Abstract. We review approaches for extending the depth of focus of different imaging systems including the human vision system. Approaches involving digital postprocessing as well as different types of all-optical techniques are discussed. © 2010 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/6.0000001]

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1 Theoretical Background

Optical imaging systems are capable of high lateral resolution only in an axial range called either the depth of focus or the depth of field, depending on whether it is measured in image space or object space, respectively. From a wave optics perspective, limited depth of focus arises because defocusing introduces an additional quadratic phase in the system pupil function, resulting in a spatial low-pass filter effect. This effect can be described mathematically by means of the optical transfer function (OTF) of the system, $H(\mu_x, \mu_y)$. The OTF of a single-lens imaging system can be expressed as a properly scaled autocorrelation of the lens pupil function $P(x, y)$ [1]:

$$H(\mu_x, \mu_y) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P \left( x + \frac{\lambda Z_i \mu_x}{2}, y + \frac{\lambda Z_i \mu_y}{2} \right) P^* \left( x - \frac{\lambda Z_i \mu_x}{2}, y - \frac{\lambda Z_i \mu_y}{2} \right) \, dx \, dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |P(x, y)|^2 \, dx \, dy},$$

where $\lambda$ is the optical wavelength, $\mu_x, \mu_y$ are the spatial frequencies, and $Z_i$ is the distance from the lens to the image plane. For a circular aperture and in the absence of aberrations, $P(x, y)$ is given by the binary circle pupil function, which equals 1 within the pupil and 0 outside. In this case the OTF can be written simply as

$$H(\mu_x, \mu_y) = \frac{\text{Area of overlap}}{\text{Total area}} = \frac{\int_{A(\mu_x, \mu_y)} dx \, dy}{\int_{A(0,0)} dx \, dy},$$

where $A(\mu_x, \mu_y)$ gives the area of overlap between two shifted pupils and $A(0,0)$ the total area of the pupil. When aberrations are introduced, the generalized pupil function takes the form

$$P(x, y) = |P(x, y)| \exp \left( i k W(x, y) \right),$$

where $W(x, y)$ is the wave aberration function and $k = 2\pi/\lambda$. In the case of defocus, $W(x, y)$ has the quadratic form

$$W(x, y) = W_m \frac{(x^2 + y^2)}{b^2},$$
where $b$ is the radius of the aperture. The coefficient $W_m$, which determines the severity of the defocusing error, is given by

$$W_m = \frac{\Psi \lambda}{2\pi},$$

(5)

where $\psi$ is defined by

$$\psi = \frac{\pi b^2}{\lambda} \left( \frac{1}{Z_i} + \frac{1}{Z_o} - \frac{1}{f} \right),$$

(6)

where $Z_o$ is the distance from the object to the lens and $f$ is the focal length of the lens. When the imaging condition is fulfilled,

$$\frac{1}{Z_i} + \frac{1}{Z_o} = \frac{1}{f},$$

(7)

and the distortion factor $\psi$ equals zero. When the imaging condition is not fulfilled, the quadratic phase factor of Eqs. (3) and (4) leads to a narrower OTF distribution, i.e., to a low-pass effect or suppression of higher spatial frequency content. For the 1-D case Eq. (1) becomes:

$$H(\mu, W_m) = \int_{-\infty}^{\infty} \frac{P \left( x + \frac{\lambda Z_i \mu}{2} \right) P^* \left( x - \frac{\lambda Z_i \mu}{2} \right) dx}{\int_{-\infty}^{\infty} |P(x)|^2 dx}$$

$$= \frac{\int_{A(\mu)} \exp \left[ \frac{ik W_m 2\lambda Z_i \mu x}{b^2} \right] dx}{2b},$$

(8)

where $A(\mu)$ is the 1-D overlap between two shifted pupil functions. Evaluating Eq. (8) gives the 1-D OTF:

$$H(\mu) = \left( 1 - \frac{\left| \mu \right|}{\mu_{c.o.}} \right) \text{sinc} \left( \frac{8W_m \pi}{\lambda} \left( \frac{\mu}{2\mu_{c.o.}} \right) \left( 1 - \frac{\mu}{2\mu_{c.o.}} \right) \right),$$

