Temporal ghost imaging with a chaotic laser

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Abstract. We use a chaotic laser, instead of thermal light, as the light source in temporal ghost imaging. This laser is generated by employing an external optical feedback. The imaging magnification is varied by adjusting the group-delay dispersion parameters of the fibers. The temporal ghost imaging result is the convolution between the transmission function of the object and the temporal correlation functions of the chaotic laser. The simulation experiment, which uses a controllable time switch as the object, shows the effectiveness of our scheme. This scheme could find applications in the time-domain tomography of pulses. © The Authors. Published by SPIE under a Creative Commons Attribution 3.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI. [DOI: 10.1117/1.OE.52.7.076103]

Subject terms: temporal ghost imaging; chaotic laser; second-order correlation; Kobayashi equation.

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1 Introduction

Ghost imaging is a type of correlation imaging that utilizes the spatial or temporal correlation of incoherent light. In this technique, light is generally divided into two paths: the signal light and the idle light. The signal light illuminates an object, and the reflected or transmitted part is collected by a bucket detector. In contrast, the idle light propagates in the free space and is collected by a scanned detector. Finally, the spatial or temporal distribution information of the object is nonlocally reappeared through coincidence measurement between the bucket detector and the scanned detector.

Klyshko proposed the earliest ghost imaging scheme, which is based on the entangled behavior of photon pairs generated by the spontaneous parametric down-conversion. Later, Pittman et al. first realized ghost imaging using a two-photon entangled source. Subsequently, ghost interference, subwavelength interference, nonlocal double slit interference, and incoherent coincidence imaging with X-ray diffraction were successively proposed. At that time, ghost imaging was deemed as a nonlocal phenomenon in quantum theory, and the entanglement was considered a prerequisite for achieving ghost imaging. However, in 2002, Bennink et al. completed a ghost imaging experiment using a mirror random reflecting laser. In 2004, the Italian Lugiatto research group proposed that ghost imaging could be achieved using thermal light. Hence, an entangled light source is not a necessary condition for ghost imaging.
Ghost imaging was generally performed using the space correlation characteristics of the entangled light and the pseudo thermal light. However, these light sources have spatial as well as temporal correlation. Hence, the temporal ghost diffraction phenomena were reported when entangled two-photon pairs were employed as the light source. In 2008, Torres-Company et al. presented a ghost interference experiment with classical partially coherent light pulses. Their results are similar to those obtained by two-photon temporal entanglement. In their setup, a white-noise source was split into two different arms, and a temporal modulator was placed in only one arm. They theoretically proved that the intensity correlation measurements yield a temporal Fraunhofer pattern of the modulation gate function.

In our scheme, the Kobayashi model, which employs a Fraunhofer pattern of the modulation gate function. In the intensity correlation measurements yield a temporal correlation. In this scheme, fibers are employed in the feedback, can be written as follows:

\[
\begin{align*}
\frac{dE(t)}{dt} &= \frac{1}{2} \left( 1 + i\alpha \right) G_N \left[ N(t) - N_0 \right] - \frac{1}{\tau_c} E(t) \\
&\quad + kE(t - \tau) \exp(-i\omega_0\tau) \\
\frac{dN(t)}{dt} &= J - \frac{1}{\tau_c} N(t) - G_N \left[ N(t) - N_0 \right] |E(t)|^2
\end{align*}
\]

where \( \alpha \) is the line width enhancement factor, \( G_N \) is the differential gain, \( N_0 \) is the transparent carriers density, \( \tau \) is the feedback delay time, \( \tau_c \) is the carrier life, \( k \) is the feedback coefficient, \( \tau_N \) is the light round-trip time in laser cavities, \( \omega_0 \) is the laser angular frequency (\( \omega_0 = 2\pi c/\lambda \), where \( \lambda \) is the laser wavelength), and \( J \) is the pump current after charge normalization.

Since an analytical solution for Eq. (1) does not exist, we solve it numerically. The employed parameters are listed in Table 1.

