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Abstract. Aperture losses and thermo-optic effects (TOE) inside optics as well as the effective beam width in far field should be taken into account in the analysis of the most appropriate laser beam profile for high-power applications. We have theoretically analyzed such a problem for a group of super-Gaussian beams taking first only diffraction limitations. Furthermore, we have investigated TOE on far-field parameters of such beams to determine the influence of absorption in optical elements on beam quality degradation. The best compromise gives the super-Gaussian profile of index \( p = 5 \), for which beam quality does not decrease noticeably and the thermo-optic higher order aberrations are compensated. The simplified formulas were derived for beam quality metrics (parameter \( M^2 \) and Strehl ratio), which enable estimation of the influence of heat deposited in optics on degradation of beam quality. The method of dynamic compensation of such effect was proposed.© The Authors. Published by SPIE under a Creative Commons Attribution 3.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI. [DOI: 10.1117/1.OE.57.4.046107]

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1 Introduction

One of the specific tasks of laser engineering is the transformation of high-power laser beam satisfying special requirements on beam waist size in far field and additionally the restrictions on beam forming optics (sizes, aperture losses, thermal effects, etc.). From the practical point of view, the main limitations in near-field (i.e., inside optics) are posed by thermal effects especially for multi-kWatt, complex, multielement optical systems. The “rule of thumb” in engineering practice is that optics should have aperture of about 2 times larger than the laser beam diameter defined for Gaussian profile at \( 1/e^2 \) level, which is not a good choice for mitigation of thermal effects by the way. Thus, the question on the optimal laser beam profile which mitigates thermal effects in near-field optics not degrading brightness and beam width in far field is still open. Let us note that the laser beam quality has to be determined according to \( D_{86.5} \) definitions (see Ref. [1]). Strehl ratio and second moment definitions are not appropriate measures for such a case. Therefore, beam parameter product (BPP) \( \pi w_{86.5} \theta_{86.5} / \lambda \) defined on the basis of \( D_{86.5} \) will be used here as a metrics.

To determine the best laser beam profile, the minimization of aperture losses and thermo-optic effects (TOEs) in near-field as well as the minimal effective width of laser beam in far field should be taken into account. Let us notice that the best from the thermo-optic limitations “top-hat” profile results in far field in multilobe, “sombrero”-like shape, which has wider \( D_{86.5} \) diameter than equivalent Gaussian one. On the other hand, for Gaussian profile of given \( 1/e^2 \) diameter in the near-field, the clear aperture should be wider (~2 times), which significantly increases TOEs. Thus, it is an evident trade-off between the above requirements in near and far fields. Moreover, we have to take into account the manufacturability of beam shaper destined to transform Gaussian beam to the most appropriate shape.

To start the analysis, we have to choose the appropriate basis of beam profiles (see Refs. [2] and [3]). The most convenient for our purposes seems to be the super-Gaussian beams (SGB)\(^2, 3\) and flattened Gaussian beams (FGB)\(^4, 5\). In both cases, the analytical method for \( M^2 \) parameter calculations was found\(^6, 7\) whereas for BPP calculations the numerical approach is required. Both approaches (SGB and FGB) should lead to the same results in principle.\(^8\) We have taken for our purposes the SGBs because of the simpler mathematical description important for numerical simulations and wide diversity of shapes especially for close to Gaussians profiles. Moreover, the technical realization of diffraction limited beam shapers transforming Gaussian beam into SGB is well known (see e.g., Refs. [8] and [9]).

In Sec. 2, we have analyzed such a problem for SGBs taking only diffraction effects into account. In Sec. 3, we have analyzed TOEs for such cases. The typical laser system consists of several elements, including mirrors and refractive elements, and each experiences the TOEs as a result of residual absorption on surfaces and in the volume. We have modeled the influence of TOEs on the far-field parameters of SGB beams applying COMSOL software\(^8\) for two cases: volume heat density source and surface heat source for idealized heat contacts. The results of numerical analysis undertaken for given heat power can be rescaled to other cases applying the simple relations given at the end of paper.

