ABSTRACT. Modern astronomical polarimeters often require simultaneous operation of multiple instruments over broad wavelength ranges. The 4 m DKIST solar telescope will soon cover 0.38 to 4.6 μm with at least 12 independent narrow band polarimeters, all in quasi-simultaneous operation. Calibration can be efficiently performed over this entire bandpass using our elliptical retarder design, achieved with just two optically contacted MgF2 crystal retarder pairs. Calibration requires very well-characterized, uniform, defect-free retarders and polarizers. I report here on the successful development of four extremely large aperture \((d = 120 \text{ mm})\) optically contacted MgF2 retarder pairs used to make a DKIST calibrator and a modulator for the Cryo-NIRSP instrument. All four crystal pairs have clear apertures free of defects. New procedures deliver fast axis alignment in the range of 0.1 deg to 0.2 deg post contact bonding. For the calibrator crystals, a new process was developed using deterministic fluid jet polishing driven by retardance mapping to achieve stringent retardance spatial uniformity. I show that transmitted wavefront error is not a sufficient proxy for retardance polishing. Polishing softer MgF2 retarder crystals required substantial development to simultaneously achieve flatness, roughness, and retardance uniformity. The optical contact bond ensures there are no bonding agents (oils, epoxies) with spectral absorption bands in the entire 0.3 to 6 μm bandpass without any possibility for leaks or degradation. These four crystals will be used in DKIST and Cryo-NIRSP in a 300 W solar beam and are anticipated to mitigate heating, stability, and UV irradiation issues. I use the Berreman calculus to compute retarder depolarization, with >10% magnitudes found at the shortest wavelengths after including typical crystal optic axis cutting errors and incidence angle variation in converging beams.

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Keywords: instrumentation; polarization; Mueller matrix; DKIST

1 Introduction

Large astronomical telescopes frequently include polarimetric instrumentation to provide imaging and spectro-polarimetric data on a wide range of targets. In the case of solar astronomy, polarimetry is often a key design driver to measure solar magnetic fields. Often, the calibration optics work in outdoor mountain environments, in converging beams, at large aperture size. Additional challenges from heating of optics are caused by absorption in a concentrated solar beam. Polarization calibration optics provide the reference for the Stokes vectors measured by the system, putting stringent requirements on these optics in terms of thermal stability, spatial uniformity, wavelength coverage, interference fringe suppression, and a host of related optomechanical constraints. Compound retarders made of millimeter scale thickness crystals can
have very strong field of view (FoV) effects, and significant depolarization when averaging over beam footprints. An observatory needs to produce accurate measurements of the polarization for every target observed and ideally does not spend much precious observing time calibrating. In this article, I show development and delivery of two 120 mm diameter MgF\(_2\) crystal retarder optic assemblies, substantially improving the performance of the largest solar telescope currently in operation. A new deterministic fluid jet polishing (FJP) process is shown using retardance spatial mapping to drive the process creating substantially more uniform retarders. New tools to derive depolarization in imperfectly cut crystals with imperfect optical alignment in non-collimated beams with imperfect orientation as a function of wavelength and footprint are developed.

1.1 Overview of DKIST
The National Science Foundation’s (NSF’s) Daniel K. Inouye Solar Telescope (DKIST) on Haleakalā, Maui, Hawai‘i, is presently beginning early operations. An observatory level summary is included in Ref. 1. The wide array of science topics that DKIST will address in the upcoming years is described in the Critical Science Plan.\(^2\) The DKIST optical path consists of six mirrors delivering solar light into a rotating coudé laboratory platform, providing flexible instrumentation capabilities.\(^1,3-11\) DKIST has a Gregorian optical system (GOS) with calibration optics, apertures, and targets built around the Gregorian focus (GF). The GOS also contains the optical station for calibration retarders, polarizers, and an artificial light source at other levels above the GF.\(^5,12-17\) The GOS enclosure is shown in figures 7 and 8 of Ref. 1.

Three polarimetric instruments are installed in the Coudé Lab that currently span the 380 to 5000 nm wavelengths. The visible spectro-polarimeter (ViSP) is a three-arm, slit-based spectropolarimeter currently on offer for science observation and summarized by de Wijn et al.\(^18\) The diffraction-limited near-infrared spectro-polarimeter (DL-NIRSP) is a fiber-optic-based integral field spectropolarimeter.\(^19\) The cryogenic near-infrared spectro-polarimeter (Cryo-NIRSP) is a slit based system optimized for the infrared.\(^20\) A fourth instrument called the visible tunable filter (VTF), planned for installation, is a tunable Fabry–Perot-type imaging spectropolarimeter.\(^21,22\) DKIST also has two high speed (30 Hz) 4K full frame cameras within the visible broadband imager red and blue instruments (VBI-red and VBI-blue).\(^23-30\)

1.2 Bandpass Coverage and Heat Loads
Many solar telescopes feed a suite of instruments that cover ultraviolet to infrared instrumentation. In several applications, the infrared absorption and heating caused by the solar beam can be reduced by rejecting longer wavelengths. Common bandpass choices include rejecting infrared wavelengths beyond \(\sim 1800\) nm given polymer optics or \(\sim 2500\) nm given typical glass (fused silica) absorption. Other instruments pursue beams out to \(\sim 5\) \(\mu m\) limited by the atmospheric absorption.

For DKIST retarders, the first light instrumentation has a requirement to operate out to at least \(5\) \(\mu m\) wavelength. This imposes multiple simultaneous constraints on polarization calibration optics of a wide bandpass (380 to 5000 nm), low IR absorption, temporal stability in a 300 W concentrated solar beam, resilience against UV damage, low beam deflection (wedge) to minimize the beam wobble as the optic rotates, high spatial uniformity, low spectral fringing, and consistent performance in an outdoor mountain environment at variable ambient temperature. I note DKIST has a requirement to deliver wavelengths out past 28 \(\mu m\) to the rotating Coudé Laboratory using an all-reflective beam path.

