Appendix A
Mathematical Notations

A.1 Definition of Nicknamed Functions

Circle function:

\[
\text{circ} \left( \frac{r}{D/2} \right) = \text{circ} \left( \frac{\sqrt{x^2 + y^2}}{D/2} \right) = \begin{cases} 
1 & r \leq D/2 \\
0 & r > D/2.
\end{cases} \tag{A.1}
\]

Cusp function:

\[
\text{cusp} \left( \frac{r}{R} \right) = \begin{cases} 
\frac{2}{\pi} \left[ \text{acs} \left( \frac{r}{R} \right) - \frac{r}{R} \sqrt{1 - \left( \frac{r}{R} \right)^2} \right] & \left| \frac{r}{R} \right| \leq 1 \\
0 & \left| \frac{r}{R} \right| > 1.
\end{cases} \tag{A.2}
\]

Rectangle function:

\[
\text{rect} \left( \frac{x}{W/2} \right) = \begin{cases} 
1 & x \leq W/2 \\
0 & x > W/2.
\end{cases} \tag{A.3}
\]

Sinc function:

\[
\text{sinc}(x) = \frac{\sin(x)}{x}. \tag{A.4}
\]

Sombrero function:

\[
\text{somb}(r) = \frac{2J_1(r)}{r}, \tag{A.5}
\]

where \( J_1(x) \) is the Bessel function of the first kind, first order.

Struve function:

\[
H_1(x) = \sum_{m=0}^{\infty} (-1)^m \left( \frac{x}{2} \right)^{2(m+1)} \frac{(m+\frac{3}{2})!}{\Gamma \left( m + \frac{3}{2} \right) \Gamma \left( m + \frac{5}{2} \right)}, \tag{A.6}
\]

where \( \Gamma(\cdot) \) is the Gamma function.
A.2 Definition of Functional Operators

Convolution:

\[ \text{conv}[f(x), g(x)] = \int_{-\infty}^{\infty} f(y) \cdot g(x - y) \cdot dy \equiv \text{conv}[g(x), f(x)]. \quad (A.7) \]

Correlation:

\[ \text{corr}[f(x), g(x)] = \int_{-\infty}^{\infty} f^*(y) \cdot g(x + y) \cdot dy. \quad (A.8) \]

Fourier transform:

\[ \mathcal{F}\{f(x)\} = \int_{-\infty}^{+\infty} f(x)e^{-2\pi j \xi x} dx = F(\xi). \quad (A.9) \]

Inverse Fourier transform:

\[ \mathcal{F}^{-1}\{F(\xi)\} = \int_{-\infty}^{\infty} F(\xi)e^{2\pi j \xi x} d\xi = f(x). \quad (A.10) \]

Hankel transform:

\[ \mathcal{H}\{f(r)\} = 2\pi \int_{0}^{\infty} r \cdot f(r) \cdot J_0(2\pi \rho r)dr = F(\rho), \quad (A.11) \]

where \( J_0(x) \) is the Bessel function of the first kind, zero order.

Inverse Hankel transform:

\[ \mathcal{H}^{-1}\{F(\rho)\} = 2\pi \int_{0}^{\infty} \rho \cdot f(\rho) \cdot J_0(2\pi \rho r)d\rho \equiv \mathcal{H}\{F(\rho)\}. \quad (A.12) \]
Appendix B
Herzberger Dispersion Formula

The basic form of the Herzberger formula is

\[ n(\lambda) = \gamma_1 + \gamma_2 \lambda^2 + \frac{\gamma_3}{\lambda^2 - \lambda_0^2} + \frac{\gamma_4}{(\lambda^2 - \lambda_0^2)^2}. \]  \hspace{1cm} (B.1)

For any dispersive medium, the four coefficients \( \gamma_1 \) through \( \gamma_4 \) have to be determined through the best fit of experimental refractive index data. Equivalently, this relationship can be cast in the form

\[ n(\lambda) = a_1(\lambda)n(\lambda_1) + a_2(\lambda)n(\lambda_2) + a_3(\lambda)n(\lambda_3) + a_4(\lambda)n(\lambda_4), \]  \hspace{1cm} (B.2)

with the four coefficients \( a_i(\lambda) \) given by

\[ a_i(\lambda) = \gamma_{1i} + \gamma_{2i} \lambda^2 + \frac{\gamma_{3i}}{\lambda^2 - \lambda_0^2} + \frac{\gamma_{4i}}{(\lambda^2 - \lambda_0^2)^2}. \]