2 Diffraction Analysis for Super-Gaussian Beams

To analyze the parameters of the laser beam of a different profile in far field, we have chosen a family of SGB defined in near-field as follows:
where $SG_p$ is the amplitude function of SGB of index $p$, $N_p(w_p) = \pi w_p^2 \Gamma(1/p) 2^{-1/p} p^{-1}$ norm of $SG_p$, accomplishing the unit power of each beam, $\Gamma(x)$ is the gamma function, and $w_p$ is the radius of beam.

Furthermore, we calculated the amplitude functions $SG_{ff,p}$ in far field applying Hankel transform valid for cylindrical symmetry (see details in Ref. 5) as follows:

$$SG_{ff,p}(r_{ff}) = c_p \int_0^{r_{ff\max}} J_0 \left( \frac{kr_{ff}\rho}{f} \right) SG_p(\rho, w_p) d\rho,$$

where $c_p$ is the constant and $f$ is the focal length.

The intensity profiles in near-field for $p = 1, 2, 5$, and 32 are shown in Fig. 1 and corresponding intensity distributions in far field are shown in Fig. 2. For each $SG_p$, the different beam radii $w_p$ were taken in such a way that it contains 99.95% of $SG_p$ power in the same aperture $D_{aper} = 2W_{aper}$ (which corresponds to $W_{aper} = 2w_1$ for Gaussian beam $p = 1$).

To define beam diameter in far field, power in bucket (PIB) distributions (see Ref. 6) were calculated as follows:

$$PIB(r_{ff}, p) = \frac{\int_0^{r_{ff}} I_{ff,p}(\rho) d\rho}{\int_0^{r_{ff\max}} I_{ff,p}(\rho) d\rho},$$

where $I_{ff,p}(r_{ff}) = |SG_{ff,p}(r_{ff})|^2$ is the intensity profile of $SG_p$, in far field and $r_{ff\max}$ is the radius in far field, beyond which the intensity is negligible for all $p$ (see Figs. 2 and 3).

Knowing PIB distributions, we have determined the beam radius $R_{ff,86.5,p}$ according to criterion of 86.5% of PIB.

Let us notice (see Fig. 2) that for $p = 1$, we have Gaussian beam, whereas for $p = 32$ we have nearly top-hat profile. The criterion of 86.5% of PIB, frequently used in laser engineering (see e.g., Refs. 6 and 4), corresponds to classical $1/e^2$ diameter definition of Gaussian beam. Strictly according to that criterion $SG_{32}$ is better, but it has multilobe shape with wide pedestals in far field (see Figs. 2 and 3), moreover, technical realization of cost-effective beam shapers for such profile is problematic. Applying higher level of criterion, e.g., 95%, the lowest diameter has the $SG_3$ beam and similar smooth profile in far field as Gaussian with <5% in pedestal (see Fig. 3).

We can now determine the optical measures being the merit functions in analysis: relative brightness $B_p$, effective beam parameter product $BPP_p$, and Strehl ratio $SR_p$ as follows:

$$B_p = A_{nf,p}A_{ff,1}/A_{nf,1}A_{ff,p},$$

$$BPP_p = \sqrt{B_1/B_p},$$

$$SR_p = I_{ff,p}(0)/I_{f,p\max}(0),$$

where $A_{nf,p} = \pi R_{nf,86.5,p}^2$ and $A_{ff,p} = \pi R_{ff,86.5,p}^2$ are the areas of $SG_p$ beam in near and far field, respectively, $R_{nf,86.5,p}$ and $R_{ff,86.5,p}$ are the beam radii of $SG_p$ beam in near and far field determined according to 86.5% criterion, respectively.

To assess the flatness of $SG_p$ profile in near-field, we have introduced additional merit function $MF_{nf,p}$ as a ratio of maximal intensity of $SG_1$ and $SG_p$ as follows:

$$MF_{nf,p} = |SG_1(0, w_1)/SG_p(0, w_p)|^2.$$
It is easy to show that for perfect top-hat beam \((p \to \infty)\) \(\text{MF}_{\text{nf,}\infty} = 8\). In Table 1, the results of calculations for a few SG\(_p\) beams were collected.