The DKIST beam creates particularly high heat loads. I summarize the largest solar telescope properties in Appendix A. I note that DKIST puts 300 W through the calibration retarder while the European Solar Telescope (EST) is planned at 53 W, the Chinese Large Solar Telescope (CLST) is near 23 W, and all others are below 10 W. The DKIST is unique in transmitting in the 2 to beyond \(5\) \(\mu m\) bandpass.

1.3 Cryo-NIRSP and Modulator Upgrade
Cryo-NIRSP (CN) can receive all wavelengths to 5000 nm and beyond by using a pick-off mirror called M9a, insertable at an optical station down stream of M9. This current M9a optic excludes CN from simultaneous use with other instruments and the AO system. A system upgrade to allow
simultaneous use of CN with other instruments is in progress using a new type of beam splitter called M9b. This M9b optic will enable DKIST to cover 380 to 5000 nm wavelengths strictly simultaneously with up to 13 separate polarimetric cameras, putting stringent requirements on calibration and modulation retarders. The CN modulator needed an upgrade after the existing oilied six MgF2 crystal modulator developed a large bubble in the oil layer and failed. Subsequent attempts to re-seal this crystal stack failed. Refractive index matching oils help suppress fringes and improve throughput, but oils come with challenges from IR absorption, leaks, degradation, UV damage, and chemical compatibility with edge seal materials. I also found that oil absorption testing with small samples did not give representative absorption on a full-sized optic, increasing IR absorption beyond acceptable limits. The new optical contact improvements shown here remove these issues by creating a permanent bond using just the MgF2 crystal itself. Additional benefits of direct contact of extremely parallel crystal surfaces come from achieving arc-second level beam deflection (wedge), minimizing the image wobble at the instrument entrance slit as the modulator rotates. See Appendix F for the alignment improvements and technique to minimize image wobble for this upgrade modulator.

1.4 Calibration and Modulation
Several types of polarization modulation and calibration strategies are required for multi-instrument systems especially with azimuth-elevation articulation. Accurate polarimetry is a key design driver for astronomical telescopes, such as DKIST. The 4-m on-axis EST project will also have calibration considerations. A full 4×4 error matrix can be used to assess the accuracy of the vector length, vector orientation, and vector zero point (c.f. Refs. 41–44). Many solar and night-time telescopes have performed polarization calibration of complex many-mirror pathways using various techniques. Polarization calibration optics are commonly mounted as far up the optical path as feasible to inject signals of known quality through the system. In the case of DKIST, calibration optics are mounted upstream of the GF after the secondary mirror but before the tertiary mirror. Modulators of different types are included within the instruments.

Modulation efficiency is shown and used in several references. Selbing Section 2.5.2 describes some simple optimization strategies for calibration efficiency that has been used at DKIST to design and optimize several retarders, including the elliptical calibrator described here. In this work, I also follow standard notation for propagation of polarization through an optical system.

1.5 Fringe Modeling
Berreman formulated a 4×4-matrix method, extending thin film calculations by Abeles and Heavens matrices based on Abel’s formula. This Berreman calculus can be used to describe wave interference in multiple birefringent layers, crystals, chiral coatings, and other complex optical configurations with many birefringent layers of arbitrary optic axis orientation (c.f. Ref. 85). Application of the Berreman calculus for collimated beams with a simple beam focal ratio-based amplitude scaling relation was shown in Ref. 86 to estimate fringe magnitudes. An upgraded modulator for ViSP was shown in Ref. 87 predicting that fringe suppression would be successful across most of the ViSP bandpass 380 to 950 nm. In actual observations with DKIST so far, no interference fringes from the modulator have been seen.

A number of references dealing with aspects of polarized spectral fringes can be found in the astronomical literature. Semel discussed fringes in an intuitive way by summing (reflected) waves inside a wave plate. Clarke has used the theory of a single Fabry–Pérot interferometer to describe interference effects in single wave plates, and Heavens matrices to describe interference effects in compound, achromatic, and Pancharatnam wave plates. Often, simple Fourier or function fitting techniques are used to remove the fringes with varying degrees of difficulty and success. Fringe removal techniques, such as pattern recognition, principal component analysis, and Fourier filtering, are used in solar applications.

Although these theories describe aspects of the interference patterns, they are mostly ad-hoc theories with several limiting assumptions. The Berreman calculus contains all polarization phenomena and is much more general. In this article, I extend the Berreman work of Refs. 100 and 86 to add additional crystal optic axis fabrication misalignments propagating at non-normal
incidence in a converging $F/13$ beam through the stack of four MgF$_2$ crystals in our new elliptical calibrator optic.

1.6 Depolarization
I present a new simulation here showing how depolarization can give rise to Mueller matrix elements of up to 15% using the depolarization degrees of freedom for an $F/13$ beam converging through our new four MgF$_2$ crystal calibration optic. Depolarization degrees of freedom are outlined in section 6.11 (Depolarization) of Ref. 73 as well as in the optics literature (c.f. Refs. 101–109). Currently, I know of no astronomical telescope including depolarization degrees of freedom in the Mueller matrix model for their retarder optics. In the calibration process, a thick crystal retarder produces significant depolarization due to the converging beam. Sueoka$^{15}$ did some depolarization modeling and decomposition for one DKIST retarder optic at one wavelength. I show a different approach here using the Berreman calculus to average over footprints in the presence of manufacturing errors and with misalignment. Including this depolarization either via modeling and look-up table estimates or by direct fitting adds significant complexity to the telescope calibration process. I use new Berreman simulations presented here to estimate the depolarization magnitude and character for these thick crystal retarders, with mitigation options planned for a future article. I also include measurements of a few common commercial depolarizers using our Mueller matrix spectral metrology tools to demonstrate the relatively straightforward measurement and fitting of these optics with our existing equipment in Appendix B. Spatially patterned liquid crystal polymer (LCP) depolarizers and also quartz retarder wedged-pair depolarizers are straightforward to measure and decompose into the depolarization degrees of freedom. Each of these depolarizer optics presents very different spectral and spatial behavior with depolarization varying with footprint size and spectral resolving power. I measure several of these for lab demonstration purposes in Appendix B.