Table 1: The parameters in the Kobayashi equation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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<tr>
<td>( \alpha )</td>
<td>4.5</td>
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<tr>
<td>( G_N )</td>
<td>( 2 \times 10^{12} )</td>
<td>m²s⁻¹</td>
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<tr>
<td>( N_0 )</td>
<td>( 10^{24} )</td>
<td>m⁻³</td>
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<tr>
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<td>2</td>
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<td>( k )</td>
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<td>( \lambda )</td>
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<td>( J )</td>
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<td>m⁻³s⁻¹</td>
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According to Fig. 1(a), waveform value randomly varies over time in 20 ns. At the same time, it is also partly cyclical. These features indicate that the signal \( E(t) \) is chaotic because chaos is just a state between random and determination. In Fig. 1(b), the frequency spectrum of the chaotic laser is shown, in which Fig. 1(b) is the frequency spectrum of chaotic laser, Fig. 1(c) is the space trajectory of chaotic laser, and Fig. 1(d) is the temporal correlation function of the chaotic laser.

2 Temporal Correlation of Chaotic Laser

There are several methods of generating a chaotic laser. In our scheme, the Kobayashi model, which employs a semiconductor laser, is used. Let \( E(t) \) represent the total electric field (including phase) in the laser cavity, and \( N(t) \) represent carrier density in the laser cavity. Then the Kobayashi equation, which is the electric field rate equation of a single-mode semiconductor laser under optical feedback, can be written as follows:

\[
\begin{align*}
\frac{dE(t)}{dt} &= \frac{1}{2} \left( 1 + i\alpha \right) G_N \left[ N(t) - N_0 \right] - \frac{1}{\tau_c} E(t) \\
&\quad + kE(t - \tau) \exp(-i\omega_0\tau) \\
\frac{dN(t)}{dt} &= J - \frac{1}{\tau_c} N(t) - G_N \left[ N(t) - N_0 \right] |E(t)|^2
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According to Fig. 1(a), waveform value randomly varies over time in 20 ns. At the same time, it is also partly cyclical. These features indicate that the signal \( E(t) \) is chaotic because chaos is just a state between random and determination. In
addition, according to Fig. 1(b), the flat and wide-band frequency spectrum illustrates that the energy is dispersed over a wide frequency range. This is one of the most important characteristics of chaotic laser.

In order to further illustrate that the solution of Eq. (1) is chaotic, its space trajectory is shown in Fig. 1(c). From this figure, we can see that its attractor is a singular attractor, and therefore, the dynamic system determined by Eq. (1) is a chaotic system. At the same time, we compute the largest Lyapunov exponent $\lambda_1 = 0.1554$, which is positive. This again confirms that the system is a chaotic system.

We now investigate the correlation function $\Gamma_0(t_1, t_2)$ of the chaotic laser, which is written as follows:

$$\Gamma_0(t_1, t_2) = \langle E^*(t_1)E(t_2) \rangle. \quad (2)$$

Because signal $E(t)$ is a stationary stochastic process, its correlation function is only related with the time difference $\tau = t_1 - t_2$. Then, Eq. (2) can be rewritten as follows:

$$\Gamma_1(\tau) = \langle E^*(t)E(t + \tau) \rangle = \frac{1}{T - \tau} \int_{0}^{T-\tau} E^*(t)E(t + \tau)dt, \quad (3)$$

where $T$ is the duration of chaotic laser. In fact, $\Gamma_1(\tau)$ and $\Gamma_0(t_1, t_2)$ are the same correlation function: $\Gamma_1(\tau) = \Gamma_0(t_1 - t_2) = \Gamma_0(t_1, t_2)$. However, in order to distinguish the number of the independent variables, we use different notations ($\Gamma_1$ and $\Gamma_0$) to represent the functions of a single variable and two variables. Using Eq. (3), we can obtain the temporal correlation function of chaotic laser, which is shown in Fig. 1(d). In this figure, we can observe an obvious peak that rapidly declines with an increase in time delay $\tau$. This indicates that when two times overlap, the chaotic laser has maximal correlation. When the time delay $\tau$ increases, the correlation quickly diminishes. These features show that the chaotic laser is suitable for use as the light source in the temporal ghost imaging.

