

# **Polarization**

of Light with Applications in

# **Optical Fibers**

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# **Polarization** of Light with Applications in **Optical Fibers**

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## Introduction to the Series

Since its inception in 1989, the Tutorial Texts (TT) series has grown to cover many diverse fields of science and engineering. The initial idea for the series was to make material presented in SPIE short courses available to those who could not attend and to provide a reference text for those who could. Thus, many of the texts in this series are generated by augmenting course notes with descriptive text that further illuminates the subject. In this way, the TT becomes an excellent stand-alone reference that finds a much wider audience than only short course attendees.

Tutorial Texts have grown in popularity and in the scope of material covered since 1989. They no longer necessarily stem from short courses; rather, they are often generated independently by experts in the field. They are popular because they provide a ready reference to those wishing to learn about emerging technologies or the latest information within their field. The topics within the series have grown from the initial areas of geometrical optics, optical detectors, and image processing to include the emerging fields of nanotechnology, biomedical optics, fiber optics, and laser technologies. Authors contributing to the TT series are instructed to provide introductory material so that those new to the field may use the book as a starting point to get a basic grasp of the material. It is hoped that some readers may develop sufficient interest to take a short course by the author or pursue further research in more advanced books to delve deeper into the subject.

The books in this series are distinguished from other technical monographs and textbooks in the way in which the material is presented. In keeping with the tutorial nature of the series, there is an emphasis on the use of graphical and illustrative material to better elucidate basic and advanced concepts. There is also heavy use of tabular reference data and numerous examples to further explain the concepts presented. The publishing time for the books is kept to a minimum so that the books will be as timely and up-to-date as possible. Furthermore, these introductory books are competitively priced compared to more traditional books on the same subject.

When a proposal for a text is received, each proposal is evaluated to determine the relevance of the proposed topic. This initial reviewing process has been very helpful to authors in identifying, early in the writing process, the need for additional material or other changes in approach that would serve to strengthen the text. Once a manuscript is completed, it is peer reviewed to ensure that chapters communicate accurately the essential ingredients of the science and technologies under discussion.

It is my goal to maintain the style and quality of books in the series and to further expand the topic areas to include new emerging fields as they become of interest to our reading audience.

*James A. Harrington  
Rutgers University*

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# Preface

The polarization phenomena associated with light waves are extremely important in many areas of photonics. Many important polarization-based devices have been developed, including Faraday rotators; in-line fiber optic components, such as polarizers, wave plates, and polarization controllers; and sensors, such as fiber optic current sensors, and fiber optic gyroscopes. In order to understand the workings of such photonic devices and to improve their design, sound knowledge of the basic concepts involved in polarization is required.

Furthermore, in optical communication systems, polarization mode dispersion has become an extremely important issue, particularly for very high-bit-rate ( $>10$  Gb) systems. Polarization mode dispersion arises because of random birefringence present in a practical optical fiber. The birefringence that causes polarization mode dispersion in optical fibers may be linear, circular, or, in general, elliptical. In order to understand the nature of polarization mode dispersion and to control or reduce it, one must know how the various types of birefringent media affect the polarization state of the guided light while it propagates through an optical fiber. Thus, it has become almost essential for most engineers (working in the general area of photonics) to have a basic knowledge of the polarization phenomena and associated concepts, as well as the basic methods of analysis, such as Jones matrices, Stokes parameters, and the Poincaré sphere.

In this book, our aim is to provide in one source all of the basic concepts and methods involved in the understanding and design of various photonic devices, keeping the discussions as succinct as possible. The Poincaré sphere representation of polarized light is a very important method that is not discussed in sufficient detail in most of the literature. Therefore, we have devoted an entire chapter to Poincaré sphere representation, including several numerical examples to make the method very clear. This book works through all steps using many examples; therefore, even undergraduate students should be able to follow along without much difficulty.

We have been teaching various aspects of polarization to our undergraduate students and to our master's students at the Indian Institute of Technology Delhi (IITD). This book has grown out of the lecture notes that we have prepared over the last 25 years. We have also used this material in several short courses organized at IITD and at other institutions.

We thank our colleagues in the physics department of IITD for many helpful discussions—in particular, we thank Profs. B. D. Gupta, B. P. Pal, A. Sharma, M. R. Shenoy, and K. Thyagarajan, and Dr. Ravi Varshney for research collab-

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We dedicate this book to the memory of our parents.

*Arun Kumar  
Ajoy Ghatak  
New Delhi  
January 2011*

# Acronyms and Abbreviations

ADP	ammonium dihydrogen phosphate
CP	circularly polarized
CT	cross talk
DGD	differential group delay
DOP	degree of polarization
DSF	dispersion-shifted fiber
DUT	device under test
EM	electromagnetic
EP	elliptically polarized
Hi-Bi	high-birefringence (fiber)
HWP	half-wave plate
JME	Jones matrix eigenanalysis
KDP	potassium dihydrogen phosphate
LA	linear analyzer
LCP	left-circularly polarized
LD	laser diode
LED	light-emitting diode
LEP	left-elliptically polarized
LHP	linear horizontal polarizer
LP	linearly polarized
LVP	linear vertical polarizer
MFD	mode field diameter
MMM	Mueller matrix method
NZDSF	nonzero-dispersion-shifted fiber
OTDR	optical time-domain reflectometer
PANDA	polarizing-maintaining and absorption-reducing (fiber)
PC	polarization controller
PMD	polarization mode dispersion
PMF	polarization-maintaining fiber
PS	Poincaré sphere
PSA	Poincaré sphere analysis
PSP	principal states of polarization
QWP	quarter-wave plate
RCP	right-circularly polarized
REP	right-elliptically polarized
rms	root-mean-square

SAP	stress-applying part
SIRF	spin-induced reduction factor
SMF	single-mode fiber
SOP	state of polarization
SP	surface plasmon
SPP	surface plasmon polariton
SPSM	single-polarization single-mode (fiber)
SSMF	standard single-mode fiber
TE	transverse electric
TGG	terbium gallium garnet
TM	transverse magnetic
TMEF	two-mode elliptic-core fiber
TMF	two-mode fiber
UP	unpolarized
YIG	yttrium iron garnet