(9)

where $\mu_{c.o.} = \frac{b}{\lambda Z_i}$. This expression for $H(\mu)$ can be approximated by

$$H(\mu) \approx \frac{\int_{A(0)} \exp \left[ \frac{ik W_m 2\lambda Z_i \mu x}{b^2} \right] dx}{2b}$$

$$= \frac{\int_{-b}^{b} \exp \left[ \frac{4\pi i W_m Z_i \mu x}{b^2} \right] dx}{2b}$$

$$= \frac{1}{2b} \cdot \int_{-\infty}^{\infty} \text{rect} \left( \frac{x}{2b} \right) \exp \left[ \frac{4\pi i W_m Z_i \mu x}{b^2} \right] dx$$

$$= \text{sinc} \left( \frac{4W_m \pi \mu Z_i}{b} \right).$$

(10)

The approximation is valid for values of $\mu$ that are not too large in comparison to $b/\lambda Z_i$, such that $A(\mu)$ can be approximated by $A(0)$, the area of the lens. This approximation is also valid for large values of $W_m$, causing the argument of the exponent to oscillate rapidly. The expression of Eq. (10) demonstrates the spectral low-pass filtering effect.
In the case of a 2-D circular aperture, the resulting OTF, which has only radial frequency dependence, is derived in Ref. 2, where it was represented as a serial expansion in $J_n$, i.e., $n^{th}$-order Bessel functions of the first type.

If one wishes to estimate the depth of focus it is easily shown to be approximately proportional to the product of the wavelength $\lambda$ and the square of the $f$ number (the ratio between the focal length and the diameter of the imaging lens), i.e., $\lambda(f/2b)^2$. The reason is easily seen from Fig. 1: Since the diffraction-determined resolution limitation is proportional to $\lambda(f/2b)$ (the dimensions of the point spread function) and the geometrical angle at which the optical rays diverge is $(2b)/f$, one obtains that the defocused spot, which equals the depth of focus range multiplied by the angle $(2b)/f$, should be proportional to $\lambda(f/2b)$. From this relation the conclusion is indeed that the depth of focus range is proportional to $\lambda(f/2b)^2$.

2 General Overview: Approaches to Extend the Depth of Focus

Following the development of photography, various methods were investigated for overcoming the defocusing limitation described in Section 1. The simplest method is simply to reduce the aperture of the imaging lens. The reduction may be gradual, as in aperture apodization, or abrupt, through the addition of a binary blocking/transmitting mask in the aperture plane [3,4]. Unfortunately smaller apertures reduce system resolution [making the OTF of Eq. (1) narrower] and also reduce the amount of light reaching the image plane. Other approaches were therefore developed. One solution relies on the addition of refractive elements in the aperture of the imaging system. One of the most popular elements is the axicon [5,6]. The operational principle of this element is illustrated in Fig. 2. In the region of overlap of the beams being diverted by the axicon, denoted by dashed lines in the figure, an extended depth of focus (EDOF) region is obtained. For comparison, the dashed blue lines show the original ray tracing in the absence of the axicon.

Another type of refractive element-based solution involves the multiplexing of several lenses, each having a different focal length. An example of spatial multiplexing is that exploited in “progressive” or multifocal spectacles (e.g., bifocal [7] or multifocal lenses [8,9]). In these lenses the two (or more) focal lengths are spatially separated and allow the required in-focus performance only over a small portion of the field of view. The different lenses divide the plane of the lens aperture and thus, since every lens covers only a limited portion of the aperture, the
system has a larger effective f number and thus reduced resolution. On the other hand, if the application is spectacles, then such a solution limits the visual field, since in order to choose the focal length allowing focusing to the desired distance one needs to choose the proper line of sight.

A different way of multiplexing several lenses is by code multiplexing. In this method the multiplexed lenses having different focal lengths are divided into very small pieces that are randomly spread over the entire aperture plane. This type of solution usually refers to a plurality of diffractive lenses. Each lens covers the full aperture, and thus the resolution of each lens is not reduced [10–12]. Such code multiplexing can be implemented either by randomly spreading the pieces of the lenses or by periodically spreading each lens over the entire aperture plane. Because of the dense spatial variations in the aperture plane due to the multiplexing of the various pieces of the lenses, a significant portion of the light energy is diverted through diffraction through large angles. The result is reduced energetic efficiency and glare effects. As in all diffraction-based solutions, the system exhibits significant chromatic aberrations, since the focal length of a diffractive lens is wavelength dependent.