3 Temporal Ghost Imaging with Chaotic Laser

3.1 Temporal Ghost Imaging Model

The setup of temporal ghost imaging is shown in Fig. 2. In this setup, we adjust the related parameters of the laser to obtain chaotic laser. A beam splitter divides the chaotic laser into two paths: the reference arm and the object arm. Both arms are sent into the fibers.

A temporal lens, whose characteristic function is $\exp(it^2/2\gamma)$, is placed on the reference arm, and an object is placed on the object arm in its optical path. The object is also called the time object because the final imaging result is the temporal characteristics of this object. The $m(t)$ is used to depict the time variation characteristics of the time object, e.g., a controllable switch on time, or lenses whose transmittance rates change over time, etc.

Two photoelectric detectors $D_1$ and $D_2$ are placed at the end of the two optical arms. The average optical intensity in the reference arm $I_1(t_1)$ ($E_1(t_1)E_1(t_1)^*$) and the total electric field in the object arm $I_2(t_2)$ ($E_2(t_2)E_2(t_2)^*$) are recorded by $D_1$ and $D_2$, respectively. The fields $E_1(t_1)$ and $E_2(t_2)$ jointly obey Gaussian statistics, and hence, the actual field coherence function $\Gamma_1(t_1, t_2)$ can be obtained from the correlation of the measured intensities in the two arms.

According to the linear systems theory of optical coherence propagation, the relationship between $\Gamma_1(t_1, t_2)$ and $\Gamma_0(t_1, t_2)$ is described as follows:

$$\Gamma(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Gamma_0(t_1', t_2')K_1(t_1-t_1')K_2(t_2-t_2')dt_1' dt_2'. \quad (4)$$

Here, $K_1(t)$ is the kernel function in the reference arm, and $K_2(t)$ is the kernel function in the object arm. In fact, the kernel function is the transmission function in the fiber channel, which can also be regarded as the unit impulse response of the system. According to Ref. 15, $K_1$ and $K_2$ can be written as

$$K_1(t,t') = \frac{1}{2\pi} \sqrt{\frac{i}{\Phi_a}} \sqrt{\frac{i}{\Phi_b}} \int_{-\infty}^{\infty} \exp \left( \frac{-it'^2}{2\gamma} \right) \exp \left[ -i \frac{(t'-t')^2}{2\Phi_a} - i \frac{(t'-t')^2}{2\Phi_b} \right] dt''. \quad (5)$$

$$K_2(t,t') = \frac{1}{2\pi} \sqrt{\frac{i}{\Phi_c}} \sqrt{\frac{i}{\Phi_d}} \int_{-\infty}^{\infty} \tilde{m}(t'') \exp \left[ -i \frac{(t''-t')^2}{2\Phi_c} - i \frac{(t''-t')^2}{2\Phi_d} \right] dt''. \quad (6)$$

where $\Phi_a$ and $\Phi_b$ are the group-delay dispersion parameters of the fiber in the reference arm, and $\Phi_c$ and $\Phi_d$ are the group-delay dispersion parameters of the fiber in the object arm.

If we directly compute $\Gamma(t_1, t_2)$ through the integration described in Eq. (4), the calculation will be related to a very complex quadruple integral. In order to reduce the calculation complexity, the following method is designed.

