Generation of partially coherent beams with controllable time-dependent coherence

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1 Introduction

Coherence is one of the most important properties of laser beams. Single transverse mode laser beams can be regarded as examples of completely coherent beams with a degree of coherence equal to 1. On the contrary, completely incoherent beams have a degree of coherence equal to 0. An extended thermal light source can be regarded as a spatially incoherent source with a short coherence time less than 0.1 ns. Beams with degree of coherence between 0 and 1 are referred to as partially coherent beams. The propagation of beams is greatly influenced by the coherence of beams. Compared with coherent beams, partially coherent beams have some advantages including spreading modulation and speckle suppression. Owing to these advantages and other applications, the generation and propagation of partially coherent beams have attracted much attention. For instance, partially coherent beams with different types of correlation, including Gaussian Schell-model arrays, multi-Gaussian correlated Schell-model beams, and cosine-Gaussian Schell-model beams, have been produced. Further to this, the propagation of partially coherent beams in atmospheric turbulence and isotropic random media has also been investigated.

A completely coherent beam can be transformed into a partially coherent beam by imposing a dynamic random phase on the completely coherent beam. Several optical elements, including rotating ground glass (RGG), dielectric elastomeric actuator, and spatial light modulator (SLM), are adapted to impose the random phase. The disadvantage of an RGG is that the random phase is difficult to be quantitatively controlled. Although using an SLM can quantitatively control the coherence of the light, the coherence of the light remains invariant in most of the theoretical simulations or focused only on the intensity distribution. To the best of our knowledge, the generation of partially coherent beam with temporally variable coherence by using an SLM has not been reported.

In this study, we proposed an effective method to generate partially coherent beams with controllable time-dependent degree of coherence. We converted a completely coherent laser beam into a partially coherent beam by imposing a dynamic random phase on the beam using an SLM. The degree of coherence of the beam is controlled by modulating the amplitude of the random phase. We showed that by introducing different types of dynamic random phase patterns, we could produce partially coherent beams with a given degree of coherence. Moreover, we also showed that the degree of coherence could be made to vary periodically in time.

2 Theoretical Analysis

It is well known that the cross-spectral density function of a partially coherent beam can be expressed as

\[ W(r_1, r_2, \omega) = \langle E^*(r_1, \omega)E(r_2, \omega) \rangle, \] (1)

where the angular brackets denote the average taken over the ensemble of the realizations of the electric field in the sense of coherence theory in the space of frequency domain. The asterisk * denotes complex conjugate. \( E(r, \omega) \) represents the electric field at a point \( r \), at frequency \( \omega \), which can be written as

\[ E(r, \omega) = A(r, \omega) \exp[i\phi(r, \omega)], \] (2)

where \( A(r, \omega) \) is the amplitude, a real valued function, and \( \phi(r, \omega) \) is the phase. Assuming that the amplitude of the electric field is deterministic but its phase is random, the cross-spectral density function can be further expressed as

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\[ W(r_1, r_2, \omega) = A(r_1, \omega)A(r_2, \omega) \exp[-i\phi(r_1, \omega)] \exp[i\phi(r_2, \omega)]. \] (3)

Since the spectral degree of coherence is defined as
\[ \mu(r_1, r_2, \omega) = \frac{W(r_1, r_2, \omega)}{\sqrt{W(r_1, r_1, \omega)W(r_2, r_2, \omega)}.} \] (4)

By substituting Eqs. (3–4) into Eq. (4) one gets that the spectral degree of coherence is reduced to
\[ \mu(r_1, r_2, \omega) = \langle \exp[-i\phi(r_1, \omega)] \exp[i\phi(r_2, \omega)] \rangle. \] (5)

In this expression, the degree of coherence ranges from 0 (when $\Delta \phi_{12}$ is constant) to 1 (when the phase difference $\Delta \phi_{12} = \phi(r_2, \omega) - \phi(r_1, \omega)$ takes a uniformly distributed random value within the interval $[0,2\pi]$). Thus, Eq. (6) states that it is possible to control the spectral degree of coherence by introducing a proper time-dependent random phase difference on a coherent beam. For this purpose, we can choose that for the coherent case the constant phase difference be at the middle of the interval. Then, allowing $\Delta \phi_{12}$ to have uniformly distributed random values within the interval $[\pi - a\pi, \pi + a\pi]$, where $a$ is a positive control parameter in the range $[0,1]$. Clearly, the phase difference mean value is $\pi$. At the ends of the latter interval, we have either a coherent or an incoherent field when $a = 0$ or $a = 1$, and a partially coherent field otherwise. In this work, we have investigated the implementation of the above proposal using an SLM to generate such type of dynamic random phase on a coherent beam and found that the degree of coherence can be controlled at will.

