

# Multiscale corner detection of contour images using wavelet transform

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**Abstract.** A new multiscale corner detection method is proposed based on dyadic wavelet transform (WT) of the orientation function of a contour image. As the decomposition of the dyadic WT is complete and its scales are sparse, all the scales are defined as natural scales for corner detection. The points that are wavelet transform modulus maxima (WTMM) at different scales are taken as corner candidates. For each corner candidate, the sum of the corresponding normalized WTMM at all the natural scales is used as significance measure of the "cornerness". The utilization of the complete information makes the performance of the proposed detector independent to the type of input images. The decomposition scales of the WT are restricted by the contour length, which makes the algorithm adaptable for both long contours and short contours. Both subjective and objective evaluation illustrate better performance of the proposed corner detector compared to the conventional methods. © 2005 SPIE and IS&T. [DOI: 10.1117/1.2076968]

## 1 Introduction

Since corner detection has a lot of applications in shape representation and image analysis, a number of methods have been proposed to detect corners of different sizes and extents for contour images. In general, they can be categorized into two groups, the support-region-based methods<sup>1</sup> and the multiscale-based methods.<sup>2,3</sup> For the support-region-based methods, for example, Teh-Chin's algorithm,<sup>1</sup> the support region of each point (its natural scale) is determined at individual point. As a result, the natural scale is optimal for the corresponding point. However, the determination of the support region is based on the raw data, which include quantization errors and possible noises. Thus, the methods are not robust. For the multiscale-based methods, there exist scale-space-based analysis and WT-based analysis. Both of them provide a simple hierarchical framework for corner detection and are robust due to their inherent smoothing property. But there exist some problems for the

existing multiscale-based methods. For the scale-space-based methods, they are computationally inefficient because they need to compute at continuous scales. Furthermore, they either utilize information on one or several heuristically selected natural scales<sup>4</sup> or utilize only location information in the transformed domain.<sup>2</sup> Without *a priori* information, no particular scales should be presupposed.<sup>5</sup> Thus, the performance of the existing scale-space-based methods is not satisfactory for different types of test images. Recently, Quddus and Gabbouj<sup>3</sup> proposed a fast and robust wavelet-based corner detection technique. The technique requires to compute *singular value decomposition* (SVD) of the dyadic WT to estimate natural scales for the contour. However, in a few cases the stop criterion for the selection of natural scales does not work. Moreover, there is some computational overhead to compute SVD.

To overcome the above problems, we propose a new contour corner detection method using dyadic WT. Although the decomposition scales of the dyadic WT are sparse, the decomposition is complete. All the possible decomposition scales of the dyadic WT are considered in this paper as natural scales for corner detection because the natural scales are the ones containing most or all the important information. For each candidate, the sum of the normalized WTMM from different scales is taken as the significance measure to differentiate the corners from the noise. The inherent smoothing and localization properties of WT make this method effective and accurate. In addition, the technique is fast due to the fast implementations of the dyadic WT computations.

The paper is organized as follows. In Sec. 2, the proposed algorithm is presented. Section 3 shows simulation results and performance evaluation. The conclusion is given in Sec. 4.

## 2 Proposed Corner Detection Algorithm Using Dyadic WT

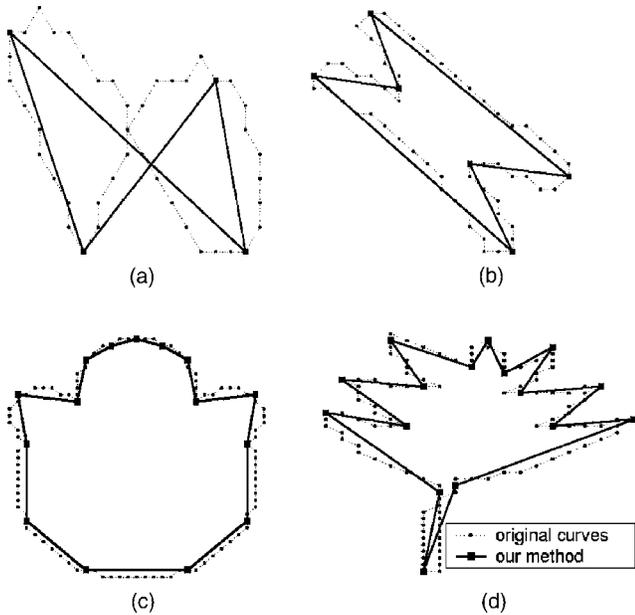
Corners are defined as high curvature points on a contour. To estimate the curvature, we select the dyadic WT using the quadratic spline mother wavelet<sup>6</sup> to decompose the orientation function because it satisfies the following necessary conditions and good properties. First, the dyadic WT is shift invariant, which is a necessary condition for feature extraction. Second, the quadratic spline mother wavelet has one vanishing moment, which is a first order differential operator on a smoothed signal. Accordingly, the curvature is approximated when the transformation is applied on the orientation function. Third, the dyadic WT is complete. Thus, it provides the decomposition at a sparse set of appropriate scales, which simplifies the following analysis and computation. Last, it has a fast implementation algorithm, which makes the proposed algorithm computationally efficient.