Another EDOF technique employs a cubic phase element attached to the imaging lens [13]. The idea involves what is basically the insertion of aberrations that are much stronger than the defocusing aberrations such that by digital postprocessing a sharp image can be reconstructed. In contrast to the previous approach, this type of solution is not an all-optical approach but rather one that requires digital postprocessing and thus does not fit to ophthalmic, i.e., vision correction, applications. Other interesting aperture coding techniques requiring digital postprocessing are discussed in Refs. 13–15 and a lens apodization technique in Ref. 16. Additional related technologies involve the tailoring of the modulation transfer functions with fractional-order phase plates [17] and with logarithmic asphere lenses [18].

A different and historically important approach to extended depth of focus involves the configuration of Scheimpflug [19], who by reorienting the imaging system provided what in theory can be infinitely extended focal depth. The sketch of Scheimpflug’s configuration is shown in Fig. 3(a) and its practical optical realization in Fig. 3(b).

In this configuration the following mathematical condition must be fulfilled:

\[ \frac{1}{a} + \frac{1}{b} = \frac{1}{d} \left( \tan B + \tan A \right) = \frac{1}{f}. \]  \hspace{1cm} (11)

The lines designated in Fig. 3(a) as Scheimpflug image and object planes are always in focus and thus objects positioned along those planes will always be in focus. The main drawback of this approach is that, as in the case of “progressive” lenses, the visual field is very limited since every axial distance is imaged at a different lateral position.

Recently several new all-optical approaches for extending the depth of focus have been developed. The development of those approaches is very important not only to digital imaging by cameras but also in ophthalmic vision correction. In one approach the depth of focus is

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**Fig. 3** Infinitely in-focus Scheimpflug planes [19]: (a) notations and (b) practical imaging configuration.
extended by the addition of a plurality of diffractive rings [20]. As previously noted, one
disadvantage in using diffractive elements is the accompanying chromatic aberrations. Another
approach, which is more of a refractive type and thus exhibits reduced chromatic aberration,
incorporates a nonbinary profile consisting of rings that is added on top of the aperture lens in
order to generate unbalanced optical path difference across the aperture [21].

A different all-optical direction providing an interference-based rather than diffractive/
refractive type of solution is suggested in Refs. 22 and 23. The basic idea is to view the
imaging lens as a component in an interferometer. Such an interpretation is possible since in
the focal point all the optical rays passing through the aperture add together. By proper addition
of an optical phase engraving on top of the imaging lens, desired constructive interference is
generated in a “focus channel” while destructive interference is created around it (see Fig. 4).
The added profile has no optical power of its own, but must be used in combination with
an imaging lens. The optical profile engraving has large spatial features and thus it is not a
diffractive type of element. Its etching depth is very small (about 1 μm) and thus it does not
add physical path difference as in refractive solutions. Because this approach is more energet-
cally efficient and exhibits reduced chromatic aberrations, it is thus suitable for ophthalmic
applications.

Another all-optical technique that extends the depth of focus involves the use of a birefringent
lens. Recently, a new approach was presented where a birefringent lens was fabricated [24] with
two focal lengths, one for the ordinary and the other for the extraordinary polarization state. It
is a monofocal lens that becomes bifocal due to the birefringence of the material from which it
is made. By proper design of the lens the two focal lengths can be chosen such that the focusing
range is extended to roughly double the focal depth [25]. However, the fabrication of such a
lens is complicated and expensive. Obviously this operation principle is valid only for the case
when the external illumination is nonpolarized, allowing the illuminating energy to be split
between the two focal points. A simplified solution includes using a regular monofocal lens and
adding a birefringent plate between the lens and the imaging plane [26]. The addition of the
plate generates different optical paths for the two polarizations and thus each one is focused a
different distance from the monofocal imaging lens. Assuming that the required difference in the
optical paths in free space is $\Delta$ and that the birefringent material has ordinary and extraordinary
refractive indexes of $n_o$ and $n_e$, respectively, then the width of the birefringent plate should be:

$$\Delta_B = \frac{\Delta}{\left(1 - \frac{n_o}{n_e}\right)}.$$  \hspace{1cm} (12)