Step 1: Through convolution, Eq. (4) can be simplified as follows:
\[ \Gamma(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Gamma_1(t_1') K_1^*(t_1', t_1) K_2(t_2, t_1') dt_1' \times dt_1/dt_2' \]
\[ = \int_{-\infty}^{\infty} (\Gamma_1(t_1') \otimes K_2(t_2, t_1')) K_1^*(t_1, t_1') dt_1', \]  

(7)

where \( \otimes \) denotes the convolution operation, which is defined follows: let \( x(t_1) \) and \( y(t_1) \) represent two signals, and the convolution between \( x(t_1) \) and \( y(t_1) \) is given as
\[ x(t_1) \otimes y(t_1) = \int_{-\infty}^{\infty} x(t_1 - t_2) y(t_2) dt_2. \]  

(8)

Step 2: We transform the continuous variable \( \Gamma_0(t_1') \) to the discrete vector \( \beta \) and transform the continuous variable \( K_1(t_1, t_1'), K_2(t_2, t_1') \) to the discrete matrices \( K_1, K_2 \).

Step 3: We record the convolution between \( \beta \) and the every row of \( K_2 \) as \( M \).

Step 4: We add zeros into \( K_1 \) and ensure that it has the same dimensionality as \( M \). The final integration result can be defined as
\[ \Gamma = K_1 M^T \Delta t, \]  

(9)

where \( \Delta t \) is the time sampling step.

According to Refs. 16 and 24, the temporal analog of the thin lens equation in spatial ghost imaging is
\[ \frac{1}{\gamma} = \frac{1}{\Phi_b} + \frac{1}{\Phi_a - \Phi_c}. \]  

(10)

When the previous condition holds, we can obtain
\[ |\Gamma(t_1, t_2)|^2 = \frac{I_0^2}{2}\frac{1}{\Phi_b}\left| \Phi_d \right|^\beta m(t_1)^{2}, \]  

(11)

where
\[ s = \frac{\Phi_b}{\Phi_c - \Phi_a}. \]  

(12)

We can find that \( |\Gamma(t_1, t_2)| \) is similar to \( m(t) \), which means that it can be considered the imaging result of the temporal object. Here, \( s \) is the magnification factor.

### 3.2 Simulation Experiments and Discussion

According to the setup proposed above, the kernel function of the reference arm \( K_1 \) is certain, which is described in Eq. (5), whereas the kernel function of the object arm \( K_2 \) is uncertain, which depends on the time object. For convenience, we choose the most simple time object, e.g., a controllable time switch described in Eq. (11).

\[ m(t) = \begin{cases} 1 & -1 < t < 1 \text{ (ns)} \\ 0 & \text{otherwise} \end{cases}. \]  

(13)

This will let the light completely pass from \(-1\) to \(1\) ns and completely block the light at other times. Through the simulation, we obtain the result of the temporal ghost imaging, which is shown in Fig. 3. Figure 3(a) shows simulated results of \( |\Gamma(t_1, t_2)| \) when \( s = 1, s = 0.5, \) and \( s = 3 \). From these figures, we can easily find that the function achieves high values at the center, and these values decrease gradually toward the edge. These features agree with the function characteristics of a controllable time switch.

From Eq. (9), we can find that each line of \( |\Gamma(t_1, t_2)| \) represents \( m(t_1)/s \). Therefore, we take out the middle line of \( |\Gamma(t_1, t_2)| \), which is shown in Fig. 3(b). From the simulated temporal ghost imaging results, we can see that when \( s = 1 \), the time object maintains its original size. When \( s = 0.5 \), the time object shrinks to half of the original size. When \( s = 3 \), the time object expands to three times its original size. These simulation results show the effect of the magnification factor \( s \) on the imaging quality. When \( s < 1 \), the image fluctuates considerably and the imaging quality is poor, whereas when \( s > 1 \), a better image can be obtained. Thus, the temporal ghost imaging achieved in this study is more suitable for enlarged imaging of an object.