In the proposed algorithm, the preprocessing steps described in Ref. 3 are adopted to get the orientation function of the contour image. Then dyadic WT is applied to the orientation function to estimate the curvature at all possible scales because no scale should be preferred without *a priori* information.

The range of the decomposition scales,  $2^j$ , for the WT is determined by the inherent property of the dyadic WT and the length of the signal,  $N$ ,<sup>6</sup>

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**Fig. 1** Results of the proposed method. The corners are indicated by squares and connected into polygons. (a) The “figure-8” curve, (b) the “chromosome” curve, (c) the “semicir” curve, (d) the “leaf” curve.

$$1 < 2^j \leq N, \quad j = 1, 2, \dots, J, \quad (1)$$

where  $J$  is the maximum level of the WT. According to Eq. (1), the decomposition scales of the WT are restricted by the signal length  $N$ , which makes the algorithm adaptable for both long contours and short contours.

Subsequently, all the decomposition scales are taken as natural scales for the corner detection. The WTMM are extracted and the points with WTMM are taken as corner candidates. Modulus maximum is any point whose absolute value is more than one of its neighborhoods and not less than the other neighborhood,<sup>6</sup> i.e.,

$$\begin{cases} |Wf(u_0, s)| > |Wf(u_1, s)| : u_1 \text{ is one neighborhood of } u_0, \\ |Wf(u_0, s)| \geq |Wf(u_2, s)| : u_2 \text{ is the other neighborhood of } u_0, \end{cases} \quad (2)$$

where  $|Wf|$  represent the modulus of  $Wf$ .

At a certain scale, the candidates with acute angles produce large WTMM, while the candidates with obtuse angles have small WTMM. To make each scale contributes the same to the final significance measure, the values of the WTMM are normalized with the respective maximum value at each scale.

Then, the values at different scales corresponding to a particular corner candidate are constructed as a WTMM sequence along the scales.

To detect corners of different subtended angles and sizes, the normalized values of each candidate at all the natural scales are used to compute the measurement as

$$\hat{W}f = \frac{1}{J} \sum_{j=1}^J |W_{2^j}f|, \quad (3)$$

where  $\hat{W}f$  denotes the average and  $W_{2^j}f$  denotes the normalized wavelet coefficients of the discrete orientation pro-

file  $f$  at the scale  $2^j$  for each corner candidate. This average includes the information considered at all the natural scales. It also suppresses the noise while strengthening the corner points.

Finally, the corners are detected by setting a predefined threshold.

The steps of the proposed algorithm are presented as follows.

1. Preprocessing steps that have been described in Ref. 3:
  - (a) Track the boundary and obtain the chain code of it;
  - (b) Compute the orientation profile.
2. Determine the natural scales according to the length of the contour image.
3. Compute the WT of the orientation profile at the natural scales.
4. Detect the corner candidates by selecting the WTMM. Normalize the values at each scale.
5. Construct the WTMM sequence for each corner candidate along the scales.
6. Compute the average of the normalized values of WTMM from all natural scales as the significance measure.
7. Normalize the significance measure to the maximum one, thus all the measure values are in the range  $[0, 1]$ . Then suppress the false corner points by setting a threshold.

