OPTICAL IMAGING AND ABERRATIONS

PART II

WAVE DIFFRACTION OPTICS

SECOND EDITION

OPTICAL IMAGING AND ABERRATIONS

PART II

WAVE DIFFRACTION OPTICS

SECOND EDITION

Virendra N. Mahajan

SPIE PRESS

Bellingham, Washington USA

Library of Congress Cataloging-in-Publication Data

Mahajan, Virendra N.
Optical imaging and aberrations / Virendra N. Mahajan. -- 2nd ed.
v. cm. -- (Press monograph ; 209)
Contents: -- pt. 2. Wave diffraction optics.
Includes bibliographical references and index.
ISBN 978-0-8194-8699-8 (pt. 2)
1. Aberration. 2. Imaging systems. 3. Geometrical optics. I. Title.
QC671.M36 2011
621.36--dc23

2011020047

Published by
SPIE
P.O. Box 10
Bellingham, Washington 98227-0010 USA
Phone: +1 360.676.3290
Fax: +1 360.647.1445
Email: Books@spie.org
Web: http://spie.org

Printed in the United States of America.
First printing

SPIE

In loving memory of my parents:

mother Shrimati **Sushila Devi**

and father Shri **Ram Chand**

FOREWORD TO THE FIRST EDITION

Three years ago Vini Mahajan published a book on the subject of Geometrical Images in the presence of aberrations. In this book, Mahajan extends this work to include the effect of wave optics. He continues his thorough tutorial on image formation with a detailed look at the approaches to calculating the form of images. Anyone interested in understanding the methods of predicting the light distribution to be expected in real imaging situations will find this book of interest.

The book begins with an exhaustive development of the basics of diffraction image formation. Mahajan covers the issues associated with the calculation of point-spread functions and discusses the accuracy of such calculations. He introduces the Optical Transfer Function as the Transform of the Point Spread Function and reviews the procedures involved in calculating the OTF. Asymptotic and approximate evaluations of the OTF are included, as are several examples throughout the book.

These approaches are then applied to some real examples of circular and annular apertures. In this discussion, Mahajan carries out in detail many of the classical computations for various image descriptors. This is a topic that is generally treated only lightly in most texts on the subject. Such issues as edge response and line spread function and encircled power are carefully considered. A good discussion of optimal balancing of aberrations is also provided. The treatment of aberration balancing and tolerances in annular pupils is unique in its completeness in this book.

Mahajan includes a detailed discussion of the effect of imaging with a Gaussian weighted aperture, a topic of much interest today. Finally the always-interesting subject of imaging through turbulence is discussed for both circular and annular pupils.

The completeness of the discussions regarding the calculation of image structure is thorough and detailed in this book. Mahajan includes in his discussion many of the "classical" computations of image quality functions that form the basis of the field today, but are seldom encountered in most texts on the subject. The student will have the opportunity to learn the details and limitations of the process. The experienced worker will find this volume useful as a reference in carrying out diffraction image calculations in a wide range of optical systems.

Tucson, Arizona R. R. Shannon
April 2001

TABLE OF CONTENTS

PART II. WAVE DIFFRACTION OPTICS

PREFACE TO THE FIRST EDITION

In Part I of this book, we discussed imaging based on ray geometrical optics. The aberration-free image of an object according to it is the exact replica of the object, except for its magnification. The aberration-free image of a point object is also a point. In reality, however, the image obtained is not a point. Because of diffraction of the object wave at the aperture stop, or equivalently, the exit pupil of the imaging system, the actual aberration-free image for a circular exit pupil is a light patch surrounded by dark and light rings. Geometrical optics is still assumed to hold from the object to the exit pupil in that rays are traced through the system to determine the shape of the pupil and the aberration across it. The determination of the characteristics of the diffraction image of an object formed by an aberrated system is the subject discussed in Part II. The emphasis of this part is on the numerical results on the effects of aberrations on a diffraction image and not on the formalism, exposition, or a critique of the variety of diffraction theories proposed over the years. It is a compilation of the works of masters like Nijboer, Hopkins, Barakat, and Fried with a sprinkling of my own work.

In Chapter 1, the diffraction theory of image formation is discussed. Starting with a brief account of the Rayleigh-Sommerfeld theory from a Fourier-transform standpoint, we derive the Huygens-Fresnel principle from it. We show that the diffraction image of a point object, called the point-spread function (PSF), is proportional to the modulus square of the Fourier transform of the complex amplitude across the exit pupil. It is shown that the diffraction image of an isoplanatic incoherent object is equal to the convolution of its Gaussian image and the diffraction PSF. The optical transfer function (OTF) of an imaging system, which is the Fourier transform of its PSF, is also discussed. The spatial frequency spectrum of the image of an isoplanatic object is shown to be equal to the product of the spectrum of the object and the OTF. The OTF based on geometrical optics is also considered, and an approximate expression valid for low spatial frequencies is given. A brief comparison of imaging based on diffraction and geometrical optics is given in terms of both the PSF and the OTF. The line of sight of a system is identified with the centroid of its PSF, which, in turn, is obtained in terms of the OTF or the aberration function. The line- and edge-spread functions are introduced and obtained in terms of the PSF or the OTF. Expressions for Strehl, Hopkins, and Struve ratios, useful for obtaining aberration tolerances, are derived. Finally, imaging of coherently illuminated objects is discussed. It is shown that the image of an isoplanatic coherent object is equal to the convolution of its Gaussian amplitude image and the coherent PSF of the imaging system. This chapter forms the foundation of Part II in that the fundamental relations derived in it, and stated as theorems, are used in the succeeding chapters to obtain some practical results for imaging systems with circular, annular, and Gaussian pupils. However, a reader need not read all of this chapter before attempting to read others.