In matrix form:

\[ a_i(\lambda) = \Delta \cdot \gamma_i = \begin{vmatrix} 1 & \frac{1}{\lambda^2 - \lambda_0^2} & \frac{1}{(\lambda^2 - \lambda_0^2)^2} \end{vmatrix} \cdot \begin{pmatrix} \gamma_{1i} \\ \gamma_{2i} \\ \gamma_{3i} \\ \gamma_{4i} \end{pmatrix}, \]

and

\[ n(\lambda) = \Delta \cdot \gamma \cdot n = \begin{vmatrix} 1 & \frac{1}{\lambda^2 - \lambda_0^2} & \frac{1}{(\lambda^2 - \lambda_0^2)^2} \end{vmatrix} \cdot \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} \end{pmatrix} \cdot \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{pmatrix}. \]
In particular, Eq. (B.2) must be verified at each wavelength \( \lambda_i, i = 1 \) to 4, which implies \( a_i(\lambda_i) = 1; a_j(\lambda_i)|_{j \neq i} \equiv 0 \) in Eq. (B.2). This translates to

\[
\begin{pmatrix}
1 & \frac{\lambda_1^2}{\lambda_1^2 - \lambda_0^2} & \frac{1}{(\lambda_1^2 - \lambda_0^2)^2} \\
1 & \frac{\lambda_2^2}{\lambda_2^2 - \lambda_0^2} & \frac{1}{(\lambda_2^2 - \lambda_0^2)^2} \\
1 & \frac{\lambda_3^2}{\lambda_3^2 - \lambda_0^2} & \frac{1}{(\lambda_3^2 - \lambda_0^2)^2} \\
1 & \frac{\lambda_4^2}{\lambda_4^2 - \lambda_0^2} & \frac{1}{(\lambda_4^2 - \lambda_0^2)^2}
\end{pmatrix}
\begin{pmatrix}
\gamma_{i1} \\
\gamma_{i2} \\
\gamma_{i3} \\
\gamma_{i4}
\end{pmatrix}
= \begin{pmatrix}
\delta_{i,1} \\
\delta_{i,2} \\
\delta_{i,3} \\
\delta_{i,4}
\end{pmatrix}, \quad (B.3)
\]

where \( \delta_{ji} \) = Kronecker's delta.

Solutions of the four linear systems in Eq. (B.3) for \( i = 1 \) to 4 give the 16 coefficients \( \gamma_{ij} \). Choosing \( \lambda_2^2 = 0.028 \) \( \mu \) \( \text{m}^2; \lambda_1 = \lambda_4 = 0.3650146 \) \( \mu \) \( \text{m}; \lambda_3 = \lambda_F = 0.4861327 \) \( \mu \); \( \lambda_C = 0.6562725 \) \( \mu \); and \( \lambda_4 = \lambda_t = 1.01398 \) \( \mu \); results in

\[
\gamma = \begin{pmatrix}
0.66149637 & -4.20170826 & 6.29866119 & -1.75844930 \\
-0.40355469 & 2.73533632 & -4.69448133 & 2.36269971 \\
-0.28047241 & 1.50549063 & -1.57515162 & 0.35013340 \\
0.03385993 & -0.11593535 & 0.10293414 & -0.02085872
\end{pmatrix}. \quad (B.4)
\]

These values coincide with those of Herzberger to the third or fourth decimal figure, with discrepancies arising primarily because of the finer definition of wavelengths \( \lambda_1 \) to \( \lambda_4 \). They are reported here to overcome a few misprints contained in the paper by Navarro, Santamaría, and Bescós (see Refs. 3,4).

