Scattering coefficient determination in turbid media with backscattered polarized light

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Abstract. A simple empirical method is presented to determine the scattering coefficient \( \mu_s \) from backscattered polarized images of turbid media. It uses the ratio, pixel by pixel, of two images that are the second and the first backscattered Stokes parameter images \( Q \) and \( I \), respectively. Taking this image ratio, then integrating it over the azimuth angle, we get a function depending on the distance from the light entrance point. This function has a maximum. Using Monte Carlo simulations, for a fixed reduced scattering coefficient \( \mu'_s \) and for an anisotropy factor \( g \) varying between 0 and 0.8, it is found a linear relationship between the scattering coefficient \( \mu_s \) and the inverse of the maximum position of this function.

Keywords: polarization; backscattering; Mie theory; multiple scattering; photon migration; turbid media.

Paper 04013 received Feb. 4, 2004; revised manuscript received Sep. 30, 2004; accepted for publication Dec. 20, 2004; published online Jun. 7, 2005.

1 Introduction

During the 1990s the use of nonpolarized backscattered light has made it possible to optically characterize turbid media with noninvasive measurements. By characterization, we mainly refer to the determination of the mean transport scattering coefficient \( \mu'_s \) and the absorption coefficient \( \mu_a \) with spatially resolved techniques. Models were generally developed from the radiative transfer equation. Then, thanks to polarized light, several studies showed that backscattered polarized light contains information that was more localized and dependent on the scattering coefficient \( \mu'_s \). It was also demonstrated that linearly polarized light propagates through a smaller region than the circular one. We concentrate here on the second parameter of the backscattered Stokes vector, that is \( Q \), in the case of linearly polarized incident light. This parameter represents the intensity difference between the parallel and perpendicular polarization direction. Parallel (respectively perpendicular) direction means that the analyzer direction is parallel (respectively perpendicular) to the polarizer one. The purpose of this paper is to utilize polarization backscattered patterns from turbid media to determine the scattering coefficient \( \mu_s \). The first part presents an experimental test of the method on polystyrene microspheres. Then, in order to determine the limits of the proposed method with respect to the reduced scattering and absorption coefficients, \( \mu'_s \) and \( \mu_a \), respectively, Monte Carlo simulations are realized.

2 Experimental Method

The experimental setup (see Fig. 1) is composed of a laser diode that has a 670 nm wavelength and a 5 mW power. The beam passes through a polarizer and is directed to the sample. The incidence angle is \( \theta=15^\circ \). The sample which is composed of 65966 Villeurbanne, France.

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Paper 04013 received Feb. 4, 2004; revised manuscript received Sep. 30, 2004; accepted for publication Dec. 20, 2004; published online Jun. 7, 2005.

1083-3668/2005/$22.00 © 2005 SPIE

Journal of Biomedical Optics 10(3), 034016 (May/June 2005)
We are interested here in the spatial extent of the vector element $Q$ with respect to $I$. In other words, we want to know the typical size of the backscattered pattern. We thus integrate the image $Q/I$ over the azimuth angle $\varphi$ [see Fig. 3(a)] to only have a dependence in $r$, which is the distance from the entrance point of light. We call this integral $q(r)$:

$$q(r) = \frac{\int_0^{2\pi} Q(r, \varphi) \frac{1}{I(r, \varphi)} d\varphi}{2\pi}.$$  \hspace{1cm} (2)

This function $q(r)$ is drawn in Fig. 3(b) for two sphere suspensions, that have different scattering coefficients $\mu_s$ of 28 and 39 cm$^{-1}$. The first feature of $q(r)$ shows that the spatial expansion is small with respect to the one of the intensity $I$ that is generally of the order of several $\lambda_s$, the transport mean free path. More interestingly this curve has a maximum, let us call $l_m$ the distance corresponding to that maximum. For our two examples, namely $\mu_s = 28$ and 39 cm$^{-1}$, we have $l_m$ that is equal to $(0.033 \pm 0.002)$ and $(0.025 \pm 0.002)$ cm, respectively. Thus, taking the inverse of these distances, we get $(30 \pm 3)$ and $(40 \pm 3)$ cm$^{-1}$, respectively, that corresponds to the scattering coefficient $\mu_s$. In order to confirm that trend, let us look at the Monte Carlo simulations in order to see the influence on that method of the reduced scattering coefficient $\mu_s$ and the absorption coefficient $\mu_a$.