Chapter 2 on systems with circular pupils starts with the aberration-free PSF and its encircled and ensquared powers. The effects of primary aberrations on its Strehl ratio are discussed, and aberration tolerances are obtained. Balanced primary aberrations are considered and identified with Zernike circle polynomials. Focused and collimated beams are discussed, and the concept of near- and far-field distances is introduced. Aberrated PSFs and their symmetry properties are discussed, and a brief comparison is made with their counterparts in geometrical optics. The line of sight of an aberrated system, identified with the centroid of the PSF, is determined for primary aberrations. The OTF for these aberrations is also discussed, phase contrast reversal is explained, and aberration tolerances for a certain value of Hopkins ratio are given. Expressions for the geometrical OTF for primary aberrations are also given. Both incoherent and coherent line- and edge-spread functions are discussed, and aberration tolerances for a certain value of the Struve ratio are given. A brief comparison of incoherent and coherent imaging is given with special reference to the Rayleigh criterion of resolution. The Fourier-transforming property of wave propagation is illustrated in altering the image of an object by spatial filtering in the Fourier-transform plane.

Systems with annular pupils are given a cursory look at best in books where imaging is discussed. Our Chapter 3 is written in a manner similar to Chapter 2 where, for example, the balanced aberrations are identified with the Zernike annular polynomials. Although the propagation of Gaussian beams is discussed in books on lasers, their treatment is generally limited to the weakly truncated aberration-free beams. In Chapter 4, we consider the effect of arbitrary truncation of beams with and without aberrations. The balanced aberrations in this case are identified with the Zernike-Gauss polynomials. It is shown, for example, that the pupil radius must be at least three times the beam radius in order to neglect beam truncation without significant error.

Finally, random aberrations are considered in Chapter 5. The effect of random image motion is considered first, and expressions and numerical values of time-averaged Strehl ratio, PSF, and encircled power are given for systems with circular and annular pupils. The random aberrations introduced by atmospheric turbulence when a wave propagates through it, as in astronomical observations by a ground-based telescope, are discussed; and expressions for time-averaged PSF and OTF are obtained. The aberration function for Kolmogorov turbulence is expanded in terms of the Zernike polynomials, and auto-correlation and cross-correlation of the expansion coefficients are given. The atmospheric coherence length is defined, and it is shown that the resolution of a telescope cannot exceed that of an aberration-free telescope of this diameter. Both the short- and long-exposure images are considered.

As in Part I, each chapter ends with a set of problems. It is hoped that they will acquaint the reader with application of the theory in terms of some practical examples. References for addtional reading are given after the Bibliography. These references represent the author's collection as the editor of Milestone Series 74 entitled *Effects of Aberrations in Optical Imaging*, published by SPIE Press in 1993.

ACKNOWLEDGMENTS

It is a great pleasure to acknowledge the generous support I have received over the years from my employer, The Aerospace Corporation, in preparing this book. My special thanks go to the senior vice president Mr. John Parsons, for his continuous interest and encouragement in this endeavor. I thank Mr. John Hoyem for preparing the figures and Mr. Victor Onouye for some figures as well as the final composition. My thanks also go to Dr. Rich Boucher for computer generating the 2D PSFs and Mr. Yunsong Huang for numerical analysis and computer plotting. The Sanskrit verse on p. xxv was provided by Professor Sally Sutherland of the University of California at Berkeley.

I am grateful to Dr. P. Mouroulis and Professor Emeritus D. Schroeder for their careful review of the manuscript. I also benefited from the comments made by many friends and colleagues over the years. Included among them are Dr. Bill Swantner, Professor A. Walther, Professor A. Marathay, and Professor J. Harvey. Of course, any shortcomings or errors in the book are my responsibility. I also thank Professor R. R. Shannon for writing the Foreword to both Parts of this book.

I thank my sister Pushpa and brother Devinder for encouraging me to study phyiscs. I can not say enough about the constant support I have received from my wife Shashi over the many years it has taken me to complete this two-part book. I dedicate this book to my departed parents who brought and nourished me in this world.

Finally, I would like to thank SPIE Press Senior Editor Dara Burrows for meticulously editing the book and SPIE Press Manager Tim Lamkins for facilitating the publication of this Second Edition. It has been a pleasure to work with them, especially because of their cooperative spirit and quality support.