### B.1 References

2. Schott Glaswerke, Optical Glass Technical Catalog, Mainz, Germany.
Appendix C

Determination Coefficient $R^2$

Given $n$ couples of observations $(x_i, y_i)$ and the model function $f = f(x, a)$, where $a$ is a set of $m$ unknown parameters, the fitting problem requires minimization of the discrepancy between observed data $y_i$ and values $f_i = f(x_i, a)$ predicted under the model. According to the method of maximum likelihood, through the assumption of a Gaussian parent distribution determining the probability of making any particular observation, the quantity to be minimized is the sum of squares of residuals:

$$SSq_{res} = \sum_{i=1}^{n} (y_i - f_i)^2. \quad (C.1)$$

The degree of variability or dispersion of the observations is accounted for by the total sum of squares (or deviance):

$$SSq_{tot} = \sum_{i=1}^{n} (y_i - \bar{y})^2 \quad (C.2)$$

where $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$ is the mean of the observed data, and $SSq_{tot}$ is proportional to the sample variance. The equivalent of Eq. (C.2) for the values predicted by the model is called the explained (or regression) sum of squares, and is given by

$$SSq_{mod} = \sum_{i=1}^{n} (f_i - \bar{y})^2. \quad (C.3)$$

The general definition of the determination coefficient $R^2$ is

$$R^2 = 1 - \frac{SSq_{res}}{SSq_{tot}}. \quad (C.4)$$
In case of linear regression—that is, when the model $f(x, a)$ is a linear function of parameter set $a$—the following partition holds:

$$SSq_{tot} = SSq_{res} + SSq_{mod},$$  \hspace{1cm} (C.5)

so that $R^2$ can be expressed as

$$R^2 = \frac{SSq_{mod}}{SSq_{tot}}.$$  \hspace{1cm} (C.6)

Eq. (C.6) gives $R^2$ in terms of the explained variance, comparing the variance of the model predictions with the total variance of the data. Under such conditions, $R^2$ equals the square of the correlation coefficient between observed and predicted data values, and its value ranges in $[0, 1]$.

If $f(x, a)$ is a nonlinear function of parameters $a$, then the partition in Eq. (C.5) does not hold, becoming

$$SSq_{tot} = SSq_{res} + SSq_{mod} + CPS,$$  \hspace{1cm} (C.7)

with

$$CPS = 2 \sum_{i=1}^{n} (y_i - f_i)(f_i - \bar{y}).$$  \hspace{1cm} (C.8)

The additional term of cross-product sum (CPS) is not identically zero as for linear functions, and can assume either sign. As a consequence, the computational definition of Eq. (C.4) can even yield $R^2$ values greater than unity or negative.

Rigorously speaking, in nonlinear regression the determination coefficient cannot be interpreted as a goodness-of-fit indicator, quantifying the fraction of deviance explained by the model relative to the total deviance of data. By willing to preserve a pregnant meaning for Eq. (C.6)—as for linear regressions—this could be evaluated as

$$R^2_{eff} = \frac{SSq_{mod}}{SSq_{tot}} = 1 - \frac{SSq_{res} + CPS}{SSq_{tot}}.$$  \hspace{1cm} (C.9)

A numerical example is helpful to clarify the situation, and the case of the chromatic difference of refraction (Section 5.1) is considered here. For the nonlinear regression of Fig. 5.1, the relevant statistical parameters are as follows: $SSq_{tot} = 56.479; SSq_{res} = 1.696; SSq_{mod} = 53.856; and CPS = 0.927$, which correctly satisfy the balance in Eq. (C.7). According
to Eqs. (C.4) and (C.8), the two values $R^2 = 0.970$ and $R_{\text{eff}}^2 = 0.954$ are obtained, which is not a significant difference.

However, the relevance of $R^2$ (or $R_{\text{eff}}^2$) must not be overrated. By looking at Eqs. (C.2)–(C.4), it can be realized that the determination coefficient quantifies how much more variation in data is explained by the model considered, compared to a null model having only a constant, equal to the data mean. There are two main concerns: how good the model is, and how good the mean is at explaining the data variation, because the better the mean is, the worse the model will look, even though the model is good.

Therefore, the goodness of fit (even for linear regression) should be jointly evaluated through a qualitative impression (by eye inspection: how well the fit curve interprets the data behavior\textsuperscript{1}), and quantitative indication (determination coefficient: how close to unity $R^2$ is). The eye can often succeed in finding out the fitting behavior that minimizes the distance from the data ($SSq_{\text{res}}$), but fails to rate the quality of the fit. $R^2$ represents a quality index of immediate comprehension, though with the limits of interpretation outlined before.

For the sake of simplicity only the $R^2$ values are provided here.

