

Field Guide to

# Polarization

Edward Collett

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## Introduction to the Series

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Welcome to the *SPIE Field Guides*! This volume is one of the first in a new series of publications written directly for the practicing engineer or scientist. Many textbooks and professional reference books cover optical principles and techniques in depth. The aim of the *SPIE Field Guides* is to distill this information, providing readers with a handy desk or briefcase reference that provides basic, essential information about optical principles, techniques, or phenomena, including definitions and descriptions, key equations, illustrations, application examples, design considerations, and additional resources. A significant effort will be made to provide a consistent notation and style between volumes in the series.

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## Field Guide to Polarized Light

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The polarization of light is one of the most remarkable phenomena in nature and has led to numerous discoveries and applications. Today it continues to play a vital role in optics. Before the 1950s there was very little activity on the foundations of polarized light. For example, answers to questions such as the nature and mathematical formulation of unpolarized light and partially polarized light were not readily forthcoming. Today there is a very good understanding of polarized light. In particular, the mathematical difficulties that had hindered complex polarization calculations were finally overcome with the introduction of the Mueller-Stokes matrix calculus and the Jones matrix calculus. Research in polarized light continues with much vigor as witnessed by the continued appearance of numerous publications and conferences.

The primary objective of this Guide is to provide an introduction to the developments in polarized light that have taken place over the past half-century. In this Guide I have tried to present the most salient topics on the subject. Hopefully, this Field Guide will enable the reader to have a good grasp of the material and most of all to allow him or her to be comfortable and even delighted with the beauty and subject of polarized light.

Finally, this Field Guide is dedicated to my wife, Mary Ann, and my children Ron and Greg. Their encouragement and support greatly simplified the task of writing this Guide.

Edward Collett  
Georgian Court University  
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## Glossary

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### Frequently used variables and symbols:

<b>B</b>	birefringence
<b><math>\mathbf{B}(\mathbf{r},t)</math></b>	magnetic induction vector
<i>c</i>	speed of light in a vacuum
<i>cp</i>	circularly polarized
<i>db</i>	decibels
<b><math>\mathbf{D}(\mathbf{r},t)</math></b>	electric displacement vector
<i>e-</i>	extraordinary ray
$\epsilon$	permittivity constant
$E_{0x}$	maximum amplitude in the <i>x</i> direction
$E_{0y}$	maximum amplitude in the <i>y</i> direction
$E_x(\mathbf{r},t)$	<i>x</i> component of the optical field
$E_y(\mathbf{r},t)$	<i>y</i> component of the optical field
<b>E</b>	Jones vector
<b><math>\mathbf{E}(\mathbf{r},t)</math></b>	electric field vector
<b>F</b>	force vector
$H_0$	Transmission of two parallel polarizers
$H_{90}$	Transmission of two crossed polarizers
<b><math>\mathbf{H}(\mathbf{r},t)</math></b>	magnetic field vector
<b>HWP</b>	half wave plate
<i>i</i>	angle of incidence
$i_B$	Brewster angle
<b><math>\mathbf{i},\mathbf{j},\mathbf{k}</math></b>	Cartesian unit vectors
<b>J</b>	Jones matrix
<b><math>\mathbf{j}(\mathbf{r},t)</math></b>	electric current density vector,
<b><math>\mathbf{J}_{POL}</math></b>	Jones matrix for a polarizer
<b><math>\mathbf{J}_{WP}</math></b>	Jones matrix for a wave plate
<b><math>\mathbf{J}_{ROT}</math></b>	Jones matrix for a rotator
<b><math>\mathbf{J}_{QWP}</math></b>	Jones matrix for a quarter-wave plate
<b><math>\mathbf{J}_{HWP}</math></b>	Jones matrix for a half-wave plate

## Glossary (cont'd)

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$\mathbf{J}(\theta)$	Jones matrix for a rotated polarizing element
$k$	wave number
$\mathbf{k}$	wave vector
$k_1$	major transmittance of a polarizer
$k_2$	minor transmittance of a polarizer
KDP	potassium dihydrogen phosphate
L-45P	linear -45 polarization
L+45P	linear +45 polarization
LCP	Left circular polarization
LHP	linear horizontal polarization
LVP	linear vertical polarization
$\mu$	permeability constant
$M$	Mueller matrix
$M_{\text{HWP}}$	Mueller matrix of a half-wave plate
$M_{\text{LP}}$	Mueller matrix of a linear polarizer
$M_{\text{POL}}$	Mueller matrix of a polarizer
$M_{\text{QWP}}$	Mueller matrix of a quarter-wave plate
$M_R$	Mueller matrix for reflection
$M_{\text{ROT}}$	Mueller matrix of a rotator
$M_T$	Mueller matrix for transmission
$M_{\text{WP}}$	Mueller matrix of a wave plate
$M(\theta)$	Mueller matrix of a rotated polarizing element
$\mathbf{n}$	complex refractive index
$n_e$	refractive index of the extraordinary ray
$n_o$	refractive index of the ordinary ray
$n_p$	parallel refractive index
$n_s$	perpendicular refractive index
$n_L$	levo-rotary refractive index

## Glossary (cont'd)

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$n_R$	dextro-rotary refractive index
$n$	refractive index
$o$ -	ordinary ray
OPS	observable polarization sphere
$p$ -	parallel polarization state
$p_x$	polarizer transmission coefficient ( $x$ )
$p_y$	polarizer transmission coefficient ( $y$ )
$\mathcal{P}$	degree of polarization
PBS	polarizing beam splitter
QWP	quarter wave plate
$r$	angle of refraction
$\mathbf{r}$	radius vector
RCP	right circular polarization
$s$ -	perpendicular polarization state
$S$	Stokes vector
$S_0$	first Stokes parameter
$S_1$	second Stokes parameter
$S_2$	third Stokes parameter
$S_3$	fourth Stokes parameter
$S_R$	Stokes vector for reflection
$S_T$	Stokes vector for transmission
TIR	total internal reflection
UNP	unpolarized
$\mathbf{v}(\mathbf{r},t)$	velocity vector
$v_x, v_y, v_z$	principal velocities
$V$	Verdet's constant
$V_\pi$	half-wave voltage
$V_m$	maximum modulation voltage
WP	wave plate
$x, y, z$	Cartesian coordinate system

## Glossary (cont'd)

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$2\alpha$	coordinate on the observable polarization sphere
$2\psi$	coordinate angle on the Poincaré sphere
$2\chi$	coordinate angle on the Poincaré sphere
$\alpha$	auxiliary angle
$\epsilon$	complex dielectric constant
$\epsilon_x, \epsilon_y, \epsilon_z$	principal dielectric constants
$\delta$	coordinate on the observable polarization sphere
$\delta$	phase difference
$\delta_x$	phase of the wave ( $x$ )
$\delta_y$	phase of the wave ( $y$ )
$\theta$	angle of rotation
$\kappa$	absorption coefficient
$\psi$	orientation angle
$\chi$	ellipticity angle
$\rho_{s,p}$	Fresnel reflection coefficients
$\rho(\mathbf{r},t)$	electric charge density
$\sigma$	conductivity
$\tau_{s,p}$	Fresnel transmission coefficients
$\phi$	phase shift
$\phi_x$	phase shift ( $x$ )
$\phi_y$	phase shift ( $y$ )
$\omega$	angular frequency
$\omega_c$	cyclotron frequency
$\omega_L$	Larmor's frequency
$\omega_m$	modulation frequency
$\nabla$	spatial vector operator