2 Contacted MgF$_2$ Crystal Elliptical Retarders: Overview
I created elliptical retarder designs by optimizing the efficiency equations$^{78,87,110}$ to provide both reasonably efficient calibration over the full 380 to 5000 nm wavelength range and modulation for the Cryo-NIRSP instrument over the 850 to 5000 nm bandpass. I explored two-pair and three-pair designs with success in both cases. The two-pair designs had expected reductions in efficiency over the entire bandpass. At the start of this development project, availability of large-diameter high quality MgF$_2$ retarder crystals was not assured. I pursued the two-pair retarder designs for this project. Optical contact bonds at smaller optic diameters are commercially available with at least one vendor. Example performance is shown in Appendix H. For optic diameters over 50 mm, I was unable to identify any vendors capable of delivering optically contacted MgF$_2$ retarders to our specifications. The procedures for ensuring tight retardance uniformity, net retardance magnitude, precise angular alignment of the crystal axes (crystal clocking alignment), and a clean optical contact bond over a 120 mm diameter optic were not clear. I note that Ref. 41 showed how the transmission losses in patches of optical contact bond failure created photometric errors larger than many other error sources. These errors caused Stokes vector calibration errors to easily exceed polarization error budget limits. Having a defect-free clear aperture over the 105 mm optic was essential for accurate calibration. The DKIST team pursued a development project to simultaneously achieve an optical contact bond on MgF$_2$ with the most challenging specifications for the calibration optic as (Figs. 1 and 2):

- 105 mm diameter defect free clear aperture at 60/40 scratch dig or better, internal and external surfaces;
- fast axis orientation (clocking) of contacted MgF$_2$ crystal pairs better than 0.3 deg, goal <0.1 deg;
- retardance magnitude aperture-averaged at specification ±0.01 waves at 633 nm wavelength;
- retardance uniformity across a 105 mm aperture at ±0.01 waves at 633 nm wavelength PV, measured over the entire CA using >200 points, uniformly spaced, with <5 mm spatial resolving power.
2.1 Elliptical Calibrator: Two Optically Contacted Pair Design

I searched over a wide range of possible elliptical pair designs for a balanced calibration efficiency. One crystal pair was clocked against the other through 180 deg in steps of 10 deg. I started with two crystals at a bias value of 2.2 mm thickness and another two near 3.5 mm thickness. Each crystal pair was polished in steps of net retardance to a total range of 4.5 waves net retardance difference on each crystal for each clocking orientation. I found a least-minimum V-efficiency search found reasonable retarder calibration efficiency when calibrating using a standard 16-state sequence. The second crystal pair fast axis was clocked 45 deg relative to the first crystal pair. Retardance magnitudes of 0.27 waves and 1.33 waves, respectively, were chosen.

I list the four crystals and the properties of the assembly in Table 1. The first column numbers the individual crystals. I also note the 5 mm thick polyether ether ketone (PEEK) spacer ring separating the optically contacted MgF\textsubscript{2} denoted as the empty P layer in the middle row. The second column shows the crystal thickness in microns with the 2200 and 3500 \( \mu \text{m} \) bias.

Fig. 1 (a) An optically contacted MgF\textsubscript{2} crystal pair at 120 mm outer diameter before bonding into the rotary stage tip/tilt cell. (b) Both MgF\textsubscript{2} crystal pairs the new CN modulator bonded into the two-axis tilt cell.

Fig. 2 The \( I \), \( QU \), and \( V \) calibration efficiency for the four crystal MgF\textsubscript{2} elliptical calibrator as the optic starting orientation is varied. Stokes \( I \) efficiency is shown in black. Stokes \( QU \) efficiency is shown in blue (\( Q \) efficiency is identical to \( U \) efficiency). Stokes \( V \) efficiency is shown in red. Left shows the efficiency when using a 16-state ASP-style calibration sequence. The right-hand graphic shows the efficiency when using the nominal DKIST 10 state calibration sequence. The thicker lines in each panel show the calibration efficiency when the retarder optic is set at a starting orientation to provide the highest minimum calibration efficiency across the entire 0.38 to 5 \( \mu \text{m} \) bandpass.

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The additional 88 and 34 μm thickness difference between the pair is called out as the thickness difference ($T_d$) in the fourth column. The third column shows the relative crystal orientation with 90 deg between subtraction pairs and an additional 45 deg between the first pair and second pair. The design retardance of the pair in subtraction is shown in the fifth column as the waves net retardance at 633 nm wavelength. The sixth column lists the actual measured aperture center net retardance for each pair. The seventh column lists the marked name of the crystal pair as engraved on the side. This fabrication used crystals from multiple sources and with different starting thicknesses. The first pair denoted CH5:CH6 is thicker and fabricated from a different supplier than second pair CN2:CN4. The eighth column shows the measured clocking misalignment as the difference between the ideal 90 deg offset and the actual offset as measured by retardance spectral oscillations. The ninth column shows the beam deflection estimated from an interferometer. The CN2:CN4 contacted MgF2 pair had a transmitted wavefront error (TWE) of 0.9 waves peak-to-valley (P-V) with 0.74 waves power and 0.22 waves root-mean-square (RMS). The CH5:CH6 contacted MgF2 pair had a TWE of 0.4 waves PV with 0.06 waves power and 0.06 waves RMS.