### C.1 References

Appendix D
Optical Parameters of the CAGE Eye Model

D.1 Geometrical Parameters

<table>
<thead>
<tr>
<th>Medium</th>
<th>Thickness (mm)</th>
<th>Curvature radius (mm)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>∞</td>
<td>7.7</td>
<td>0.72</td>
</tr>
<tr>
<td>Cornea</td>
<td>0.5</td>
<td>6.8</td>
<td>0.78</td>
</tr>
<tr>
<td>Aqueous humor</td>
<td>3.1</td>
<td>10</td>
<td>-0.89</td>
</tr>
<tr>
<td>Anterior lens cortex</td>
<td>0.546</td>
<td>7.911</td>
<td>1.2</td>
</tr>
<tr>
<td>Lens nucleus</td>
<td>2.419</td>
<td>-5.76</td>
<td>-0.64</td>
</tr>
<tr>
<td>Posterior lens cortex</td>
<td>0.635</td>
<td>-6</td>
<td>-1.3</td>
</tr>
<tr>
<td>Vitreous body</td>
<td>17.185</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

D.2 Chromatic Dispersion Parameters

Using Eq. (5.1),

\[
n(\lambda) = a_1(\lambda)n_i + a_2(\lambda)n_F + a_3(\lambda)n_C + a_4(\lambda)n_t, \quad (D.1)\]

with
$a_i(\lambda) = \gamma_{1i} + \frac{\gamma_{2i}}{\lambda^2 - \lambda_0^2} + \frac{\gamma_{3i}}{(\lambda^2 - \lambda_0^2)^2}$

and coefficients $\gamma_{1i}$ given by the matrix in Eq. (B.4).

**D.3 Paraxial Properties at Five Wavelengths**

**D.3.1 Dioptric Powers of Individual Interfaces and Components**

<table>
<thead>
<tr>
<th>Optical element</th>
<th>Wavelength (nm)</th>
<th>400</th>
<th>490</th>
<th>587.6</th>
<th>680</th>
<th>770</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ant. corneal surf.</td>
<td></td>
<td>50.429</td>
<td>49.432</td>
<td><strong>48.831</strong></td>
<td>48.148</td>
<td>47.302</td>
</tr>
<tr>
<td>Post. corneal surf.</td>
<td></td>
<td>−5.979</td>
<td>−5.947</td>
<td><strong>−5.882</strong></td>
<td>−5.923</td>
<td>−6.053</td>
</tr>
<tr>
<td>Cornea</td>
<td></td>
<td>44.559</td>
<td>43.592</td>
<td><strong>43.053</strong></td>
<td>42.329</td>
<td>41.354</td>
</tr>
<tr>
<td>Ant. lens cortex surf.</td>
<td></td>
<td>5.264</td>
<td>5.055</td>
<td><strong>5</strong></td>
<td>4.991</td>
<td>4.996</td>
</tr>
<tr>
<td>Ant. lens nucleus surf.</td>
<td></td>
<td>2.793</td>
<td>2.662</td>
<td><strong>2.528</strong></td>
<td>2.605</td>
<td>2.851</td>
</tr>
<tr>
<td>Post. lens nucleus surf.</td>
<td></td>
<td>3.836</td>
<td>3.657</td>
<td><strong>3.472</strong></td>
<td>3.578</td>
<td>3.915</td>
</tr>
<tr>
<td>Post. lens cortex surf.</td>
<td></td>
<td>8.773</td>
<td>8.425</td>
<td><strong>8.333</strong></td>
<td>8.319</td>
<td>8.327</td>
</tr>
<tr>
<td>Eye</td>
<td></td>
<td>61.113</td>
<td>59.515</td>
<td><strong>58.636</strong></td>
<td>58.084</td>
<td>57.653</td>
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</table>
D.3.2 Separations Between Cardinal Points

In millimeters, with $F$ as focal point, $P$ as principal point, $N$ as nodal point, $E$ as pupil, $V$ as vertex, with suffixes $f$ for front and $b$ for back, where, for example, $E_f$ corresponds to the entrance pupil and $E_b$ to the exit pupil.

