Applied
Prismatic and
Reflective Optics

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Preface

This text deals primarily with the optics of refracting and reflecting planar surfaces in the form of prismatic refracting and reflecting components, and the design, analysis, and applications of these components. Optical prisms consist of multiple planar surfaces, constructed to a specified geometry and formed from optical glass or plastic. The surfaces may have thin-film coatings that contribute to their functionality. Optical prismatic elements can be classified into two general types: those that are used in imaging systems, such as binoculars or projectors, and those used in nonimaging systems, such as spectrometers, illuminators, and solar concentrators. In addition to well-known prism systems, new applications of prisms are being introduced in the fields of electro-optics, metrology, prismatic films and arrays, projection displays, and others.

Chapter 1 introduces and reviews the optical concepts that are useful for the topics developed in the succeeding chapters. In Chapter 2, some better-known prism types are discussed, along with the essential ray-trace equations that define their specific properties. This includes both single and compound prisms, along with cube-corner retroreflectors. Birefringent prisms and polarizing beam-splitting prisms that produce polarized light are discussed in Chapter 3, including prisms that affect the polarization state of light, such as polarization-preserving prisms and prisms that rotate the plane of polarization. Prisms with collinear and coaxial dispersion properties, achromatic multiprisms, and anamorphic designs for beam expansion and compression are examined in Chapter 4. In Chapter 5, several methods of prism design are reviewed, including some of the more recent methodologies. This chapter also covers prism fabrication, tolerancing, choice of optical material, and some mounting methods. Specific uses of prisms in optical systems, such as scanning, beam steering, spectroscopy, interferometry, light coupling and switching, and viewing and illumination are presented in Chapter 6. Chapter 7 covers the use of prisms as dichroic color beamsplitters and combiners, polarizers, and light-beam homogenizers and integrators in projection displays. Microprism arrays are very useful for light guides, luminaires, brightness-enhancement sheets, backlight displays, and sheet polarizers. These applications are detailed in Chapter 8. Last, Chapter 9 covers Fresnel lens optics and the use of both refractive and reflective lenses in illumination, solar concentration, and direct-view displays. Several design methods for producing achromatic and phase-corrected Fresnel lenses are also presented.

References and examples are drawn from specialized texts, journal articles, conference proceedings, trade publications, and patent literature. I wish to acknowledge the editorial assistance of Gwen Weerts of SPIE Press for her suggestions and contributions during the composition of this book.

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