Modeling the Optical and Visual Performance of the Human Eye
Modeling the Optical and Visual Performance of the Human Eye

Pier Giorgio Gobbi
In grateful memory of

CARLO ALBERTO SACCHI (1937–1989)
PASCAL ROL (1956–2000)
GIAN PIERO BANFI (1946–2002)

three smart scientists, three good friends
## Contents

Preface ........................................................................................................... xv

Part I Chromatic Aspherical Gullstrand Exact (CAGE) Eye Model for Imaging Purposes ................................................................. 1

Part IA Assessment of Optical Parameters for the CAGE Eye Model ................................................................................................. 3

Chapter 1 Schematic Eye Models and Foveal Image Measurements ............................................................................................................................... 5

1.1 Review of Schematic Eye Models .......................................................... 5
1.2 Foveal Image Measurements ............................................................... 7
1.3 Campbell and Gubisch Experiment ..................................................... 8
1.4 Chapter Summary ................................................................................ 11
1.5 References ......................................................................................... 11

Chapter 2 Choice of Eye Models for Optical Evaluation ....................... 15

2.1 Gullstrand Exact Eye Model ............................................................... 16
2.2 Gullstrand Graded-Index Eye Model .................................................. 16
2.3 Aspherizing Interfaces ...................................................................... 19
2.4 Amplitude Spread Function ............................................................... 22
2.5 Chapter Summary ................................................................................ 24
2.6 References ......................................................................................... 25

Chapter 3 Modeling Foveal Reflection ..................................................... 29

3.1 Signature of Directionality ................................................................. 29
3.2 Backward-Pass ASF .......................................................................... 31
3.3 Foveal Curvature ............................................................................... 32
3.4 Chapter Summary ................................................................................ 33
3.5 References ......................................................................................... 33
## Chapter 4  Illumination: Coherence Features

4.1 Spectral Coherence ................................. 35
4.2 Spatial Coherence ........................................... 36
  4.2.1 Lamp to source slit ................................. 37
  4.2.2 Source slit to retina ................................. 40
  4.2.3 Retina to aerial image ............................. 46
  4.2.4 Double-pass image ................................. 47
4.3 Chapter Summary ........................................... 50
4.4 References .................................................... 50

## Chapter 5  Monochromatic to Broadband Optical Model

5.1 Dispersion Relations .................................... 53
5.2 Chromatic Model .......................................... 58
  5.2.1 Chromatic ASF ........................................ 58
  5.2.2 Partially coherent foveal image .................. 60
  5.2.3 Chromatic double-pass image .................... 60
5.3 Broadband Double-Pass Image ........................ 62
5.4 Chapter Summary ........................................... 62
5.5 References .................................................... 62

## Chapter 6  Numerical Algorithms

6.1 Ray Tracing .................................................. 65
6.2 Core Algorithm ............................................. 66
6.3 Chapter Summary ........................................... 68
6.4 References .................................................... 68

## Chapter 7  Convergence to the CAGE Eye Model

7.1 Fitting of Campbell–Gubisch LSF Data ............... 69
7.2 True Single-Pass LSF ...................................... 74
7.3 Comments on Surface Asphericities .................... 77
7.4 CAGE Eye Model ............................................. 81
7.5 Chapter Summary ........................................... 82
7.6 Conclusion of Part IA ..................................... 82
7.7 References .................................................... 82
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.4</td>
<td>Chapter Summary</td>
<td>156</td>
</tr>
<tr>
<td>12.5</td>
<td>Conclusion of Part IB</td>
<td>156</td>
</tr>
<tr>
<td>12.6</td>
<td>References</td>
<td>157</td>
</tr>
</tbody>
</table>

**Part II CAGE–Barten Eye Model for Contrast Perception**  
159

**Part IIA Assessment of the CAGE–Barten Model Psycho-physical Parameters**  
161

**Chapter 13 Optics and Psychophysics**  
163

- 13.1 Chapter Summary  
  - 13.2 References  

**Chapter 14 Neurophysical Model by Barten and Its Development**  
169

- 14.1 Total MTF  
  - 14.1.1 Optical MTF  
  - 14.1.2 Retinal MTF  
  - 14.1.3 Neural MTF  
- 14.2 Ocular Internal Noise  
  - 14.2.1 Photon noise  
  - 14.2.2 Neural noise  
  - 14.2.3 Integration constraints  
- 14.3 Complete Model  
- 14.4 Chapter Summary  
- 14.5 References  