<table>
<thead>
<tr>
<th>Separation</th>
<th>Wavelength (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>400</td>
</tr>
<tr>
<td>$V_fV_b$</td>
<td>7.2</td>
</tr>
<tr>
<td>$V_fF_b$</td>
<td>23.688</td>
</tr>
<tr>
<td>$V_fP_f$</td>
<td>1.37</td>
</tr>
<tr>
<td>$V_fP_b$</td>
<td>1.636</td>
</tr>
<tr>
<td>$V_fN_f$</td>
<td>7.058</td>
</tr>
<tr>
<td>$V_fN_b$</td>
<td>7.325</td>
</tr>
<tr>
<td>$V_fE_f$</td>
<td>3.031</td>
</tr>
<tr>
<td>$P_fN_f = P_bN_b$</td>
<td>5.689</td>
</tr>
<tr>
<td>$F_fP_f = N_bF_b$</td>
<td>16.363</td>
</tr>
<tr>
<td>$F_fN_f = P_bF_b$</td>
<td>22.052</td>
</tr>
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</table>

D.4 Ray-Transfer Matrix Elements

From anterior cornea to posterior lens surface

<table>
<thead>
<tr>
<th>Matrix element</th>
<th>Unit</th>
<th>Wavelength (nm)</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>400</td>
</tr>
<tr>
<td>$A_E$</td>
<td>–</td>
<td>0.74768</td>
</tr>
<tr>
<td>$B_E$</td>
<td>m</td>
<td>$5.1527 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$C_E$</td>
<td>–</td>
<td>$-61.113$</td>
</tr>
<tr>
<td>$D_E$</td>
<td>D</td>
<td>0.9163</td>
</tr>
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</table>
Appendix E
Visual Acuity Lines

Qualitative VA evaluations such as counting fingers and hand motion have been quantitatively measured to correspond to logMAR values of +1.85 (range 1.7 to 2) and +2.28 (range 2.05 to 2.48), although very different equivalences have also been proposed.

E.1 References


Table E.1 Comparison of VA levels (lines) in different notations, together with values of MAR and maximum angular frequency $\psi_M$. The relevant equations connecting all of these quantities are reported in Section 18.2.

<table>
<thead>
<tr>
<th>LogMAR</th>
<th>VA notations</th>
<th>Snellen fraction</th>
<th>MAR (arcmin)</th>
<th>Max frequency $\psi_M$ (cpd)</th>
<th>Notes</th>
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<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+2</td>
<td>0.01</td>
<td>0.1/10</td>
<td>6/600</td>
<td>20/2000</td>
<td>$100^m$</td>
</tr>
<tr>
<td></td>
<td>0.0125</td>
<td>0.125/10</td>
<td>6/480</td>
<td>20/1600</td>
<td>$79^{m.26}$</td>
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<tr>
<td></td>
<td>0.016</td>
<td>0.16/10</td>
<td>6/380</td>
<td>20/1250</td>
<td>$63^{m.06}$</td>
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<tr>
<td>+1.7</td>
<td>0.02</td>
<td>0.2/10</td>
<td>6/300</td>
<td>20/1000</td>
<td>$50^{m.07}$</td>
</tr>
<tr>
<td></td>
<td>0.025</td>
<td>0.25/10</td>
<td>6/240</td>
<td>20/800</td>
<td>$39^{m.48}$</td>
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<tr>
<td></td>
<td>0.032</td>
<td>0.32/10</td>
<td>6/200</td>
<td>20/630</td>
<td>$31^{m.37}$</td>
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<tr>
<td></td>
<td>0.04</td>
<td>0.4/10</td>
<td>6/150</td>
<td>20/500</td>
<td>$25^{m.07}$</td>
</tr>
<tr>
<td>+1.6</td>
<td>0.063</td>
<td>0.63/10</td>
<td>6/100</td>
<td>20/320</td>
<td>$15^{m.50}$</td>
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<tr>
<td></td>
<td>0.08</td>
<td>0.8/10</td>
<td>6/75</td>
<td>20/250</td>
<td>$12^{m.35}$</td>
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<tr>
<td></td>
<td>0.1</td>
<td>1/10</td>
<td>6/60</td>
<td>20/200</td>
<td>$10^{m}$</td>
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<tr>
<td>+1.3</td>
<td>0.05</td>
<td>0.5/10</td>
<td>6/120</td>
<td>20/400</td>
<td>$8^m$</td>
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<tr>
<td></td>
<td>0.125</td>
<td>1.25/10</td>
<td>6/48</td>
<td>20/160</td>
<td>$6^{m.18}$</td>
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<tr>
<td></td>
<td>0.16</td>
<td>1.6/10</td>
<td>6/38</td>
<td>20/125</td>
<td>$5^{m.18}$</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>2/10</td>
<td>6/30</td>
<td>20/100</td>
<td>$4^{m}$</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>2.5/10</td>
<td>6/24</td>
<td>20/80</td>
<td>$3^{m.09}$</td>
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<tr>
<td>+0.9</td>
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<td>6/20</td>
<td>20/63</td>
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<td></td>
<td>0.4</td>
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<td>6/15</td>
<td>20/50</td>
<td>$2^{m.30}$</td>
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<tr>
<td></td>
<td>0.5</td>
<td>5/10</td>
<td>6/12</td>
<td>20/40</td>
<td>$2^{m}$</td>
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<td></td>
<td>0.63</td>
<td>6.3/10</td>
<td>6/10</td>
<td>20/32</td>
<td>$1^{m.35}$</td>
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<td>6/7.5</td>
<td>20/25</td>
<td>$1^{m.15}$</td>
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<td></td>
<td>1</td>
<td>10/10</td>
<td>6/6</td>
<td>20/20</td>
<td>$1^{m}$</td>
</tr>
<tr>
<td>−0.1</td>
<td>1.25</td>
<td>12.5/10</td>
<td>6/5</td>
<td>20/16</td>
<td>$0^{m.47}$</td>
</tr>
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<td>−0.2</td>
<td>1.6</td>
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<td>6/3.75</td>
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# Appendix F
## List of Acronyms