### Table 1

<table>
<thead>
<tr>
<th>C</th>
<th>$T_h$ (μm)</th>
<th>$\theta_n$</th>
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<td>—</td>
</tr>
<tr>
<td>4</td>
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<td>72.7</td>
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<td>1.332</td>
<td>CN2:CN4</td>
<td>0.10 deg</td>
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The additional 88 and 34 μm thickness difference between the pair is called out as the thickness difference ($T_d$) in the fourth column. The third column shows the relative crystal orientation with 90 deg between subtraction pairs and an additional 45 deg between the first pair and second pair. The design retardance of the pair in subtraction is shown in the fifth column as the waves net retardance at 633 nm wavelength. The sixth column lists the actual measured aperture center net retardance for each pair. The seventh column lists the marked name of the crystal pair as engraved on the side. This fabrication used crystals from multiple sources and with different starting thicknesses. The first pair denoted CH5:CH6 is thicker and fabricated from a different supplier than second pair CN2:CN4. The eighth column shows the measured clocking misalignment as the difference between the ideal 90 deg offset and the actual offset as measured by retardance spectral oscillations. The ninth column shows the beam deflection estimated from an interferometer. The CN2:CN4 contacted MgF2 pair had a transmitted wavefront error (TWE) of 0.9 waves peak-to-valley (P-V) with 0.74 waves power and 0.22 waves root-mean-square (RMS). The CH5:CH6 contacted MgF2 pair had a TWE of 0.4 waves PV with 0.06 waves power and 0.06 waves RMS.

#### 2.2 Elliptical Calibrator: Efficiency with 10- and 16-State Sequences

This elliptical retarder design can calibrate the entire 0.38 to 5 μm bandpass in a single, strictly simultaneous sequence. As expected, there are significant changes in the calibration efficiency when the number of input states is reduced and the optic starting orientation is varied. This 4 – MgF2-crystal retarder has calibration efficiency very similar to the other DKIST retarders shown in Refs. 110, 87, and 41. The calibration efficiency for a common 16-state sequence is shown in Fig. 3(a). The best balance in calibration efficiency is achieved at a particular starting orientation so this retarder produces a diversity of Stokes parameters throughout the calibration sequence. Figure 3(b) shows the calibration efficiency for the nominal 10-state calibration sequence outlined in Refs. 44 and 111. I note that having fewer states does result in a relative change in signal-to-noise ratio (SNR) due to the wavelength dependent changes in calibration efficiency. However, I have generally found that systematic errors and observatory efficiency losses are a far greater source of error than a slight efficiency loss. By accomplishing a calibration faster, the losses due to issues, such as software lag times, solar temporal evolution, thermally induced retarder changes, etc., are reduced far below these slight changes in SNR caused by fewer states in a calibration sequence. System calibrations are hardly ever shot-noise-limited.

#### 2.3 Elliptical Calibrator: Measured Retardance and Predicted Efficiency

Our lab measurements of retardance fit a refractive index based model of retardance very well over the 0.36 to 1.7 μm measurement bandpass. I show here a single measurement to demonstrate the success of this design and fabrication of the optic. Figure 3(a) compares the best fit elliptical retardance to the measured Mueller matrix and a refractive index based birefringence model. The close agreement between a refractive index model and the measurements shows that our thickness difference estimates and crystal orientations will be a good predictor for infrared wavelengths as well as for the more complex phenomena modeled later in the paper. Figure 3(b) shows the calibration efficiency for a 16-state sequence and a nominal starting orientation. The solid lines showing the efficiency when using the measured elliptical retardance agree well with the refractive index based model shown in dashed lines.
I identified a four-crystal design that would have sufficient modulation efficiency over the 0.85 to 5 \( \mu \text{m} \) bandpass. The design involves a 1.64 wave net magnitude compound pair followed by a 0.64 wave net magnitude compound pair. This corresponds to roughly 88 \( \mu \text{m} \) of thickness difference for pair 1 crystals and 34 \( \mu \text{m} \) for pair 2 crystals. The relative clocking angle is 140 deg between the two pairs. I list an example crystal thickness configuration in Table 2 using a nominal crystal bias thickness. I also include that the 5.3 mm thick PEEK spacer ring separating the two optically contacted MgF\(_2\) crystal pairs is denoted as the empty P layer in the middle row. The second column shows the individual crystal thickness in microns. The first pair has a 2100 \( \mu \text{m} \) bias and the second pair a 3500 \( \mu \text{m} \) bias. The additional 88 and 34 \( \mu \text{m} \) representing the zero-order retardance thickness is called out as the thickness difference (\( T_d \)) in the fourth column. The third column shows the relative crystal orientation with 90 deg between the individual compound subtraction pairs, and an angle of 140 deg between the first pair and second pair. The net retardance of the pair in subtraction is shown in the fifth column as the waves net retardance at 633 nm wavelength. The sixth column (\( \text{Act} \)) shows the actual measured aperture average retardance. The seventh column names the individual crystal pairs as pair 1 = C1a:C1b and pair 2 = CH3:CH4. The eighth column of Table 2 shows the angular rotational error (clocking offset) between the crystal fast axes, both meeting the \(<0.3\) deg goal. The ninth column lists the beam deflection in

**Table 2** Four MgF\(_2\) crystal CN mod.

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<th>( \theta_h )</th>
<th>( T_d )</th>
<th>Wvs'</th>
<th>( \text{Act} )</th>
<th>( N )</th>
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<tr>
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<td>2100</td>
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<td>88</td>
<td>1.64</td>
<td>1.614</td>
<td>C1a:C1b</td>
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</tr>
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<td>0.64</td>
<td>0.618</td>
<td>CH3:CH4</td>
<td>0.08 deg</td>
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</table>

2.4 CN Modulator: Two Optically Contacted Pair Design and Modulation Efficiency

Fig. 3 (a) The elliptical retardance measurements are shown as as solid lines and the refractive index model as dashed lines. The measurements stop at 1.7 \( \mu \text{m} \) as the long wavelength limit of the NDSP4 system used in the lab. The refractive index model uses the terms measured in Chapter 2 of Ref. 15 covering wavelengths out to 5 \( \mu \text{m} \), with good agreement over the entire measurement bandpass. (b) The \( QU \) and \( V \) calibration efficiency for the 4-crystal MgF\(_2\) elliptical calibrator. Stokes \( QU \) efficiency is shown in blue (\( Q \) efficiency is identical to \( U \) efficiency). Stokes \( V \) efficiency is shown in red. The Stokes \( I \) efficiency is a constant 50% at all wavelengths reflecting the ideal assumptions of 50% transmission for the calibration polarizer, which is always in the beam for this 16-state sequence. The horizontal magenta line shows a spectrally constant 18% efficiency limit. Retarder transmission losses through surface reflection reduce the efficiency by an additional fraction.