Let us notice that taking into account only the maximum of brightness (or equivalent minimum of BPP), the best is Gaussian beam \((p = 1)\) evidently. For the almost top-hat \((p = 32)\) beam of equivalent aperture, 43% drop in brightness and 33% increase in BPP were found. However, if we additionally have to consider the profile flatness in near-field, which affects temperature profile and thermally induced distortions, the answer becomes more complicated. Taking both near- and far-field requirements, the best compromise seems to be SG\(_5\) \((p = 5)\) beam (blue curves in Figs. 3(a)-3(b)), which has smooth profile in far field, slightly increased BPP \((\text{BPP}_5 = 1.259)\), but much more flattened profile in near-field \((\text{MF}_{\text{nf,5}} = 4.77)\). Moreover, as we will show in Sec. 3, such profile enables nearly compensation of higher order TOEs.

### Table 1 Results of calculations of merit functions for \(p = 1, 2, 5, \) and 32.

<table>
<thead>
<tr>
<th>(p)</th>
<th>(p = 1)</th>
<th>(p = 2)</th>
<th>(p = 5)</th>
<th>(p = 32)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{MF}_{\text{nf,p}})</td>
<td>1.000</td>
<td>2.55</td>
<td>4.77</td>
<td>6.75</td>
</tr>
<tr>
<td>(B_p)</td>
<td>1.000</td>
<td>0.85</td>
<td>0.63</td>
<td>0.569</td>
</tr>
<tr>
<td>(\text{BPP}_p)</td>
<td>1.000</td>
<td>1.085</td>
<td>1.259</td>
<td>1.326</td>
</tr>
<tr>
<td>(\text{SR}_p)</td>
<td>0.576</td>
<td>0.758</td>
<td>0.945</td>
<td>1.000</td>
</tr>
</tbody>
</table>

3 Modeling of Thermal Optics Effects for SG Beams

To investigate the influence of TOEs on beam quality, we have taken the typical optical element of 50-mm diameter, 10-mm thickness made of fused silica. The “idealized” heat contacts (constant temperature on side, and negligible heat transfer to rear and front facet) were assumed. The eight different heat sources (for each SG\(_p\), pair of volume or surface source) were considered.

To compare the effects of such eight different heat sources, we have assumed that the same heat power of 1 W (corresponding to 10-ppm heat conversion for incident 100 kW of laser power) was deposited in optical element. Let us notice that in the best quality fused silica glass, the absorption coefficient is much lower than 10\(^{-5}\) 1/cm. The surface absorption is determined by quality of surface itself and the absorption in dielectric coatings. It is possible to achieve the same level of absorption losses in the highest quality mirrors. However, as a result of technical imperfections and superposition of different factors, the realistic value of absorption is estimated of a few dozens of ppm.

For each case, the problem was solved applying COMSOL software and three-dimensional (3-D) maps of temperature increase were calculated [see examples for SG\(_1\) and SG\(_{32}\) beams in Figs. 4(a)-4(d)].

Next, we calculated the profile of averaged temperature \(\Delta T_{\text{avg},i}\) versus radius \(r_i\) integrating temperature profile over \(z\)-depth. Then, multiplying by effective thermal dispersion coefficient \(\chi_T\) and \(b\)—thickness of sample, the thermally induced optical path difference (OPD) was determined

\[
\text{OPD}(r, P) \equiv \chi_T \cdot \Delta T_{\text{avg},i}(r, P) \frac{b}{\lambda}.
\]

where the effective thermal dispersion for transmission is defined as

\[
\chi_T = \frac{dn}{dT} + \left(1 + v_{\text{Poisson}}\right)\alpha_T(n - 1) + \cdots.
\]

The OPD can be divided into paraxial component \(\text{OPD}_{\text{par}}\) and residual nonparaxial \(\text{OPD}_{\text{np}}\) according to the following equation:

\[
\text{OPD}(r, P) = \text{OPD}_{\text{par}}(r, P) + \text{OPD}_{\text{np}}(r, P).
\]

where \(M_T\) is the paraxial thermal optical power.

To determine OPD dependence on radius, the mean square approximation of data array \((r_i, \Delta T_{\text{avg},i})\) calculated in COMSOL was applied as follows:

\[
\Delta T_{\text{approx}} = \sum_{m=1}^{N} a_m r^{2m}.
\]

After a few simple transformations

\[
M_T = 2 \cdot \chi_T b \cdot a_1,
\]

where \(a_1\) is the quadratic coefficient of power series of mean square approximation of data array \((r_i, \Delta T_{\text{avg},i})\).