Figure 1 shows the typical examples of phase difference $\Delta \phi_{12}$. The phase of Fig. 1(a) is a uniform constant phase with a value of $\pi$. The phase of Fig. 1(b) is an example of random phase with $a = 0.5$, which indicates that the phase of each point is a random value within the range of $0.5\pi - 1.5\pi$. The phase of Fig. 1(c) is a random value within the range of $0$ to $2\pi$. Below we will see that the degree of coherence decreases with the increasing amplitude of the control parameter $a$. It should be pointed out that the phase distributions in Fig. 1 are static examples of a dynamic process. In the experiments, we will use a dynamic random phase to realize the ensemble average of a partially coherent beam.

Since the random phase is a $\pi$ mean random variable that is characterized by uniform probability distribution in the range from $\pi - a\pi$ to $\pi + a\pi$, we have
\[ p[\phi(r)] = \begin{cases} \frac{1}{2\pi a}, & \text{for } \pi - a(r)\pi \leq \phi(r) \leq \pi + a(r)\pi \\ 0, & \text{otherwise}. \end{cases} \] (6)

The ensemble average of random phase can be calculated as
\[ \langle \exp[-i\phi(r)] \rangle = \int_{-\infty}^{\infty} \exp[-i\phi(r)] p[\phi(r)] d\phi(r) \]
\[ = \frac{1}{2\pi a(r)} \int_{\pi-a(r)\pi}^{\pi+a(r)\pi} \exp[-i\phi(r)] d\phi(r) \]
\[ = \sin c[a(r)], \] (7)

where \( \sin c(x) \) is the normalized sinc function and equals to 1 for $x = 0$ and $\sin(n\pi) / \pi$ otherwise.

During the phase modulation process, the phase change of each point is statistically independent, the coherence of the partially coherent beam in Eq. (3) can be expressed as follows:
\[ \mu(r_1, r_2, \omega) = \langle \exp[-i\phi(r_1, \omega)] \rangle \langle \exp[i\phi(r_2, \omega)] \rangle. \] (8)

Using Eq. (5), Eq. (6) can be further written as
\[ \mu(r_1, r_2, \omega) = \begin{cases} \sin c[a(r_1)] \sin c[a(r_2)], & \text{if } r_1 \neq r_2 \\ 1, & \text{if } r_1 = r_2. \end{cases} \] (9)

Although the phase change of each point is random, the randomness of the phase change is independent of the position because the coefficient $a$ of the random phase is a constant parameter. Thus, from Eq. (9) it is clear that the coherence of the beam can be modified by the control parameter $a$ of the random phase. Figure 2 plots the dependence of coherence with control parameter $a$.

**3 Experimental Measurement**

According to the previous analysis, the coherence of a laser beam can be controlled by imposing dynamic random phase onto the beam. Figure 3 shows the experimental setup to produce partially coherent beams with time-dependent coherence. He–Ne laser beam (HNL-150L-NC, Thorlabs) with a wavelength of 632.8 nm, and a bandwidth of 0.2 nm is used as the incident beam. The expanded laser beam is reflected by a liquid-crystal SLM (PLUTO-VIS, Holoeye). The SLM strongly depends on the polarization, i.e.,
only the component of the incident electric field parallel to the liquid-crystal director (representing the direction along which the LC molecules are aligned) can be purely phase modulated, whereas the perpendicular component can not be modulated. Therefore, a polarizer is inserted in front of the SLM. In the experiment, the SLM is employed to load the dynamic random phase, and the maximal phase shift for the wavelength of 632.8 nm is $2.1\pi$. The coherence of the beam is measured by double-pinhole interference experiment.