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arc seconds caused by residual wedge in the thickness of the crystal pair. Beam deflection measurement error is at the level of an arc second or two with the particular technique used, so these values are effectively near zero within measurement uncertainty. Final MgF₂ crystal thicknesses are measured by a Heidenhain depth gauge on a flat polished optical glass block. Thicknesses are CH₄: 3.558 mm, CH₃: 3.525 mm, C₁A: 2.1889 mm, C₁B: 2.1006 mm with some microns of uncertainty due to the air layer and crystal flatness on the optical block.

The TWE on the modulator assembly is between 0.68 and 0.81 waves RMS. This was measured for all three polarization states at 632.8 nm wavelength (circular and horizontal and vertical linear polarization states). The RMS TWE values are between 0.16 and 0.19 waves. The beam deflection is derived from a sub-aperture based interferometric measurement done at the vendor Meadowlark Optics (MLO). The deflection for the final assembly was 1.28 arc seconds at a wavelength of 632.8 nm. The C₁a:C₁b pair had a deflection measured at 1.45 arc seconds. The CH₃:CH₄ pair has a measured deflection of 1.34 arc seconds. The C₁a:C₁b crystal pair has roughly 16 waves PV of surface figure almost entirely as power. The CH₃:CH₄ pair has closer to five waves PV surface figure as power but also with some astigmatism. There is a PEEK based spacer ring used to set the air gap between optically contacted MgF₂ crystal pairs. The PEEK spacer is stated to have parallel faces to roughly 8 μm. These irregularities should help mitigate fringes in the 5 mm thick cavity formed by the PEEK spacer. See Appendix C for fringe calculation details.

2.5 CN Modulator: Efficiency with 12 Deg Clocking Error: Mueller Matrix Measurements

Our lab Mueller matrix measurements fit a refractive index based Mueller matrix optical model very well over the 0.36 to 1.7 μm bandpass. I show here a single measurement to demonstrate the success of this design and fabrication of the optic. I measured the optic with the NDSP4 system mid-January, 2022. Figure 4 shows the best fit refractive index based optic model compared with the measurements. The measured Mueller matrix shows expected modulation efficiency at all wavelengths. I find a very good fit to the data provided; the two crystal pairs were rotated −12 deg from the specified nominal rotation angle. I note that this 12 deg error is consistent with an MLO mistake in identifying the fast axis as the slow axis. The efficiency curves are broadly high and insensitive to an assembly misalignment. The net result of this mounting error

![Fig. 4](a) The best-fit elliptical retardance model (solid lines) is compared with aperture-center measurements (dashed lines). Each color shows a different component of retardance. Black is the magnitude of elliptical retardance. Red is the circular retardance component. Blue is the first component of linear retardance. Green is the second component of linear retardance. The refractive index based model fits very closely over the measured 0.38 to 1.7 μm wavelength range. (b) The best-fit modulation efficiency model determined from the measured Mueller matrix. Stokes Q,U efficiency is shown in blue (Q efficiency is identical to U efficiency). Stokes V efficiency is shown in red. The Stokes I efficiency does not include any transmission losses for the retarder. The reduction of I efficiency at 0.7 μm wavelength is consistent with the poorly conditioned demodulation and degeneracy between I and Q,U inputs. I note that this best fit model had a −12 deg clocking error relative to the nominal design. The Q,U efficiency is 18% at 4.6 μm and 27% at 3.9 μm.
is increased efficiency in the 0.8 to 1.5 μm bandpass and a roughly 3% efficiency loss at 4.9 μm wavelength. I decided to not attempt any re-adjustment as the efficiency curves were acceptable. This rotation increases modulation efficiencies around 1 to 1.5 μm wavelength, while slightly decreasing efficiency at 4.6 μm wavelength. See Sec. 12.4 for details.

2.6 CN Modulator: Complexities and Upgrade Pathways
The QU efficiency is 18% at 4.6 μm and 27% at 3.9 μm. This is less than the ideal balanced modulation efficiency of 57% per state, but this was achievable before the start of DKIST observing commissioning using just two contacted MgF2 crystal pairs using an air gap between pairs. The optic was installed July, 2022, into the instrument and aligned to minimize image motion at the slit per Appendix F. I detail in Appendix G how the optically contacted MgF2 crystals have a few waves of surface deformation, and an additional wave of astigmatism change per degree Celsius temperature change. This strong and variable lack of surface flatness in the contacted MgF2 crystal pairs prevents optically contacting of these already-contacted pairs of crystals. I anticipate verifying the interference fringe magnitudes in the Appendix C. In particular, the prediction that fringes will be significant at the longest CN science wavelength of 4.6 μm. Now that the DKIST team has a pathway for 120 mm diameter contacted MgF2 crystal pairs with both low clocking errors and a clean aperture, DKIST has an obvious upgrade path. An oiled assembly with only one oil layer between these two pairs would further suppress fringes per the F/18 marginal ray scaling estimates of Ref. 86 in Appendix G. I also made designs for more efficient modulation using three contacted pairs. There are several challenges (cost, FoV dependence, uniformity, yield, and complexity) in a three-pair design opposed to a two-pair design. However, de-modulation can calibrate out many issues in a non-uniform, thick modulator crystal stack. Reducing the oil from five layers to only one or two layers is possible, though I note that the many stated challenges when using oils would remain.