3 Monte Carlo Simulations

The Monte Carlo code$^8$ simulates the propagation of polarized light through a scattering medium having a slab shape, that is composed of homogeneously distributed spheres. This code considers a polarized light source illuminating one point of the turbid medium surface. This Monte Carlo simulation is based on a previous one$^9$ that described propagation of unpolarized light thanks to the radiative theory using the absorption and scattering coefficients, $\mu_a$ and $\mu_s$, respectively. In addition to the position of the photon and its propagation direction, the photon packet is characterized here by its Stokes vector which is defined in the photon local frame when this photon packet is inside the medium. Thanks to the Mie theory$^{10}$ we know the anisotropy factor $g$ of the spheres and the Mueller matrix for a single scattering. This makes it possible to statistically generate the photon direction for each scattering. Thus, we follow the modification of the Stokes vector and as the photon escapes the medium, either in the backscattering case or the transmission one, this vector is expressed in the detector frame and recorded. Notice that the detector is assumed lying on the medium surface. We end up with a backscattering (and transmission) images for each element of the Stokes vector. An example of backscattering images is given in the Fig. 4 for the $I$ and $Q$ Stokes vector elements. The medium parameters are the following: $\mu_a = 0$ cm$^{-1}$, $\mu_s = 57$ cm$^{-1}$, $g = 0.65$, the thickness of the slab is $d = 0.5$ cm, the sphere refraction index is $n_s = 1.58$, these
spheres are immersed in a medium with a refraction index of \( n_m = 1.33 \), and the source wavelength is equal to 670 nm. These images are normalized by the number of the photons used for that simulation, namely 100 millions. In our simulations, we set the value of the reduced scattering coefficient \( \mu_s' \) equal to 20 cm\(^{-1}\) and we vary the value of the anisotropy factor \( g \) from 0.04 (equivalent to \( \mu_g = 21 \text{ cm}^{-1} \)) to 0.8 (equivalent to \( \mu_g = 84 \text{ cm}^{-1} \)). The absorption is still null. We follow the evolution of the inverse of the length \( l_m \) with respect to the scattering coefficient \( \mu_s \). The slab parameters are the following: medium refraction index \( n_m = 1.33 \), sphere refraction index \( n_s = 1.58 \), slab thickness \( d = 0.5 \text{ cm} \) and the source wavelength is 670 nm. The backscattered \( Q \) image is divided by the \( I \) image, pixel by pixel, and then integrated to obtain \( q(r) \) (see Fig. 5). If we draw the maximum position \( l_m \) as a function of \( \mu_s \) in the case where the anisotropy factor \( g \) is kept constant, we can see a linearity between \( l_m \) and \( \mu_s \). The corresponding linear regression coefficient \( r^2 \) is equal to 0.985. The evolution of the relationship between \( l_m \) and \( \mu_s \) is shown on Fig. 7 where the reduced scattering coefficient \( \mu_s' \) has been changed to 10 and 40 cm\(^{-1}\), respectively. We have a linear regression coefficient \( r^2 \) that is equal to 0.995 for \( \mu_s' = 10 \text{ cm}^{-1} \) and to 0.977 for \( \mu_s' = 40 \text{ cm}^{-1} \). Considering the coefficient \( r^2 \), it is difficult to conclude on the influence of the transport scattering coefficient \( \mu_s' \) on the linear relationship between \( 1/l_m \) and \( \mu_s \). But its dependence remains weak compared to the one with the anisotropy factor \( g \).

The influence of the absorption coefficient \( \mu_a \) on the relationship between the maximum position \( l_m \) of \( q(r) \) is shown on Fig. 8 when \( \mu_s' = 20 \text{ cm}^{-1} \), for two examples of absorption: \( \mu_a = 1 \text{ cm}^{-1} \) and \( \mu_a = 5 \text{ cm}^{-1} \). The linear regression coefficient \( r^2 \) for the first example is 0.966 and 0.659 for the second case. Therefore considering the diminishing linear regression coefficient \( r^2 \), the linearity between \( 1/l_m \) and \( \mu_a \) disappears as the absorption coefficient \( \mu_a \) increases. We notice that as the absorption increases, there is a region (corresponding to \( g < 0.7 \)) where the length \( l_m \) is smaller than expected.

**4 Discussion**

The advantage of this method is its weak dependence with the transport mean free path \( \lambda_s' \) in the case of a negligible absorption coefficient \( \mu_a \) compared to \( \mu_s' \). These coefficients can be known, for example, with a reflectance technique using unpolarized light. As the anisotropy factor \( g \) tends to 1, we see [Fig. 9(left)] that the distance \( l_m \) does not vary anymore. Its inverse tends approximately to the value of 80 cm\(^{-1} \) [Fig. 9(right)]. This cannot be explained by the thickness of the slab since the latter has been modified and increased to 10 cm in order to see if it influences the backscattered intensity as the anisotropy factor \( g \) increases to 1. The results of the simula-

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**Fig. 4** Simulated backscattered \( I \) and \( Q \) images for a scattering medium with the following coefficients: \( \mu_s = 0 \text{ cm}^{-1} \), \( \mu_g = 57 \text{ cm}^{-1} \), \( g = 0.65 \), thickness=0.5 cm, wavelength=670 nm. Image size 0.35 cm \( \times 0.35 \text{ cm} \).

