Coherent-Mode Representations in Optics

Andrey S. Ostrovsky
To the memory of my mother, Alla Burjinskaya Ostrovskaya, and my father, Sergey Ostrovsky.
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Preface

Everyone knows the fundamental role that the Fourier transform plays in optics, representing a monochromatic light field as a linear superposition of plane waves propagating in different directions. Perhaps, the coherent-mode representation of the optical field broached for the first time by H. Gamo in his *Matrix Treatment of Partial Coherence* (Progress in Optics III, E. Wolf, ed., North-Holland, Amsterdam, 1964), which was later developed by E. Wolf in his “New theory of partial coherence in the space-frequency domain” (*J. Opt. Soc. Am. A*, Vol. 72, No. 3, 1982, and Vol. 3, No. 1, 1986), plays a not less important role in contemporary optics. From a physical point of view, the coherent-mode representation describes an optical field of any state of coherence as a linear superposition of uncorrelated, completely coherent modes, a fact that gives new insight into the physics of generation, propagation, and transformation of optical radiation. From a mathematical standpoint, it expresses the cross-spectral density function of an optical field as a sum of terms that are separable in space, a fact that allows significant simplification of the analysis of statistical optical processes and systems. However, to my mind, the coherent-mode representation of optical fields, despite its power and attractiveness, has not yet found its due place in optical science and practice. This is affirmed, in particular, by a relatively small number of publications where the coherent-mode representation is treated. Even in a monumental treatise like *Optical Coherence and Quantum Optics* by L. Mandel and E. Wolf, less than two dozen pages are dedicated to this subject.

The present book represents a modest attempt to make up, to a certain extent, for a deficiency in possible applications of the coherent-mode representations in several areas of optics. This book is mainly based on the original results obtained by the author and his postgraduate students but, to ensure a thorough coverage of the total scope of the subject, it also contains some results of other authors, which are properly referenced. I tried to present this book in a brief recapitulative form, handy for both professionals and postgraduate students in physical optics. I hope that the book will be interesting for the reader and will stimulate the subsequent development of the coherent-mode representations in optics and their practical applications.

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Andrey S. Ostrovsky
Autonomous University of Puebla
Puebla, Mexico
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