I also emphasize that spinning modulators induce beam wobble through both translation and tilt. The beam is translated from imperfect flatness of rotation stages inducing tilts of the optic. The beam is made to wobble from the non-zero wedge in any optic, even the few arc seconds of wedge for these very flat MgF2 crystals. I detail in Appendix F a common procedure to compensate for non-zero wedge by tilting the optic. The thicker the optic substrate, the worse the beam translation is during rotation. This complicates the tradeoffs between using more and thicker crystals to boost efficiency and fringe suppression while introducing image motion causing polarization artifacts based on image intensity gradients.

3 Retardance Based Fluid Jet Deterministic Polish
Polishing these relatively thick crystal retarder substrates to very stringent uniformity specifications is key to achieving accurate polarization calibration. Uniformity issues are exacerbated at shorter wavelengths given the increased crystal birefringence. I report here that to achieve very tight retardance uniformity, I required polishing using retardance metrology instead of more typical transmitted wavefront measurements. The correlation breaks down between net crystal pair retardance and the crystal thickness variation typically derived using interference fringes in a circularly polarized beam as used in most transmitted wavefront interferometers. I had to work with vendors to develop a procedure to drive the polishing based on retardance spatial mapping in order to achieve our tight retardance uniformity specifications.

I worked together with Meadowlark to improve the polishing uniformity procedures for the multiple steps at their facility. DKIST has now successfully pursued deterministic FJP by funding multiple rounds of polishing under different circumstances to achieve uniform net retardance. The vendor and I have continued this work to achieve improved retardance uniformity in the softer MgF2 material with the correspondingly worse uniformity. I also have done sufficient testing and metrology development to polish already contacted crystal pairs to net retardance. This allows for polishing the final parts much more accurately.

3.1 Lack of TWE Correlation with Retardance Uniformity: SiO2 OCcal
In Ref. 87 (appendix C and figure 37), the authors showed the first moderately successful fluid jet polish (FJP) test on an individual 4.2 mm thick quartz retarder crystal. The vendor had achieved...
TWEs better than $\lambda/52$ showing 7.7 nm of crystal thickness variation. However, intrinsic materials properties caused retardance uniformity errors at $\pm 1$ deg magnitudes when the wavefront errors converted to thickness variation would only produce 0.04 deg retardance error. Figure 37 of Ref. 87 showed a crystal fluid jet polished still retained a linear ridge type feature over a 120 nm aperture at roughly 1.5 deg magnitude at 633 nm wavelength. In Fig. 5(a), I convert the retardance spatial map to a material thickness removal map that would be required to spatially flatten the retardance across the aperture. The color scale is $\pm 220$ nm of material. In Fig. 5(b), I show the vendor physical thickness metrology based on a spectral fringe measurement technique. The thickness map had 26.5 nm of wedge removed on an axis 39.5 deg toward the lower right quadrant. The color scale shows this wedge-removed thickness is shaped mostly like power on a color scale of $\pm 7$ nm, 30X smaller than (a). The thickness derived uniformity looks entirely unlike the retardance uniformity at these small magnitudes.

3.2 Lack of TWE Correlation with Retardance Uniformity: MgF$_2$

Different crystal materials from different suppliers can have varied intrinsic variation and defects. As an example, I detail here the difference between retardance and transmitted wavefront as part of polishing one of our contacted MgF$_2$ crystal pairs named CN2:CN4. This pair has a net retardance magnitude of 1.33 waves at 633 nm wavelength in the design. I show in Fig. 6 how the vendor achieved retardance uniformity by polishing using retardance spatial mapping. Figure 6(b) shows the crystal removal derived from TWE. There is a significant center to edge deviation when removal is derived using TWE. Figure 6(a) shows the residual material removal derived from retardance spatial mapping. This removal corresponds to $<0.25 \mu m$ P-V after the FJP with a much more uniform spatial pattern especially in the center of the aperture. The small deviation right near aperture center is a few millimeters in size but is a very repeatable crystal defect, detailed more in Appendix D.

There were ultimately three rounds of FJP required on this crystal pair, with two rounds being done in October, 2021. The TWE in November, 2021, post-FJ polish was roughly $\pm 250$ nm or $\pm 0.4$ waves per the MLO interferometry using a circularly polarized probe beam. Similar results were seen when TWE interferometry was also done using both horizontal and vertical linear polarization states. With a refractive index of $\sim 1.38$, this corresponds to roughly
±180 nm of MgF₂ material that would create this TWE. With a birefringence of ∼0.01178 this is about ±2 nm of phase retardance. At 633 nm wavelength, this 2 nm of phase is ±0.003 waves retardance.

The MLO retardance mapping from October, 2021, did show a retardance non-uniformity of ±0.005 waves. This corresponds to roughly double the material removal derived from the TWE. I note that the linear fast axis orientation measured on the same system at the same time does not vary by more than ±0.02 deg. The MLO retardance spatial map gives polishing errors that look like wedge without substantial power. The MLO retardance map does not appear to be dominated by power shaped variations as the transmitted wavefront would suggest. The particular interferometer used at MLO for transmitted wavefront is not large enough to distinguish wedge, so this relative comparison is fairly limited.

This polishing disagreement between interferometry TWE, thickness measurement using fringe-based techniques and retardance metrology is substantially above the DKIST error budget limits. The disagreement of 2x for MgF₂ crystal retarders is far smaller than the 20x disagreement seen for SiO₂. The ridge-like features seen in the prior DKIST SiO₂ retarder crystals are nowhere near as large in our MgF₂ crystals. This FJ polish procedure driven by retardance metrology presented here is a key component to achieving very uniform retardance over large crystal retarders for DKIST polarization calibration.

