them.

The designs produced using the methods in this text can also be used as initial designs for computer refinement and synthesis algorithms. With some commercially available thin-film design software, some designs are produced with no starting design or specified layers; the algorithm adds layers of preselected materials until the desired spectral performance is achieved. Still, a good starting design helps to reduce the time or effort required to produce a final design that (1) achieves desired performance specifications; (2) is insensitive to layer thickness errors; and (3) can be manufactured. The design methods presented here are also expected to accomplish these tasks for some applications that require stopbands.

In general, the analytical design methods presented were developed using the following methodology. First, layer thicknesses of an arbitrary quarter-wave stack were modulated using various mathematical functions (e.g., sinusoidal). Next, a computer program was written to determine the existence of all stopbands produced from the modulated design. The resulting patterns of stopbands were evaluated graphically as functions of modulation parameters and spectral frequency.
Then, based on these graphical patterns of stopbands, analytical (linear) equations were tested by direct calculation of spectral performance to see if the stopbands could be reproduced analytically.

Several variations on the modulation of layer thickness are presented in this text, including an inhomogeneous rugate design. The last chapter presents a related, novel design method where one quarter-wave stack is linearly transformed into another. Here, empirical testing of layer thickness was used to develop general transform equations. A summary of each chapter and the appendices follows:

Chapter 1 reviews the fundamental mathematics for thin-film design that applies to the proposed methods. Again, the objective here was to keep this chapter brief since this information can be found in many texts.

Chapter 2 introduces sinusoidal thickness modulation of quarter-wave stacks. First, stopbands and passbands are defined. Next, modulation parameters are assessed and many designs are evaluated. Then linear equations are determined that predict all possible stopbands. The last section evaluates the electric fields and the reflected differential phase shifts of some modulated designs.

Chapter 3 introduces discrete apodization of the modulated designs from Chapter 2. Two specific apodization functions are evaluated: amplitude modulation functions and Gaussian envelope functions. Linear equations are again determined that predict all possible stopbands.

Chapter 4 describes two variations of the modulation scheme from Chapter 3. First, chirped-modulation designs are evaluated for spectral performance. Next, a half-modulation is discussed where every other layer of a quarter-wave stack is modulated. Both of these methods are applied to dispersion-controlled mirrors used to produce femtosecond laser pulses. Two design examples and the limitations of these modulation schemes are covered.

Chapter 5 presents a novel, linear transform method that can be used to partially transform a given quarter-wave stack into a second quarter-wave stack. This transformation is accomplished by adjusting the individual layer thickness while the total thickness of the original quarter-wave stack remains constant. General transform equations are developed by direct numerical testing of the transform method. The purpose of this transform is to obtain, or achieve, some of the spectral properties of both quarter-wave stacks (i.e., stopbands).

The five appendixes provide some useful thin-film equations, the Chebychev polynomials used in Chapter 2, the FORTRAN source code used to determine all possible stopbands of modulated designs, several graphs of stopband positions, and a summary of the linear equations that predict stopband positions and the general linear transform equations.

Hopefully this text will provide readers with some new thin-film design tools, further insight to design methods, and inspiration for further research on thin-film design.

I would like to thank my family for their support of my research and writing of this text. I greatly appreciate several helpful discussions with Dr. Philip Baumeister, Dr. Angus Macleod, and Dennis Fischer on these modulation design methods. I also acknowledge the collaborative investigation of rugate versions of these
modulated designs, and the rugate designs and calculations, provided by Dr. Pierre Verly at the National Research Council Canada. I would also like to thank the SPIE reviewers for suggesting several improvements to this text. I would like to thank Coherent, Inc. for supporting this work. Lastly, I would like to acknowledge my graduate professor, Dr. Rasheed M. A. Azzam, for his inspiration to continue my research of thin films.

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Definitions

Additive fractional units ($S^+$)  The number of fractional parts of a quarter-wave layer(s) that combine to produce a new quarter-wave layer, as part of a new quarter-wave stack (see fractional units).

Amplitude-modulated TMD (AM-TMD)  A variation of a thickness-modulated design where the modulation amplitude of the TMD is also modulated at a lower frequency.

Apodization  Envelope function of refractive-index profiles of rugate filters used to suppress passband ripples; also used as envelope functions for layer thickness of TMDs.

Base period (TMD)  The integer number of layers that have a nonrepeating thickness pattern (see modulation period).

Chirped TMD (C-TMD)  A thickness modulated design where the modulation frequency varies or changes as a function of layer number.

Degenerate TMD  Typically, a thickness modulated design that has no modulation (i.e., a quarter-wave stack).

Fractional units ($U$)  The integer number of fractional parts that a quarter-wave thick layer must be subdivided into in order to transform a quarter-wave stack into a second stack (see LOST equations).

Gaussian envelope function TMD (G-TMD)  A thickness-modulated design with the modulation amplitude determined by a Gaussian function.

Half-modulated TMD (H-TMD)  A variation of a thickness-modulated design where the modulation is applied to every other layer; the layers that are not modulated all have the same optical thickness.

HWOT  Half-wave optical thickness.

Linear optical stack transform (LOST)  A linear transform of a quarter-wave stack where the layer thicknesses are adjusted to produce a second quarter-wave stack; a partial transform results in some properties from both quarter-wave stacks.

LOST equations  Two general equations used to linearly transform the additive and subtractive fractional units of the quarter-wave layers of one quarter-wave stack into a second quarter-wave stack.

Modulation amplitude ($k$)  The amplitude at which layer thicknesses are modulated (0–1).

Modulation frequency ($f$)  The frequency at which layer thicknesses are modulated.

Modulation period ($T$)  The number of layers of a modulated design that produces a unique layer pattern.

Quarter-wave stack transform  See linear optical stack transform.
QWOT  Quarter-wave optical thickness.

**Rugate TMD**  An inhomogeneous design in which the continuously refractive index profile is determined using a Fourier transform of the desired transmission produced from a discrete-layered thickness-modulated design.

**Subtractive fractional units** ($S^-$)  The number of fractional parts of a quarter-wave layer(s) that is subtracted from additive fractional units to produce a new quarter-wave layer, as part of a new quarter-wave stack (see fractional units).

**Thickness-modulated design (TMD)**  A quarter-wave stack that has the optical thickness (QWOT) of its layers modulated by a periodic function, for example, a sinusoidal function.

**Universal stopband equation (USE)**  A linear equation that is a function of modulation frequency, which predicts all possible stopbands for thickness-modulated designs.