### 3 Simulation Results and Performance Evaluation

#### 3.1 Subjective Evaluation

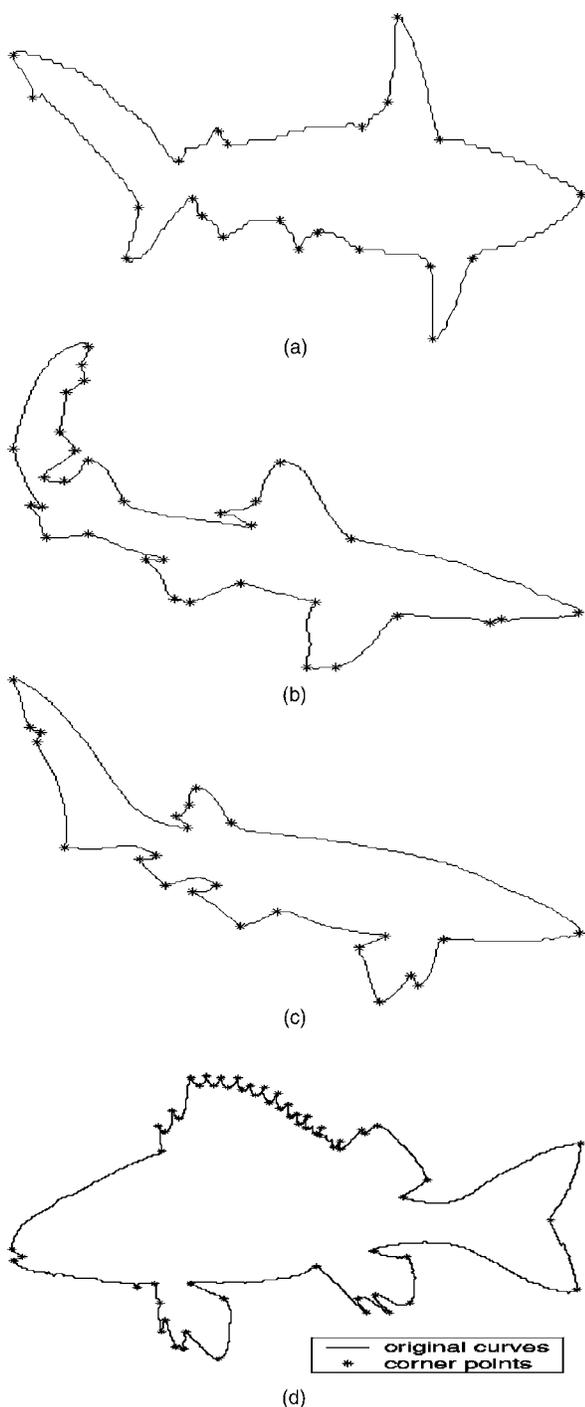
The simulation results of the proposed method are shown in Fig. 1 and Fig. 2. The fixed threshold, which is set empirically, is 0.08 for all the simulations shown in this paper. The lengths of the curves in Figs. 1(a)–1(d) are 45, 60, 102, and 120, respectively. The lengths of the curves in Figs. 2(a)–2(d) are 563, 854, 872, and 1104, respectively.

The test images in Fig. 1 are quite short in length, and consequently, the proposed method detects fewer corners compared to the results in Fig. 2. From the simulation results, we see that the proposed method provides satisfactory performance for both long and short contours. This feature makes it more suitable in practical applications.

#### 3.2 Objective Comparison Using Rosin’s Method

The quantitative measurements using Rosin’s evaluation method<sup>7</sup> are shown in Table 1 for the test images in Fig. 1.  $Merit_2$  and  $Merit_\infty$  measures the compression ratio as well as the error measurements, for which the integral square error  $E_2$  and the maximum deviation error  $E_\infty$  are adopted, respectively. For  $Merit_2$  and  $Merit_\infty$ , the larger the values, the better the performance.

From the results, we see that performance of all the selected methods is good. Among them, the proposed method provides better performance in general. Teh-Chin’s method is based on the support region determination. This method is classical but not so effective compared to the other three



**Fig. 2** Results of the proposed method. The corners are indicated by asterisks.

methods as shown in Table 1. Rattarangsi-Chin’s method<sup>2</sup> is based on the scale-space analysis. It shows better performance measurement than the proposed method for the “semicir” curve using  $E_2$ . But the proposed method obtains better results for all the other measures. Quddus-Gabbouj’s method<sup>3</sup> is based on WT and SVD. However, in a few cases the stop criterion for the selection of natural scale does not work, e.g., for the “figure-8” curve. Moreover, there is some computational overhead to compute the SVD. The

**Table 1** Quantitative results of the test curves in Fig. 1 using  $E_2$  and  $E_{\infty}$ , respectively.

Test Curves	Algorithm	$Merit_2$	$Merit_{\infty}$
“figure-8”	Proposed method	86.84	88.55
	Teh-Chin’s method <sup>1</sup>	46.0	49.8
	Rattarangsi-Chin’s method <sup>2</sup>	74.0	67.6
	Quddus-Gabbouj’s method <sup>3</sup>	—	—
“chromosome”	Proposed method	100	96.16
	Teh-Chin’s method <sup>1</sup>	61.8	84.2
	Rattarangsi-Chin’s method <sup>2</sup>	58.5	71.9
	Quddus-Gabbouj’s method <sup>3</sup>	87.6	92.6
“semicir”	Proposed method	35.4	63.9
	Teh-Chin’s method <sup>1</sup>	44.9	60.0
	Rattarangsi-Chin’s method <sup>2</sup>	57.7	63.7
	Quddus-Gabbouj’s method <sup>3</sup>	33.4	56.7
“leaf”	Proposed method	86.27	93.82
	Teh-Chin’s method <sup>1</sup>	50.1	59.3
	Rattarangsi-Chin’s method <sup>2</sup>	68.9	77.8
	Quddus-Gabbouj’s method <sup>3</sup>	77.3	77.8

length of the “figure-8” curve is 45 pixels. It is relatively short, which might be the possible reason that Quddus-Gabbouj’s method fails.

**4 Conclusions**

In this paper, we propose a new corner detector for the contour images based on dyadic WT. The maximum decomposition level of the dyadic WT is imposed by the contour length, which makes the algorithm suitable for both long and short contours. Unlike the existing methods, the information at all dyadic scales have been used for detection, which makes the proposed method independent to the type of contour images. This method is also computationally efficient due to the fast implementation of the dyadic WT. The objective evaluation reveals improved performance by the proposed method compared to the classical methods.

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