4 Retarder Performance: Spatial Uniformity

The calibration retarder (and polarizer) provides the reference Stokes vectors for every FoV. During the calibration process, the calibration retarder is almost always assumed to have exactly the same retardance at all field angles and at all orientations (rotation). Having a calibration retarder that meets stringent uniformity requirements at the shortest (most difficult) wavelengths is key to accurate polarization calibration. I show the improved retardance spatial uniformity provided by using this new retardance-based FJP process on the two MgF₂ crystal pairs used in the new DKIST elliptical calibrator. Improvement in spatial uniformity is the most necessary and the most obvious at the shortest wavelengths, such as the CaII H & K lines at 393 and 396 nm wavelength. I show here the achievement of ~8 deg PV retardance uniformity at 396 nm wavelength in this MgF₂ calibration retarder over a 110 mm diameter aperture.

4.1 Elliptical Calibrator: Spatial Maps of Spectral Retardance

The retarder centering and symmetry of the spatial pattern of retardance impacts calibration accuracy through changing retardance. A footprint-based model for Stokes vector measurement errors as the retarder spins throughout a calibration process was outlined in sections 4, 5, and appendix A of Ref. 41. Asymmetric and de-centered illumination of the calibration retarder coupled with...
spatially variable retardance creates systematic errors in the calibration leading to inaccurate Stokes vector measurements during science observations.

Figure 7 shows the elliptical retardance spectra fit to the Mueller matrix from the 5378 individual spatial locations measured in our NDSP5 mapping setup. I choose a nominal elliptical retardance model that keeps the total retardance magnitude between quarter wave and three quarter waves. There is very good agreement between the many repeated measurements in our spatial mapping data. The non-uniformity of this retarder mostly seen only as a widening of the line caused by over-plotting 5378 individual spectra. The spatial uniformity generally improves inversely with the wavelength. The retardance RMS spatial variation follows the $\lambda^{-1}$ trend but with substantial deviations. The mixing between the components of retardance (ellipticity, fast axis orientation) varies more significantly than the magnitude variation.

Figure 8 shows the spatial variation of retardance for the elliptical calibrator for all 5378 spatial locations at a wavelength of 396.9 nm. The elliptical retardance design and the differing
spatial uniformity from the two individual contacted crystal pairs causes certain components of retardance to show relatively larger or smaller errors as a function of wavelength and aperture. The elliptical magnitude spatial variation shown in Fig. 8(a) does not look at all similar to the circular retardance spatial pattern. I note that the metrology equipment has sufficient spatial resolution and stability to measure multiple small crystal defects seen as the little blue dots in the circular retardance map. These features are very stable and persistent in our lab data collected over at least 3 weeks. The features are present at spatial scales near or below our spatial resolution limit of a few millimeters. I show maps at other wavelengths for this optic with very different spatial patterns in the detailed metrology collected in Appendix D. With this new fluid jet retardance-based polishing, the new DKIST elliptical calibrator is several times more spatially uniform than another four MgF$_2$ crystal retarder I detail in the next section.

4.2 CN Modulator: Spatial Maps of Spectral Retardance

For comparison, I show the spatial and spectral metrology on the MgF$_2$ based CN modulator. None of the four MgF$_2$ crystals in this optic were fluid jet polished as the demodulation process calibrates each optic orientation. The retardance uniformity follows the expected spatial pattern resembling wedge across the aperture as the dominant error. I show in Sec. 15.3 the example retardance maps of a single crystal quartz window from another vendor also showing wedge shaped polishing errors. When polishing without an added deterministic FJP step driven by retardance metrology, achieving both retardance net magnitude and minimum wedge simultaneously is quite difficult.

I performed a similar Mueller matrix spatial and spectral mapping campaign on this CN modulator. Figure 9 shows the spectral dependence and uniformity of elliptical retardance derived from the measured Mueller matrices. I choose an elliptical retarder model with a magnitude that stays between 90 deg and 300 deg magnitude for clarity. In Fig. 9, the spectra from many individual spatial points illustrate the changing retardance spatial uniformity as particular wavelengths showing a significant widening of the family of curves. The wedge shaped spatial variation of crystal thickness is mostly manifest as variation among elliptical parameters, not as much with the total overall retardance magnitude. For instance, the circular retardance (red curves) of Fig. 9 varies most at shorter wavelengths while the first component of linear retardance (blue curves) varies more at 450 and 600 nm wavelength. There is much less variation in overall retardance magnitude (black curves). This optic is almost entirely a circular retarder near 700 nm.

![Fig. 9](image-url) The retardance spectra illustrating spatial uniformity for the CN modulator. Retardance is derived from NDSP4 Mueller matrix spectra on a 217 point map at 6.5 mm spatial sampling to a 104 mm aperture diameter. Black shows the elliptical retardance magnitude. Magenta shows the linear retardance magnitude. Blue, green, and red show the components of elliptical retardance. There are 217 curves for each type of retardance. The spatial uniformity can be seen as the spread in the lines, changing strongly with wavelength.
wavelength as the magenta lines showing linear retardance magnitudes approach zero. At 1 μm wavelength, this modulator is almost entirely a linear retarder as the magenta curves match the black curves in Fig. 9.

The spatial variation of retardance for a single wavelength of 1083.5 nm is shown in Fig. 10. The elliptical magnitude varies in Fig. 10(a). Figure 10(b) shows the fast axis orientation for linear retardance. Figure 10(c) shows the circular component of retardance. I note that for a modulator, the spatial variation of retardance parameters is automatically calibrated during the measurement of the modulation matrix. The main impact of the retardance spatial variation here is in a mild change of modulation efficiency from depolarization caused by the spatial average over a beam footprint. See section 2.6 of Ref. 87 for an example calculation of efficiency loss and modulation matrix variation for a substantially less uniform polycarbonate retarder used in the DKIST ViSP instrument.