**Chapter 15 Convergence to the CAGE–Barten Eye Model**  
185

- 15.1 Experimental CSF Database  
- 15.2 Pupil Light Response  
- 15.3 Numerical Fitting of CSF Data  
- 15.4 Data Alignment  
- 15.5 Chapter Summary  
- 15.6 References  

**Chapter 16 Application of the CAGE–Barten Model to Extended Contrast Sensitivity Data**  
203

- 16.1 Comparison with Data from van Nes and Bouman  
- 16.2 Comparison with Data by Luntinen, Rovamo, and Näsänen  

---

**Downloaded From:** https://www.spiedigitallibrary.org/ebooks/  
**Terms of Use:** https://www.spiedigitallibrary.org/terms-of-use  
**Downloaded on:** 18 Jan 2020
16.3 Comparison of Sinusoidal and Square-Wave Gratings ...... 207
16.4 Comparison with Defocused CSF Data ........................................ 210
16.5 Comparison with Barten’s Results ............................................ 219
16.6 Chapter Summary ........................................................................ 220
16.7 References ................................................................................ 220

Chapter 17 Comments on the CAGE–Barten Eye Model ............ 223
17.1 Discussion of CAGE–Barten Results ........................................ 223
17.2 Evaluation of Signal-to-Noise Ratio .......................................... 226
17.3 Parameter Variability ................................................................ 230
17.4 Comparison with Other Visual Perception Models .............. 232
17.5 Chapter Summary ........................................................................ 236
17.6 Conclusion of Part IIA ................................................................. 236
17.7 References ................................................................................ 236

Part IIB Visual Performances of the CAGE–Barten Eye Model 239

Chapter 18 Characterization of Visual Performance ............... 241
18.1 Eye as a Photocamera ................................................................. 241
18.2 Visual Performance Metrics ...................................................... 243
18.3 Visual Performance Metrics and Image Quality Perception ........................................ 248
18.4 BLINCS: A Visual Specific Metric ........................................... 254
18.5 Chapter Summary ........................................................................ 259
18.6 References ................................................................................ 260

Chapter 19 CAGE–Barten Eye Model Visual Performances ...... 263
19.1 Reference Visual Condition .......................................................... 263
19.2 Steady-State Pupil Light Response ............................................ 264
19.3 Natural Pupil Visual Performance ............................................. 265
19.4 Visual Performance versus Spherical Aberration ................. 270
19.5 Out-of-Focus Visual Performance ............................................ 273
19.6 Visual Performance versus Stimulus Parameters ................ 277
19.7 Monocular and Binocular Visual Performance ................. 281
19.8 Visual Performance versus Neurophysical Parameters .... 284
19.9 Chapter Summary ........................................................................ 287
19.10 References ................................................................................ 287

Chapter 20 Discussion of Visual Performance Results .......... 289
20.1 Previous Visual Acuity Modeling .............................................. 289
Appendix C Determination Coefficient $R^2$ ........................................ 363
    C.1 References .................................................................................. 365

Appendix D Optical Parameters of the CAGE Eye Model ............ 367
    D.1 Geometrical Parameters ............................................................... 367
    D.2 Chromatic Dispersion Parameters ............................................. 367
    D.3 Paraxial Properties at Five Wavelengths .................................... 368
        D.3.1 Dioptric Powers of Individual Interfaces and Components .... 368
        D.3.2 Separations Between Cardinal Points .................................... 369
    D.4 Ray-Transfer Matrix Elements .................................................. 369

Appendix E Visual Acuity Lines ......................................................... 371
    E.1 References .................................................................................. 371

Appendix F List of Acronyms ............................................................ 373

Index .................................................................................................... 375
Preface