**Fig. 5** Simulated \( q(r) \) for different \( \mu_s \) and with a constant \( \mu_s' = 20 \text{ cm}^{-1} \).

**Fig. 6** \( 1/l_m \) with respect to \( \mu_s \) (corresponding anisotropy factor \( g \) varies between 0.04 and 0.8). \( \mu_s' = 20 \text{ cm}^{-1} \). Linear regression coefficient \( r^2 = 0.985 \).
tion, in the case where \( g = 0.92 \), give no difference concerning the behavior of the curve \( q(r) \), that is the position of the maximum is not altered. Consequently, the distance \( l_m \) remains unchanged even if the thickness represents a large number of times \( \approx 2500 \) the mean free transport \( \lambda'/s \). An explanation can be given by the Mie distribution, that generates the propagation direction of the photons. Indeed if we assume a single scattering, we have the following Mueller matrix:

\[
\begin{pmatrix}
  m_{11}(\theta) & m_{12}(\theta) & 0 & 0 \\
  m_{12}(\theta) & m_{11}(\theta) & 0 & 0 \\
  0 & 0 & m_{33}(\theta) & m_{34}(\theta) \\
  0 & 0 & -m_{34}(\theta) & m_{33}(\theta)
\end{pmatrix},
\]

where the elements depending of the polar angle \( \theta \) of this matrix are defined by the Mie theory.\(^\text{10}\) If the initial polarization is linear, the incident Stokes vector can be chosen as \((1,1,0,0)\), and if we put the azimuth angle \( \varphi = 0 \), the scattered intensity \( I' \) is

\[
I' = m_{11}(\theta) + m_{12}(\theta).
\]

Then we look, with respect to \( g \), at the Mie distribution for a single scattering which means that \( \theta = 180^\circ \) (see Fig. 10). To change \( g \), we vary the radius of spheres, keeping constant the other sphere characteristics. We observe that this function is bijective with \( g \) if only the latter is under 0.65. Then the evolution of the Mie function tends to zero and the bijectivity disappears. That could explain the \( q(r) \) behavior for high anisotropy factors.

We notice that if the integration over the azimuth angle is done on the backscattered image \( Q \) normalized by the total backscattered intensity \( I \), the dependence of \( 1/l_m \) with respect to \( m_s \) is not seen. We therefore need to combine the spatial evolutions of both Stokes vector element images \( I \) and \( Q \) by dividing them pixel by pixel. Moreover, since we consider lengths that are of the order of \( l_s \), an experimental problem that can occur is the entrance diameter of the light beam which will modify the backscattered image near the entry point. Thus the profile of the light source has to be known in order to correctly deconvolute the image and to get rid of the beam finite size influence.

Model describing qualitatively low scattering backscat-

![Fig. 7](image1.png)  
**Fig. 7** \( 1/l_m \) with respect to \( \mu_s \). For \( \mu'_s = 10 \, \text{cm}^{-1} \), \( r^2 = 0.995 \). For \( \mu'_s = 40 \, \text{cm}^{-1} \), \( r^2 = 0.977 \). The corresponding anisotropy factor \( g \) varies between 0.04 and 0.8 for each transport scattering coefficient.

![Fig. 8](image2.png)  
**Fig. 8** \( 1/l_m \) with respect to \( \mu_s \) for \( \mu'_s = 20 \, \text{cm}^{-1} \). For \( \mu_s = 1 \, \text{cm}^{-1} \), \( r^2 = 0.966 \). For \( \mu_s = 5 \, \text{cm}^{-1} \), \( r^2 = 0.659 \). The corresponding anisotropy factor \( g \) varies between 0.04 (\( \mu_s = 21 \, \text{cm}^{-1} \)) and 0.8 (\( \mu_s = 84 \, \text{cm}^{-1} \)) for each absorption coefficient.

![Fig. 9](image3.png)  
**Fig. 9** (Left) \( q(r) \) for different \( m_s \) and with a constant \( \mu'_s = 20 \, \text{cm}^{-1} \), \( \mu_s = 0 \, \text{cm}^{-1} \); and (right) \( 1/l_m \) with respect to \( \mu_s \). \( \mu'_s = 20 \, \text{cm}^{-1} \) and \( \mu_s = 0 \, \text{cm}^{-1} \). Corresponding anisotropy factor \( g \) varies between 0.04 (\( \mu_s = 21 \, \text{cm}^{-1} \)) and 0.93 (\( \mu_s = 282 \, \text{cm}^{-1} \)).
tered polarized light already exists. In order to extend this method for higher anisotropy factors, it is necessary to have a quantitative model that precisely describes the polarized back-scattered light.

5 Conclusion
Thanks to experiments and Monte Carlo simulations, we have seen that the spatial information contained in the backscattered image ratio $Q/I$ allowed to see the influence of the scattering coefficient $\mu_s$ and to determine the latter when the anisotropy factor $g$ ranges between 0 and 0.8. A quantitative theoretical model is necessary to understand the multiple scattering regime and to be able the treatment of higher anisotropy factors, in order to be closer to biological tissue conditions.

References