Residual, nonparaxial part of \(\text{OPD}_{\text{np}}\) corresponds to higher order thermally induced distortions [see Figs. 5(a) and 5(b) resulting in beam quality degradation.

The surface heat sources [Fig. 5(b)] result in larger OPDs comparing to volume absorption, which agrees well with intuition and engineering practice. The magnitude of near top-hat \(\text{OPD}_{\text{np}}\) \([\text{SG}_{32}\) black curves in Figs. 7(a) and 7(b)] is much smaller and has the opposite sign regarding \(\text{OPD}_{\text{np}}\) calculated for Gaussian beam. Therefore, we can suppose that the proper choice of beam profile can result in near compensation of nonparaxial OPDs at least. It was shown in Figs. 7(a) and 7(b) (green curves) that for SG\(_5\) beam such effect exists, which gives another argument for its beneficial properties in a case of high-power applications. Moreover, having beam shaper transforming SG\(_1\) \(\rightarrow\) SG\(_5\) and playing with reflecting and refractive elements, we can achieve dynamic compensation of nonparaxial OPDs of all system for variable incident laser power.

To determine the impact of TOEs on laser beam metrics, we have to calculate in first step variance \(\sigma = \text{rms(\text{OPD}_{\text{np}})}\) taking into account weighting functions corresponding to given beam profile. Furthermore, we calculated Strehl ratio \(\text{SR}_{\text{apr}}\) and \(M^2_{\text{apr}}\) parameter applying the following approximated equation:

\[
\text{SR}_{\text{apr}} = \exp[-(2\sigma)^2]; \quad M^2_{\text{apr}} = \exp[2(\sigma)^2].
\]
calculations made for lower power of 0.5 W showed the similar dependencies, only magnitudes have changed proportionally to absorbed power. Thus, we can conclude that the OPDs shown in Figs. 5(a) and 5(b) are typical for given type of heat source in the framework of linear heat diffusion approximation.

According to theoretical predictions and numerical examples showed here, the values of resulting rms(OPD) and
Table 2 Results of calculations for eight types of heat sources.

<table>
<thead>
<tr>
<th>Type of source</th>
<th>( M_T ) (1/km)</th>
<th>rms OPD&lt;sub&gt;np&lt;/sub&gt;</th>
<th>( M_{\text{appr}} )</th>
<th>SR&lt;sub&gt;appr&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG&lt;sub&gt;1&lt;/sub&gt;volume</td>
<td>2.62</td>
<td>0.128</td>
<td>1.378</td>
<td>0.526</td>
</tr>
<tr>
<td>SG&lt;sub&gt;1&lt;/sub&gt;surface</td>
<td>4.05</td>
<td>0.198</td>
<td>2.16</td>
<td>0.214</td>
</tr>
<tr>
<td>SG&lt;sub&gt;2&lt;/sub&gt;volume</td>
<td>0.59</td>
<td>0.012</td>
<td>1.003</td>
<td>0.994</td>
</tr>
<tr>
<td>SG&lt;sub&gt;2&lt;/sub&gt;surface</td>
<td>2.32</td>
<td>0.047</td>
<td>1.045</td>
<td>0.916</td>
</tr>
<tr>
<td>SG&lt;sub&gt;5&lt;/sub&gt;volume</td>
<td>0.17</td>
<td>0.0016</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>SG&lt;sub&gt;5&lt;/sub&gt;surface</td>
<td>1.23</td>
<td>0.012</td>
<td>1.003</td>
<td>0.994</td>
</tr>
<tr>
<td>SG&lt;sub&gt;32&lt;/sub&gt;volume</td>
<td>0.074</td>
<td>0.0017</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>SG&lt;sub&gt;32&lt;/sub&gt;surface</td>
<td>0.77</td>
<td>0.017</td>
<td>1.006</td>
<td>0.998</td>
</tr>
</tbody>
</table>

Note: \( M_T \), optical power; rms OPD<sub>np</sub>, root mean square of nonparaxial OPD<sub>np</sub>; SR<sub>appr</sub>, Strehl ratio; \( M_{\text{appr}} \), beam quality parameter; \( P_{\text{heat}} = 1 \) W, cylinder sample made of fused silica, \( D_{\text{appr}} = 50 \) mm, and \( b = 10 \) mm.