- Can a schematic eye model reproduce the foveal images recorded in human eyes, and if so, to what degree of accuracy?
- To accomplish this, is it necessary to develop a new eye model?
- What is the physical approach required?
- What is the optimum defocus that corresponds to the maximum performance of ocular optics?
- What is the typical performance of ocular optics at different pupil sizes, and how far is it from the diffraction limit?
- How do spherical and chromatic aberration affect optical performance of the human eye?
- What is the ultimate optical performance of the eye?
- Do the international standards on safety from optical radiation properly estimate retinal irradiance?
- Can the optical performance of an emmetropic eye be improved further by means of optical aids or refractive surgery?
- Can a neurophysical model of the human eye simulate its visual performance with satisfactory accuracy?
- How can human visual performance be characterized quantitatively beyond visual acuity?
- What is the typical visual performance of an average human eye on axis in different visual conditions?
- Which are the most relevant optical factors limiting human visual performance?
- What is the ultimate visual performance of the eye?
- Is it possible to enhance the visual performance of the human eye through either optical aids or surgery?
- Is the existence of spatial frequency channels compatible with the neurophysical model here developed?

These basic questions are addressed in this book from a deterministic approach. Quantitative answers are given through the development of physical models that describe the optical process of image formation on the fovea, and the subsequent neural processing of visual information gathered by photoreceptors.
A faithful and robust simulation of the optical and visual performance of the human eye is provided for axial vision of distant objects in a variety of visual conditions. The book moves from intrinsically theoretical aspects (optical and neurophysical models of the eye) to include a large number of experimental measurements from within scientific literature. The model parameters are tuned to the observed phenomenology, in order to validate the predictive power of the models. The results turn out to be very satisfactory in terms of quantitative and qualitative adherence of the model predictions to field measurements.

The majority of material in this book is original and is the result of investigations made by the author during the last decade. The most relevant achievement of this work is the capacity to evaluate visual acuity for a range of visual conditions, such as variations in pupil size, refractive error, and ambient illumination.

The material is organized into two parts: optical and neurophysical aspects of the eye model. Each part is then divided into two sections. The first sections are devoted to assessment of the specific models through derivation of parameters from the best-fitting of experimental data. The second sections contain descriptions of the relevant properties derived from the models, together with discussions and connections to real-life situations. The reader should note that chapters and paragraphs with high-level mathematical and physical optics content can be safely skipped without compromising overall comprehension. To this end, a brief summary is provided at the end of each chapter.

Part IA defines the optical eye model that is used throughout the book—the chromatic aspherical Gullstrand exact (CAGE) eye model, which is developed from the Gullstrand exact eye model with the introduction of aspherical interfaces and chromatic index dispersion. Surface asphericities are derived from the best-fitting of line images recorded in a classical double-pass experiment, with similar images obtained from the CAGE model. Theoretical modeling of the double-pass experiment requires a complex physical optics analysis, including directionality of foveal reflection and spatial partial coherence of illumination light. The procedure is supported by the available accurate reporting of experimental conditions. The result is an excellent match-up of model predictions with measurements at all pupil sizes ($R^2 > 0.92$). The values of surface asphericities match well with independent measurements performed in vivo.

Part IA demonstrates the feasibility of using schematic eye models not only for estimating first-order geometrical optics properties and aberrations, but also for evaluating and reproducing the actual retinal images recorded by human eyes with high accuracy. The physical optics approach is attractive, since the starting point for the calculation is not the
usual wave aberration at the exit pupil (estimated from aberration data), but a well-defined optical scheme. This approach allows for the joint treatment of monochromatic and chromatic aberrations, as well as diffraction. As a consequence, the CAGE model is representative of the average human eye for distance foveal imaging.

Part IB provides a detailed presentation of optical performances exhibited by the CAGE model. The model’s paraxial properties at the central wavelength coincide with those of the Gullstrand exact model, but vary with wavelength. The CAGE eye model is characterized through the analysis of spherical aberration, point and line spread functions at variable pupil sizes, relative energy content, and modulation transfer function. Single-valued parameters are extracted for a simpler, direct description of optical behavior, including Strehl and Struve ratios, optimum defocus, full widths at half maximum for point and line images, spatial frequency bandwidths, and retinal gain. The entire characterization is illustrated by the continuous comparison between monochromatic and white light performances, as well as by comparison with two diverging behaviors: the diffraction-limited model and the purely spherical model (Gullstrand exact). CAGE model predictions are successfully compared with independent in-vivo measurements of spherical aberration and psychophysical modulation transfer function.