El Segundo, California　　　　Virendra N. Mahajan
April 2011

PREFACE TO THE SECOND EDITION

Ten years have passed since the publication of the first edition in April 2001. Many of the typographical errors were corrected in the Second Printing that took place after 3 years in 2004. Only a small amount of new material, approximately 11 pages, was added at that time. It included Appendix B in Chapter 1, Gaussian OTF in Chapter 4, and an extension of the discussion of the short-exposure image in Chapter 5. Additional corrections were made in the e-version of the book in 2009, and the discussion of Zernike circle, annular, and Gauss polynomials was streamlined utilizing an abbreviated notation with emphasis on their orthonormal form. However, a considerable amount of new material amounting to another 88 pages has been added in this Second Edition. Besides correction of some residual typographical errors, a Summary section has been included in Chapters 2, 3, and 5 for consistency with Chapters 1 and 4. Any compound references have been split into single ones. The new material is primarily in Chapters 4 and 5.

In Chapter 4, a Gaussian pupil obtained by apodization is compared with that of a Gaussian beam. The optimum value of the Gaussian radius relative to that of the pupil to yield the maximum focal-point irradiance is derived. The discussion of the standard deviation, aberration balancing, and Strehl ratio for primary aberrations for different values of the ratio of the beam and pupil radii has been expanded. It is shown that the approximate expression for Strehl ratio in terms of the aberration variance is not suitable for very narrow Gaussian beams. The aberration-free OTF has been extended to that of a defocused system. The problem of balancing defocus aberration with spherical aberration or astigmatism is discussed, illustrating that an aberrated beam can yield a higher axial irradiance in a certain defocused region than its aberration-free focal-point value. The PSFs aberrated by spherical aberration are considered to illustrate the loss of advantage of the Gaussian pupil in reducing the secondary maxima when the aberration is present. The characterization of the width of a multimode beam compared to that of a Gaussian beam is discussed briefly.

In Chapter 5, the effect of random transverse image motion on the PSF has been supplanted by a discussion of the effect of random longitudinal defocus. The coherence length of atmospheric turbulence is calculated for the Hufnagel-Valley model of the refractive index structure parameter, for both looking up and down through the atmosphere. The angle of arrival of the light wave propagating through turbulence is discussed for both the Zernike tilt as well as the centroid of the PSF, showing that they are nearly equal. A brief discussion of lucky imaging is also given, where better quality short-exposure images are selected, aligned, and added to obtain a high-quality image.

El Segundo, California — Virendra N. Mahajan
April 2011

SYMBOLS AND NOTATION

a	pupil radius
a_i	aberration coefficient
A	amplitude
A_i	peak aberration coefficient
B_d	defocus coefficient
B_t	tilt coefficient
D	pupil diameter
$\mathcal{D}$	structure function
F	focal ratio
ESF	edge transfer function
H	Hopkins ratio
I	irradiance
Im	imaginary part
k	wavenumber
l	distance, log amplitude
LSF	line spread function
m_p	pupil magnification
M	magnification
MCF	mutual coherence function
MTF	modulation transfer function
OTF	optical transfer function
P	object point
P'	Gaussian image point
P_{ex}	power in the exit pupil
P_i	image power
$P(\cdot)$	pupil function
PSF	point-spread function
PTF	phase transfer function
r	radial coordinate
r_c	radius of circle
r_0	atmospheric coherence diameter
$\vec{r}_i$	image point position vector
$\vec{r}_p$	pupil point position vector
R	radius of reference sphere
Re	real part
$R_n^m(\rho)$	Zernike radial polynomial
S	Strehl ratio
S_{en}	area of entrance pupil
U	complex amplitude
S_{ex}	area of exit pupil
W	wave aberration
$x,\ y$	rectangular coordinates of a point
z	optical axis, axial distance
$Z_n^m(\rho,\theta)$	Zernike circle polynomial
$\vec{v}_i$	image spatial frequency vector
$\vec{v}_o$	object spatial frequency vector
v	normalized spatial frequency
τ	optical transfer function
Ψ	phase transfer function
$\rho = r/a$	normalized radial coordinate
θ	polar angle of a position vector
ϕ	polar angle of frequency vector
ϵ	obscuration ratio
$\delta(\cdot)$	Dirac delta function
η	relative irradinace
Φ	phase aberration
r,θ	polar coordinates of a point
λ	optical wavelength
ξ,η	normalized frequency coordinates
σ_W	standard deviation of wave aberration

अनन्तरत्नप्रभवस्य यस्य हिमं न सौभाग्यविलोपि जातम् ।
एको हि दोषो गुणसन्निपाते निमज्जतीन्दोः किरणेष्विवाङ्कः ॥

Anantaratnaprabhavasya yasya himaṃ na saubhāgyavilopi jātam |
Eko hi doṣo gūṇasannipāte nimajjatīndoḥ kiraṇesvivāṅkaḥ ‖

The snow does not diminish the beauty of the Himālayan mountains which are the source of countless gems. Indeed, one flaw is lost among a host of virtues, as the moon's dark spot is lost among its rays.

Kālidāsa *Kumārasambhava* 1.3