\( M_T \) is proportional to effective absorption or more generally to heat power deposited in the element. Thus, knowing parameter \( M_{T,0} \) calculated for given heat power \( P_{\text{heat},0} \), we can determine the optical power \( M_{T,1} \) for the different heat power \( P_{\text{heat},1} \) as follows:

\[
M_{T,1} = \frac{\kappa_{1,0}}{\kappa_{0,0}} M_{T,0},
\]

where \( \kappa_{1,0} = \frac{P_{\text{heat},1}}{P_{\text{heat},0}} \) is the ratio of heat powers for cases 0 and 1.

The remaining metrics of optical quality (Strehl ratio SR<sub>appr</sub> and parameter \( M_{\text{appr}}^{2} \)) are highly nonlinear with respect to \( \sigma \). However, applying the same approach, we can determine the similar relations for \( SR_{\text{appr},1}^{2}, M_{\text{appr},1}^{2} \) knowing the parameters \( SR_{\text{appr},0}^{2}, M_{\text{appr},0}^{2} \) as follows:

\[
M_{\text{appr},1}^{2} = \left( M_{\text{appr},0}^{2} \right)^{2}\frac{\kappa_{1,0}^{2}}{\kappa_{0,0}^{2}}, \quad SR_{\text{appr},1} = \left( SR_{\text{appr},0} \right)^{2}\frac{\kappa_{1,0}}{\kappa_{0,0}}.
\]

Let us notice that the simplest way to mitigate TOEs is increasing the laser beam size in aperture, e.g., taking collimator of longer focal length and wider aperture. As a rule, lowering power density, the temperature increase basically diminishes.\(^{10,14}\) As was shown in Refs.\(^{15,16}\), the average temperature increase is proportional to absorbed power and has nonlinear dependence on laser beam diameter.

An alternative method of TOEs diminishing is application of more flattened than Gaussian laser beam profiles. As shown above, the profile \( SG_{3} \) should give much lower rms(OPD<sub>np</sub>) comparing to Gaussian one. On the other hand, the typical beam shaper used to transform \( SG_{1} \rightarrow SG_{3} \) consists of at least two optical elements (see Refs.\(^{8}\) and\(^{10}\), with the first one exposed to Gaussian beam. For complex, multielement optical trains, or multipass systems, e.g., beam cleaners,\(^{13}\) dynamic mitigation of TOEs of all system can be achieved by application of appropriate beam shaper and playing with mirrors and refractive elements. It is also important for transient, nonstationary regime of operation typical, e.g., laser weapon engagement.

In summary, we have prepared the numerical method to estimate the TOEs for wide class of optical elements valid for linear heat equation and small absorption approximation. In first step, we have to solve numerically the problem for given type of heat source, absorption efficiency, and sample geometry. The results can be rescaled for different absorptions or heat powers applying Eqs. (14) and (15). The paraxial thermal lensing (determined by \( M_T \)) can be compensated by defocusing. The thermo-optic distortions can be transient, time-dependent functions, and for short operation time and relatively low duty, factor can be low as well. The beam quality deteriorates during operation up to the worst case of stationary value typical, e.g., for industry applications. Equations (14) and (15) can be applied also for the estimation of those unstationary, transient effects, knowing additionally time constants of elements. Let us insist on the limits of above approximations. It is valid only for a case of linear, heat equation of constant coefficients, small stresses, and low absorption. Moreover, for higher (comparable with wavelength) variances of OPD, the approximated Eq. (13) is not valid.

### 4 Conclusions

To achieve the high beam quality and flattened profile in near-field, several beam profiles were analyzed. The SGB \( SG_{p} \) of index \( p = 5 \) was found as the best compromise.

Furthermore, we have developed the simplified numerical-analytical model of TOEs to estimate acceptable level of heat power dissipated on surfaces and in volumes of optics. Such analysis was performed for several \( SG_{p} \) profiles. In first step, we have to solve problem numerically for given type of heat source, absorption, and sample geometry. The results can be rescaled for different absorption or heat powers applying approximated formulas derived in the paper. The model can be applied also for the estimation of unstationary, transient effects. Moreover, the method of dynamic compensation of TOEs due to application of the appropriate beam profile and combinations of lenses and mirrors was proposed.

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### References


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