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<td>2AFC</td>
<td>Two alternatives forced choice</td>
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<tr>
<td>2D</td>
<td>Bidimensional</td>
</tr>
<tr>
<td>AECD</td>
<td>Airy-disk energy content diameter</td>
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<tr>
<td>ALSF</td>
<td>Amplitude line spread function</td>
</tr>
<tr>
<td>ASF</td>
<td>Amplitude spread function</td>
</tr>
<tr>
<td>BF</td>
<td>Binocularity factor</td>
</tr>
<tr>
<td>BFD</td>
<td>Back focal distance</td>
</tr>
<tr>
<td>BLINCS</td>
<td>Bilogarithmic integral of normalized contrast sensitivity</td>
</tr>
<tr>
<td>CAGE</td>
<td>Chromatic aspherical Gullstrand exact</td>
</tr>
<tr>
<td>CDR</td>
<td>Chromatic difference of refraction</td>
</tr>
<tr>
<td>ChTF</td>
<td>Coherent transfer function</td>
</tr>
<tr>
<td>CIE</td>
<td>Commission Internationale de l’Éclairage</td>
</tr>
<tr>
<td>cpd</td>
<td>Cycles per degree</td>
</tr>
<tr>
<td>CPS</td>
<td>Cross products sum</td>
</tr>
<tr>
<td>CRT</td>
<td>Cathode ray tube</td>
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<tr>
<td>CS</td>
<td>Contrast sensitivity</td>
</tr>
<tr>
<td>CSF</td>
<td>Contrast sensitivity function</td>
</tr>
<tr>
<td>CTF</td>
<td>Contrast transfer function</td>
</tr>
<tr>
<td>DDD</td>
<td>Discriminable difference diagrams</td>
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<tr>
<td>DL</td>
<td>Diffraction limited</td>
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<tr>
<td>DLHE</td>
<td>Diffraction limited with hard-edge pupil</td>
</tr>
<tr>
<td>DLSC</td>
<td>Diffraction limited with Stiles–Crawford tapered pupil</td>
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<tr>
<td>DoG</td>
<td>Difference of Gaussians</td>
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<tr>
<td>ESA</td>
<td>External spherical aberration</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier transform</td>
</tr>
<tr>
<td>FT</td>
<td>Fourier transform</td>
</tr>
<tr>
<td>FWHM</td>
<td>Full width at half maximum</td>
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<td>GRIN</td>
<td>Graded index</td>
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<tr>
<td>GVA</td>
<td>Grating visual acuity</td>
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<tr>
<td>HE</td>
<td>Hard edge</td>
</tr>
<tr>
<td>ICS</td>
<td>Integrated contrast sensitivity</td>
</tr>
<tr>
<td>IFT</td>
<td>Inverse Fourier transform</td>
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<tr>
<td>INCS</td>
<td>Integral of normalized contrast sensitivity</td>
</tr>
<tr>
<td>IQ</td>
<td>Image quality</td>
</tr>
<tr>
<td>ISA</td>
<td>Internal spherical aberration</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
<td>-------------</td>
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<tr>
<td>jnd</td>
<td>Just noticeable difference</td>
</tr>
<tr>
<td>LCA</td>
<td>Longitudinal chromatic aberration</td>
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<tr>
<td>LGN</td>
<td>Lateral geniculate nucleus</td>
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<tr>
<td>LI</td>
<td>Lateral inhibition</td>
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<tr>
<td>LINCS</td>
<td>Logarithmic integral of normalized contrast sensitivity</td>
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<tr>
<td>LSA</td>
<td>Longitudinal spherical aberration</td>
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<tr>
<td>LSF</td>
<td>Line spread function</td>
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<tr>
<td>MAR</td>
<td>Minimum angle of resolution</td>
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<td>MCF</td>
<td>Mutual coherence function</td>
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<td>MIF</td>
<td>Mutual intensity function</td>
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<td>MIS</td>
<td>Mutual intensity spectrum</td>
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<tr>
<td>MON</td>
<td>Monochromatic illumination spectrum</td>