Note that the last equation is valid for normal incidence of the incoming beam and that non-
normal incidence angles (mainly corresponding to the edges of the field of view) will not have
proper extension in the focal depth.
As previously mentioned, using a pinhole reduces the resolution and the energetic efficiency but significantly increases the depth of focus of a system. Thus, one possible improvement for achieving extension in the focal depth involves the usage of a random pinhole plate instead of an imaging lens [27,28]. In this case the energetic efficiency is increased, since there are many pinholes covering the lens aperture, and the resolution, although reduced, can be recovered by a proper digital inverse filtering operation. A nice feature of such a configuration is that, since the pinhole mask is random, the OTF has a random phase distribution that varies with distance. Therefore, the distance to various objects in the image can be extracted by properly correcting the phase of the OTF. Each phase correction will result in in-focus objects positioned at different axial distances. If digital postprocessing is considered, then the amount of defocusing can be estimated digitally from the image by observing the zeros generated in its Fourier plane [29,30].

The topic of extended depth of focus is strongly linked to beam shaping, and approaches for this purpose are relevant to this review. Numerical Gerchberg–Saxton-based [31,32] iterative algorithms [33] as well as analytically optimized 3-D point spread function approaches have been demonstrated as useful in properly designing the point spread function. Controlling the 3-D shape of the point spread function can be used for beam shaping as well as for extending the focal depth. The optimization procedure may either be obtained by descriptions based on generalized propagation-invariant wave fields [34,35] or by 3-D optimization using the calculus of variations [36,37]. The desired beam shaping can also be obtained by proper phase and amplitude apodizing of the aperture of the imaging lens as described in Refs. 38–42. In Ref. 41, for instance, the authors show how to tailor the depth of focus for an optical system using pupil functions obtained by applying the Fourier transformation tool.

3 Presbyopia and Human Vision

The human eye can focus on objects at different axial distances. This capability is called accommodation. The required range of accommodation is approximately 3.00 diopters, allowing focus from about 30 cm out to infinity. After the age of 45 the range of accommodation is noticeably reduced until by age 65 or so no such ability remains and the eye operates like a monofocal lens with fixed focal length. This reduction in accommodation power is called presbyopia. Bifocal and progressive spectacle lenses are designed to shift the direction of gaze and thus severely limit the functional visual field [7]. Diffractive optics-related solutions for presbyopia in contact lenses have not been able to penetrate the market.

The existing multi-focal-length contact lenses can be divided into the two categories of soft and rigid lenses. With soft contact lenses the operation principle is usually related to refractive rings for only two focal lengths or progressive transition through the radius of the lens [43]. The most popular lenses are PureVision® made by Bausch and Lomb. These are aspheric lenses with progressive change of the focal length. There are other bifocal spherical lenses made by Johnson & Johnson that are also based on refractive rings. In those bifocal solutions there is always one image in focus (the one corresponding to the object positioned in the relevant distance) and one is defocused. Thus, the brain must “learn” how to suppress the undesired defocused image while reinforcing the relevant in-focus image. This “learning” procedure require adaptation time.

With rigid contact lenses the operation principle is based on the fact that when one reads, the eyes are directed downward and the contact lens can be caught by the eyelid. The person can thus look through the peripheral part of the lens, which is designed to have a different focal length that is suitable for focusing at close ranges. This operation requires considerable practice from the subject. Examples of such types of lenses are Lifestyle and Gelflex. The main problem with these technologies is that they do not function well in low-light environments and have visual artifacts associated with glare and halo.

In cataract surgery the crystalline lens is usually replaced by a fixed monofocal intraocular lenses (IOL). The ophthalmic challenge is thus similar to that encountered in the case of presbyopia. With IOLs one of the most common approaches to extend the depth of focus is by...
physically reducing the aperture of the lens by means of a hole with reduced radius \([44–47]\). The problem, of course, is significantly reduced energetic efficiency.

There are also diffractive optics IOLs that are bifocal \([48–50]\) and that thus allow close-and distance-focused vision, but they exhibit large chromatic aberrations. They function well for green light but in other wavelength bands they lose their multifocal property and function as almost monofocal lenses. Examples of multifocal diffractive lenses are the ReSTOR® lens made by Alcon and the Acri.LISA lens made by Zeiss. There are also refractive lenses, such as the ReZoom® made by AMO \([51]\), but these lenses, as well as those based on diffraction, produce a discrete number of focal lengths and thus they provide no solution for the intermediate range, i.e., they allow reading and looking far away but not working at a computer because their depth of focus extension is not continuous as, e.g., in the all-optical solution of Refs. 22 and 23.