From Fig. 3(b), we can observe that the function values are not smooth and have a certain degree of fluctuations. This phenomenon indicates that the imaging result is not the ideal image of the original object described in Eq. (9). When Eq. (9) is deduced, the light source considered is the ideal thermal light whose correlation function is an ideal unit impulse pulse. Its correlation function is described by the following formula:

\[ \Gamma_0(t_1', t_2') = I_0 \delta(t_1' - t_2'). \]  

(14)

In this case, the temporal ghost imaging can recover the original image perfectly. However, the light source in this article is a chaotic laser whose temporal correlation function is shown in Fig. 1(d), so Eq. (11) needs to be revised. From Eq. (7), we can obtain

\[ \Gamma(t_1, t_2) = \int_{-\infty}^{\infty} (\Gamma_1(t_1') \otimes K_2(t_2, t_1')) K_1^*(t_1, t_1') dt_1' \]
\[ = \Gamma_1(t_1) \otimes \int_{-\infty}^{\infty} K_2(t_2, t_1') K_1^*(t_1, t_1') dt_1'. \]  

(15)
According to Ref. 16, when Eq. (10) holds, we can obtain
\[
\int_{-\infty}^{\infty} K_z(t_2, t'_2) K_z^*(t_1, t'_1) \, dt'_1 = \frac{m(s)}{\sqrt{-2\pi s}} \exp(i\psi') \exp \left[ -\frac{i}{2\Phi_d} \left( \frac{t_1}{s} - t_2 \right)^2 \right] \left( \frac{1}{2\Phi_b} \left( 1 - \frac{1}{s} \right) \right) \Gamma(t_1) \Gamma(t_2).
\]
where \(\psi'\) is a time-independent phase term. Substituting Eq. (16) into Eq. (15) gives
\[
[\Gamma(t_1, t_2)]^2 = \frac{1}{2\pi} \left| \frac{1}{\Phi_d \Phi_b} \right| m(s) \Gamma(t_1) \Gamma(t_2).
\]

Equation (17) shows that final imaging result \(\Gamma(t_1, t_2)\) is the convolution between the original image, \(m(t_1/s)\), and the temporal correlation function of the chaotic laser, \(\Gamma(t_1)\).

In order to verify Eq. (17), a simulation experiment is performed. The time object chooses a cycle of the sine function as the transfer function. This is more complex than the time switch, and it can be written as follows:
\[
m(t) = \begin{cases} 
\sin(10^8 \times at) & -1 < t < 1 \text{(ns)} \\
0 & \text{otherwise}
\end{cases}
\]

The simulation result is shown in Fig. 4, in which Fig. 4(a) shows the original image of the time object, Fig. 4(b) shows the absolute value of the original image, Fig. 4(c) shows the absolute value of the convolution between \(m(t)\) with \(\Gamma(t)\), and Fig. 4(d) shows the temporal ghost imaging result obtained using the method proposed in this article.

By comparing Fig. 4(c) and 4(d), we observe that these two figures are almost the same. This shows that the temporal ghost imaging result is indeed the absolute value of the convolution between the original image and the temporal correlation function, thus verifying Eq. (13). In addition, because the final imaging result is the absolute value of the original image, the negative part in the original image cannot be distinguished and restored, i.e., temporal ghost imaging can only obtain the absolute value image of the time object.

4 Conclusion
We have investigated the temporal correlation characteristics of a chaotic laser and found that its correlation time is very short. This indicates that a chaotic laser can be used as a new light source in temporal ghost imaging. By adjusting the group-delay dispersion parameters \(\Phi\), we can change the magnification factor \(s\), so as to magnify or shrink the resultant image. However, because the correlation function of the chaotic laser is not an ideal unit impulse pulse, imaging results cannot accurately restore the time objects. In fact, the direct temporal ghost imaging result is the convolution between the original image and the temporal correlation function of the light source. In order to recover the original image, deconvolution must be performed on the imaging results, which involves postprocessing and image recovery. It should be noted that temporal ghost imaging virtually obtains the amplitude image of a time object. In the future, temporal ghost imaging with chaotic laser may be applied in some practical fields such as biomedical sensing and time-domain tomography of pulses.

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References

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