

Wavefront Optics for Vision Correction

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视力矫正之波前光学

戴光明

For God so loved the world that he gave his only Son, that whoever believes in him should not perish but have eternal life.

John 3:16

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Preface

In the past three to four decades, adaptive optics has evolved from a technology solely for compensating atmospheric turbulence in astronomy to a promising technology with additional applications in the military, vision correction, and laser propagation. A key component of an adaptive optics system is the wavefront sensor, which measures the aberrations of an optical system in real time. Wavefront technology is thus a major component in the research and development of many optical system applications.

The application of wavefront technology in vision was pioneered in the early 1990s by Prof. Josef F. Bille and Dr. Junzhong Liang, then Bille's PhD student, at the University of Heidelberg. They built the world's first aberrometer for measuring the low- and high-order aberrations of human eyes. The application of wavefront sensing and adaptive optics in vision science expanded rapidly after Dr. Liang, with colleagues in Prof. David R. Williams' lab at the University of Rochester, built the world's first adaptive optics system for retinal imaging. At the same time, laser vision correction was growing rapidly. In the early years of the new century, several manufacturers had obtained FDA approval for wavefront-guided LASIK procedures in the US. Active research in ocular wavefront technology has expanded to include many ophthalmologists, optometrists, vision scientists, optical scientists, and refractive laser engineers.

Along with the development of the wavefront-driven refractive surgical techniques, a new field of research that studies the optics of ocular wavefronts with a consistent mathematical treatment has also emerged. The key to this mathematical approach is the use of orthonormal basis functions, namely Zernike polynomials. Other sets of orthonormal basis functions (e.g., Fourier series) and sets of nonorthonormal basis functions (e.g., Taylor monomials and Seidel series) can also be used. With this consistent mathematical approach in mind, I have arranged this book to answer the following questions: How is the ocular wavefront represented? How is it obtained with wavefront sensing and reconstruction? How are the coefficients of different representing basis functions converted to each other? How do the coefficients change when an ocular wavefront changes due to pupil constriction, cyclorotation, or decentration? How do the coefficients change when the wavefront propagates? How is the ocular wavefront evaluated? What is the clinical impact of the ocular wavefront?

Over the course of my research during the past 15 years, several individuals have

influenced me and deserve special appreciation. My first thanks go to Prof. Arne Arderberg, who introduced me to the field of astronomy and adaptive optics. Together with Prof. Mette Owner-Peterson, I enjoyed reading Robert J. Noll's famous paper on Zernike polynomials and atmospheric turbulence. His paper took me a long time to fully understand, but the result is my increased interest in mathematics and optics. I was lucky enough to work for Prof. David R. Williams at the University of Rochester and Dr. Junzhong Liang at Visx (now Advanced Medical Optics)—two important figures in the development of adaptive optics in vision science. During the past couple of years, I have had a very fruitful collaboration with Dr. Virendra N. Mahajan during our spare time. Our published papers on orthonormal polynomials lay the foundation for the representation of ocular aberrations.

This book would not have been possible without the generous support of my employer, Advanced Medical Optics (AMO). I am very grateful to Leonard Borrmann, Tom Shoup, and Carol Harner for their executive support. During my employment with AMO and while writing this book, I have enjoyed working with some of the world's leading refractive surgeons: Noel Alpins, Eric Donnenfeld, Ken Greenberg, Jack Holladay, Bruce Jackson, Douglas Koch, Martin Mainster, Marguerite McDonald, Marc Odrich, Luis Ruiz, Steve Schallhorn, Kerry Solomon, Julian Stevens, Gustavo Tamayo, and Steve Trokel. I am indebted to Profs. David A. Atchison and Jim Schwiegerling for reviewing the entire manuscript on a tight schedule and for their valuable suggestions. Drs. Linda Lundström and Sverker Norrby carefully read every chapter and provided me with detailed comments. Many of my colleagues at AMO provided helpful comments and suggestions. Of course, I am responsible for any error or omissions still remaining in the book. It has been a pleasure to work with the SPIE Press staff, in particular Scott Schrum and Tim Lamkins, due to their professionalism and enjoyable cooperation.

Last but not least, I am extremely grateful to my lovely family—my wife Wendy and my sons Percy and Perry—for their understanding and support during my career and, in particular, during the nights and weekends over the past year while this book was being written.

Guang-ming Dai
Fremont, California
December, 2007

Symbols, Notations, and Abbreviations

A	ablation zone diameter	$a(k, \phi)$	Fourier coefficients
$a(u, v)$	direction factor	b	boundary factor
b_i	Taylor coefficients	C	cylinder power
C	conversion matrix	$C(f)$	contrast sensitivity function
c_i	Zernike coefficients	D	wavefront diameter
d	vertex distance	ϵ	pupil resizing ratio
$\{F_i\}$	orthonormal polynomials	F_u^v	Fourier series
f	focal length	ϕ	aberration function
ϕ	cylinder axis	g_i	elliptical coefficients
$\mathcal{G}_n^i(\epsilon)$	Zernike resizing polynomials	h	optical transfer function
H	modulation transfer function	i	single index of polynomials
$i(\alpha, \phi)$	point spread function	j	imaginary symbol ($\sqrt{-1}$)
(k, ϕ)	polar coordinate in frequency domain	l	optical path length
$\mathcal{L}_p(\epsilon)$	Taylor resizing monomials	l	ablation depth
M	conversion matrix for orthonormal polynomials	$L(\alpha)$	encircled energy
n	refractive index of stroma	m	Zernike azimuthal frequency
P	wavefront power	n	Zernike radial order
ψ	wavefront slope angle	O	optical zone diameter
\mathcal{Q}	Fourier transform of Taylor monomials	p	Taylor radial order
(r, θ)	polar coordinates in spatial domain	q	Taylor azimuthal frequency
R_{mar}	visual acuity in LogMAR	R	Wavefront radius
R_1	corneal radius of curvature	\mathcal{R}	Zernike radial polynomials
S	Strehl ratio	(ρ, θ)	normalized polar coordinates in spatial domain
S_n^m	Seidel series	R_f	visual acuity in fraction
σ	wavefront RMS	S	sphere power
		S_v	visual Strehl ratio
		SE	spherical equivalent
		σ^2	wavefront variance

T_p^q	Taylor monomials	t	lens thickness
Θ	triangular functions	U_i	Fourier transform of Zernike polynomials
V_i	conjugate Fourier transform of Zernike polynomials	$V(f_1 : f_2)$	MTF volume
u, v	Fourier double-index	(u, v)	normalized Cartesian coordinates
Δu	normalized x shift	Δv	normalized y shift
W	ocular wavefront	WRx	wavefront refraction
(x, y)	Cartesian coordinates	Z_n^m	Zernike polynomials
$\langle \cdot \cdot \rangle$	inner product of functions	\otimes	convolution

AO	adaptive optics
BSCVA	best spectacle corrected visual acuity
CCD	charge-coupled device
CMTF	compound modulation transfer function
CSF	contrast sensitivity function
FDA	United States Food and Drug Administration
FFT	fast Fourier transform
FSS	fixed-size scanning
FWHM	full width at half maximum
LASIK	laser-assisted <i>in situ</i> keratomileusis
MRSE	manifest refraction in spherical equivalent
MTF	modulation transfer function
OPD	optical path difference
OTF	optical transfer function
PMMA	polymethyl methacrylate
PRK	photorefractive keratectomy
PSF	point spread function
PV	peak-to-valley
RMS	root mean square
SSED	summary for safety and effectiveness data
SVD	singular value decomposition
UCVA	uncorrected visual acuity
VSS	variable spot scanning