Another type of IOL is in fact accommodative \([52]\). In these lenses, the subject can achieve some accommodation after the implantation. There are two types of technological approaches for this category. In the first, a monofocal lens is positioned on an axial pivot such that when the lens is pressed with the muscles of the eye it is axially shifted \([53,54]\). An example is the CrystaLens lens made by Bausch and Lomb. In a variation of this method, a doublet is composed of two monofocal lenses designed in such a way that, when they are pressed by the muscles of the eye, their separation changes and, thus, the overall focal length changes as well \([55]\). The second technology includes construction of a lens made of a flexible material whose curvature, and thus its focal length, changes when the lens is pressed by the muscles of the eye \([56]\). The problem with these lenses is that they are unable to provide more than 1.00 diopter of accommodation, and even this small amount decreases with time. Also, this type of lens functions differently for different people, since each subject applies different force on the lens.

Another depth-of-focus-related ophthalmic aberration is astigmatism, observed when the eye has different focal lengths for transverse different axes. In regular astigmatism \([57]\), the meridians in which the two different curves lie are located 90 deg apart. In irregular astigmatism \([58]\), the two meridians may be located at something other than 90 deg apart; or there are more than two meridians. In their medical definitions, regular astigmatism is an astigmatism in which the refractive power of the eye shows a uniform increase or decrease from one meridian to another while an irregular astigmatism is when the curvature varies in different parts of the same meridian or in which refraction in successive meridians differs irregularly.

Irregular astigmatism is a common problem in cases when the astigmatism results from keratoconus or refractive surgery \([59,60]\). Cylindrical lenses or toric contact lenses provide a solution for regular astigmatism. However, no common solution for irregular astigmatism is currently available. The interferometric-based solution of Refs. 22 and 23, because it is rotation-invariant, allows regular and irregular astigmatism correction because, due to the extended depth of focus, if the astigmatism is lower than the extension in the focal depth (which in this approach was limited to 3.00 diopter \([61–65]\)), there will always be an axial position where the two focal points (after the extension) will coincide. This operation principle is schematically demonstrated in Fig. 5. In this figure one may see a lens with different horizontal and vertical focal lengths. An EDOF technique that is implemented in the lens design generates focal extension around each one of the two focal planes corresponding to each one of the two axes (each having its own focal length). Due to the EDOF there is a plane (marked in the figure) in which the two extensions overlap. For this axial position an image without astigmatic aberration will be formed.

Note that the extension in depth of focus can also be applied for correction of myopia \([66]\). This is especially important in the case of children, since children who start using lenses at an early age experience accelerated development of their myopia. The use of the interferometric EDOF solution, having no real optical power, can stop the progress of myopia \([66]\).

There are other EDOF techniques for addressing the presbyopia problem by modifying the corneal profile. One important method relates to laser-based refractive surgery \([67–70]\). One of the most popular refractive surgery procedures is laser-assisted in situ keratomileusis (LASIK). During the LASIK procedure, refractive surgeons reshape the cornea by removing precise
amounts of corneal tissue to correct the patient’s degree of refractive error or presbyopia. Today’s custom LASIK procedure incorporates the use of a wavefront map, which provides the LASIK surgeon with a 3-D map of the eye that can be transferred directly to the laser. IntraLASIK is similar to traditional ‘LASIK in the sense that it also involves corneal reshaping. The difference lies in the method used to create the flap during the first part of the procedure. The laser-assisted sub-epithelial keratomileusis (LASEK) procedure is another variation of LASIK. The main difference between LASIK and LASEK takes place when the flap is created. The LASEK procedure allows refractive surgeons to save more corneal tissue, making it an excellent treatment option for patients with thin corneas. Epi-LASIK is another variation of the LASIK procedure. During Epi-LASIK, an epikeratome is used to detach a thin layer of tissue in the epithelium. Once this layer of tissue is moved aside, the refractive surgeon can reshape the cornea as done in the traditional LASIK procedure. When corneal reshaping is complete, the refractive surgeon replaces the epithelial tissue and a special contact lens is introduced to promote healing.

4 Conclusions

In this paper we reviewed a variety of techniques for extending the depth of focus of imaging systems. This field is important for different types of imaging applications involving digital cameras and those designed to cope with presbyopia and regular/irregular astigmatism aberrations or for stopping the progress of myopia in children.

References


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