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<tr>
<td>MTF</td>
<td>Modulation transfer function</td>
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<tr>
<td>MTFA</td>
<td>Modulation transfer function area</td>
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<tr>
<td>OC</td>
<td>Optimum cornea</td>
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<tr>
<td>OPD</td>
<td>Optical path difference</td>
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<tr>
<td>OTF</td>
<td>Optical transfer function</td>
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<td>PF</td>
<td>Psychometric function</td>
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<td>PSF</td>
<td>Point spread function</td>
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<tr>
<td>PTF</td>
<td>Phase transfer function</td>
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<tr>
<td>REC</td>
<td>Radial energy content</td>
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<tr>
<td>RF</td>
<td>Receptive field</td>
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<tr>
<td>RG</td>
<td>Retinal gain</td>
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<tr>
<td>RMSE</td>
<td>Root mean square error</td>
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<td>SA</td>
<td>Spherical aberration</td>
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<tr>
<td>SECD</td>
<td>Sinc-lobe energy content diameter</td>
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<tr>
<td>SLE</td>
<td>Spectral luminous efficiency</td>
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<tr>
<td>SNR</td>
<td>Signal-to-noise ratio</td>
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<tr>
<td>SPH</td>
<td>Spherical Gullstrand eye model</td>
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<tr>
<td>SQF</td>
<td>Subjective quality factor</td>
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<tr>
<td>SQRI</td>
<td>Square root integral</td>
</tr>
<tr>
<td>SR</td>
<td>Strehl ratio</td>
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<tr>
<td>TA</td>
<td>Ray transverse aberration</td>
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<tr>
<td>TEC</td>
<td>Transverse energy content</td>
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<tr>
<td>TMF</td>
<td>Threshold modulation function</td>
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<tr>
<td>TSA</td>
<td>Transverse spherical aberration</td>
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<tr>
<td>UR</td>
<td>Struve ratio</td>
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<tr>
<td>UVL</td>
<td>Ultimate visual limit</td>
</tr>
<tr>
<td>VA</td>
<td>Visual acuity</td>
</tr>
<tr>
<td>vpt</td>
<td>Visual performance tile</td>
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<tr>
<td>WHT</td>
<td>Broadband illumination spectrum</td>
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<td>WRC</td>
<td>Windowing resolution correction</td>
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The Pizzomunno monolith emerging from the Adriatic Sea at Vieste, Italy. The very embryo of this book was conceived while swimming in these wonderful waters. The photo is a tribute to the genius loci, and to his gift of serendipitous inspiration.
Pier Giorgio (“Giò”) Gobbi was born in 1953 in Mantua, Italy, where he completed his classical studies at the Liceo-Ginnasio Virgilio. In 1976 he graduated cum laude in Electrical Engineering at the University of Pavia as an alumnus of the historical Collegio Ghislieri. He has been involved in various research fields including physics of laser-produced plasmas, physics and technology of laser sources, design of medical optoelectronic instrumentation, biomedical applications of lasers, physics of visual refraction, and eye modeling. He is author/co-author of more than 70 publications in scientific journals and books, more than 50 presentations at scientific conferences, and 10 national and international patents. He is currently with the Scientific Institute Hospital San Raffaele in Milan.