From Eq. (9), we deduced that the coherence of the beam decreases gradually with the increasing control parameter $a$. Thus, a partially coherent beam with coherence decreasing with time can be generated when the random phase on the SLM is modified with $a$ that also increases with time. We set the control parameter $a$ to increase from 0 to 1 with a step of 0.1, producing 11 dynamic processes. This is crated following the next procedure. Start generating 200 random phase maps for each given control parameter. Then, a 5-s dynamic process with a frame rate of $33/s$ is produced by randomly combining these 200 phase maps. Another 5-s dynamic process is produced for another control parameter and so on. Finally, a 55-s movie was produced by combining these 11 dynamic processes with $a$, which gradually increased from 0 to 1. A partially coherent beam can be produced by reflecting a laser beam from the SLM displaying the movie. The result is shown in Fig. 4. As expected, a partially coherent beam was created with its coherence kept nearly invariant within a lapse of 5 s for a given $a$. The increase in this value decreased the coherence degree to a new value, which remained constant through 5 s interval.

The coherence is measured by the average intensity of interference fringes of double-pinhole experiment within each 5 s. The time range is much longer than the refresh time of SLM with a frame rate of 60 Hz. This can be seen as the realization of the ensemble average of partially coherent beam. The coherence curve was measured for 10 times, and the error bar represents the maximum and minimum coherence of the measurement. Moreover, the dot in the curve is the average of these 10 measurements.

According to the theoretical prediction, the coherence degree of a beam is 0 when the control parameters $a$ is equal to 1. It is seen that in experiment the minimum value of degree of coherence is about 0.2. Some experimental limitations might have caused this difference: first, the refresh speed of the SLM was too slow to achieve a completely incoherent light, due to the fact that the frame rate of the SLM is only 60 Hz. Second, in theory the light source is assumed to be ergodic. However, the random phase we generated in experiments by MATLAB® is discrete but not continuous. At the last, the partially coherent beam is generated through an SLM imposing a random phase onto the coherent laser beam. This means that the partially coherent light is produced, as the light beam reflected just from the SLM. The experimental measurement for the coherence degree of the generated partially coherent light beam is performed by double-pinhole interference experiment, which is placed behind the SLM. This distance may slightly affect the coherence properties of the light at the position of the pinhole.

Now, we showed that in a similar manner, according to our theoretical analysis above, the coherence of the beams can be gradually increased when the control parameter of the random phase decreases. Following an analogous process,
we produced a second movie. The control parameter of the dynamic phase was kept fixed for 10 s, and then it is decreased to a new value for the next 10 s. Figure 5 shows the evolution of the degree of coherence of a laser beam reflected by the SLM that imprinted the dynamic random phase on the beam. During the time of 0 to 10 s, the control parameter of the dynamic phase is chosen as 1, and the corresponding coherence is 0.17. We then decreased the control parameter of the phase, which led to an increment of the measured coherence (Fig. 5). When the control parameter was 0, the coherence of the beam was 0.996. This finding indicates that the beam was practically coherent.

In the final example we show that the beam coherence can be modulated periodically by selecting an appropriate dynamic random phase. For this case, we produced a 300-s dynamic random phase movie. Using the same steps of the control parameter, the dynamic random phase was changed in periods of 30 s, which was initially increased up to its fifth value and then decreased until the last one. The coherence of the beam imposed by this dynamic random phase shows a V-shape, as shown in Fig. 6. Repeating this process would show a periodic behavior.

4 Conclusions
In conclusion, we produced partially coherent beams with time-dependent coherence. We theoretically predicted that a partially coherent beam could be generated by imposing dynamic random phase on to a completely coherent beam. The value of the degree of coherence of the partially coherent beam depends on a given parameter that allows control of the range of the dynamic random phase. This was confirmed experimentally. A completely coherent beam reflected by an SLM with dynamic random phase transforms into a partially coherent beam with a previously selected degree of coherence. Therefore, creating partially coherent beams with decreasing, increasing, and periodic degree of coherence as function of time is feasible. Time-dependent partially coherent beams can have potential applications in laser processing, imaging, and particle manipulation.

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References

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