The most important innovative contributions from Part IB are as follows:

- Optimum defocus is effective in maximizing the foveal performance against spherical aberration (explaining the hyperopic choice operated by Gullstrand in his model).
- Retinal gain in conditions of optimum defocus is much larger than that assumed in international standards for laser safety.
- Chromatic aberration is the major limiting factor of optical performance.
- The eye behaves as a poor optical system in monochromatic illumination, but in white light it performs only 50% worse than a diffraction-limited eye.

In Part IIA, the CAGE optical eye model is merged with a neurophysical model of the eye from Barten, which describes the psychophysical response of the eye to sinusoidal bar stimuli with variable frequency, contrast, and luminance (ocular contrast sensitivity). The Barten model is based on the estimate of noise level generated internally in the eye. It depends on a few scalar parameters related to the integration properties of the eye, and on the ocular modulation transfer function. Modifications to the original Barten model have been introduced for physical consistency and improved phenomenological representation. The main modification
involves the modulation transfer function of the eye, which is calculated by means of the CAGE optical model. The joint CAGE-Barten model can provide estimates of the contrast sensitivity function (CSF) for a wide range of ambient and subject conditions. Values of the model parameters are derived from the best-fitting of 15 experimental data series on CSF, taken from the literature. The overall agreement obtained is excellent ($R^2 > 0.96$), providing good predictability in a variety of test conditions.

The main achievement of Part IIA is the development of a physical model that can predict human contrast sensitivity for a large number of conditions (including pupil size and refractive error of the subject; spatial frequency, spectrum, size, and duration of the stimulus; and ambient luminance). Results are obtained by following a deterministic physical pathway, without any ad-hoc heuristic assumptions (as in the original Barten model). Furthermore, values of the psychophysical parameters (obtained from the best-fitting procedure) help to define both structural properties of the eye (photoreceptor quantum efficiency, neural noise spectral density) and features of the integration capability of the visual system (temporal, spatial, and frequency integration limits, lateral inhibition cutoff). Thus, the CAGE-Barten model represents an effective tool for evaluating optical and perceptive properties of the human visual system.

In Part IIB, visual performances of the CAGE-Barten model are analyzed, starting from the evaluation of the entire perceptive region in the contrast-spatial frequency plane, which characterizes the quality of vision for any visual condition. The analysis is based on two single-valued parameters—grating visual acuity and bilogarithmic area of the perceptive region—which are evaluated as a function of pupil size and pupil response, illumination spectrum, spherical aberration, defocus, stimulus properties, and psychophysical parameters. The results are satisfactorily compared with the experimental measures of Snellen visual acuity and image quality. As an example, model grating visual acuity at 3.3-mm pupil size and 160-cd/m² luminance is $-0.14$ logMAR (20/14.5 Snellen fraction), which well overlaps with analogous measurements performed in young subjects. The CAGE-Barten model allows analysis of visual performance in relation to the fundamental limits placed by diffraction and noise, thus quantifying potential margins of improvement. Despite being based on a single filter-detector unit, the CAGE-Barten model is compatible with the existence of a plurality of spatial frequency channels; also, fitting such channels into the CSF evaluated by the model helps to shed light on their nature and structure.

The main contribution of Part IIB is unification of the optical and psychophysical descriptions of vision under a single model, with high predictability of mean performances in the human eye. In addition
to providing access to the neural image, the model provides local and integrated metrics for the quantitative evaluation of vision quality, related to variations of observing conditions. The CAGE–Barten model represents an effective tool for reproducing and analyzing both imaging and perception behaviors of an average human eye.

I am indebted to Dr. Laura Galli, Scientific Institute Hospital San Raffaele, for precious statistical advice. I thank Prof. Gianni Gilardi, Department of Mathematics F. Casorati, University of Pavia, for providing me with useful analytical formulas. Finally, I am grateful to my wife Mara and my children Alessandra and Francesco for their confident and patient waiting for this laborious delivery.

Pier Giorgio Gobbi
Milan, Italy
November 2012