5 Crystal Optic Axis and Incidence Angle: Depolarization and FoV

The variation of retardance as a function of incidence angle represents one of the major causes of depolarization and variation across a beam footprint. An accurate model of system polarization calibration errors must include not only the retardance spatial variation of the calibrator but also any issues related to alignment. Imperfect alignment of the crystal material optic axis is expected when cutting and orienting a crystal from the raw boule. This represents another source of systematic error. Even for a collimated beam perfectly aligned with the optic polished surface normal, the crystal optic axis can deviate from the nominal alignment, perpendicular to the beam propagation direction. This crystal optic axis misalignment will cause retardance variation as the optic spins. Most astronomical calibration procedures must assume and fit a constant retardance for the calibration retarder as it spins, representing an uncorrected error in the calibration process. Fitting for too many variables in the calibration process can cause systematic errors to be introduced, reducing the accuracy of the calibration itself.

In this section, I estimate the magnitude and behavior of these crystal retarder properties using realistic parameters drawn from the metrology in our new MgF2 elliptical calibrator fabrication. I have now several errors that can be combined. The spatial variation of retardance is known given both thickness and material inhomogeneity. The crystal optic axis can be modeled as non-normal to the optical surface and non-normal to other crystals in the stack. This represents retardance variation even for a perfectly aligned optic along the optic axis of the incoming beam. The beam can be modeled as non-collimated ($F/13$ for DKIST), giving a range of incidence angles across any individual footprint. I can combine all these effects to estimate the polarization calibration inaccuracies, the depolarization caused by averaging over a non-uniform beam, and the dependence of the calibration on FoV. I show images of incidence angle effects on compound MgF2 crystal stacks using our lab polariscope. I then include several of these angle-dependent and orientation-dependent errors in a new Berreman calculus-based model. I highlight errors as the calibration retarder spins, even if perfectly aligned, with imperfectly cut crystals in a non-collimated, field-dependent beam.
5.1 AOI for Single MgF$_2$ Compound Zero Order Retarder Crystal

In typical compound retarder stacks, the impacts of incidence angle and crystal material orientation (crystal optic axis misalignments) can cause substantial deviations from the theoretical on-axis performance. For compound retarders with thicknesses of millimeters or more, <1 deg of incidence angle change creates a significantly elliptical deviation from the nominal design retardance. I show an example of the wavelength and incidence angle dependence of retardance in Fig. 11. This image shows a single MgF$_2$ crystal near 4 mm thickness placed between crossed polarizers (polariscope) illuminated with white light. A hand held cell phone camera is imaging conjugate to infinity with a uniform diffuse background. The crystal optic axis is specified to be within ±1 deg of the perpendicular to the surface normal. MLO has measured these crystals to be roughly 0.5 deg out of plane. This colored pattern is not interference fringes, just integer multiples of half wave retardance as a function of wavelength and incidence angle through a crystal retarder.

5.2 MgF$_2$ Crystal Optic Axis Angle with Surface Normal

During the crystal manufacturing process, a grown crystal boule has to be cut and ground so the optic axis of the crystal is parallel to the optic surfaces. Crystal optic axes are typically specified to be within ±1 deg of the plane of the surface though specifications from a variety of vendors can vary. Orientation verification of the crystal optic axes can include x-ray diffraction and also retardance metrology. DKIST funded an independent assessment of six of our new ~4 mm thick...
MgF$_2$ crystals. Retardance versus incidence angle measurements were collected along both fast and slow axes as an independent verification of the x-ray diffraction results of one of the crystal growers used for this project. Figure 12 shows the measured optic axis angle with respect to the polished optic surface. These are two main axes. For axis 1, the tilt is about axis perpendicular to optic axis, i.e., the optic axis is in the plane of incidence. For axis 2, the tilt is about optic axis, i.e., the optic axis is perpendicular to the plane of incidence. The retardance was measured at many incidence angles and a parabola fit to the resulting retardance. The crystal optic axis is within a $\pm 1.3$ deg plane after convolving the results with measurement uncertainty. I note this crystal optic axis variation is roughly half the incidence angle range for the DKIST beam at a focal ratio of $F/13$. Marginal rays converge at $\pm 2.2$ deg for an $F/13$ beam near the DKIST GF calibration station.

5.3 Polariscope Images: Elliptical Calibrator AOI and Clocking Dependence

Visual inspection images of the 120 mm diameter optic mounted between crossed polarizers and illuminated by white light immediately give a sense of the strong incidence angle dependence of a compound crystal retarder. Figure 13 shows a series of polariscope images as the 4 MgF$_2$ crystal elliptical calibrator is rotated through 90 deg. The combined AOI dependence of the two individual compound retarders creates an asymmetric pattern especially when the linear polarization states are not aligned with the crystal axes of either compound retarder.

5.4 Four MgF$_2$ Crystal Elliptical Retarder: Crystal Axis Imperfections with Incidence

I show here a very brief overview of the modeling done using the Berreman calculus to account for incidence angle and crystal material axis misalignments. I use our four crystal MgF$_2$ elliptical calibrator upgrade optic as an example. The Berreman calculus can be run for any arbitrary crystal axis alignment imperfections at any incidence angle. I implement a sum over a range of misalignments, optic orientations, and incidence angles to simulate a more realistic set of errors within DKIST polarization calibration.

5.4.1 Berreman models of the elliptical calibrator with incidence angle and clocking

I show an example in Fig. 14 where I model the nominal four crystal optic design with incidence angle and clocking along with another model of the optic with one simple perturbation. I model...
the retardance dependence on incidence angle over a ±3 deg range while clocking the optic through a full 360 deg rotation. I note that the $\frac{F}{13}$ beam at DKIST GF represents ±2.2 deg incidence for the on-axis beam. The full 5 arc minute field of DKIST gives another 0.3 deg incidence angle variation at the Gregorian calibration station. This is due to the 8x pupil demagnification at this particular station in the DKIST beam.

Figure 14(a) shows the perfectly aligned nominal crystal design behavior with incidence angle and crystal clocking orientation. The left-hand graphics of Fig. 14 show the first component of linear retardance. The middle graphics show the second linear retardance component. The right-hand graphics show the circular component. Each graphic has $X$ and $Y$ axes representing incidence angle ranges of ±3 deg. This angle range covers the full FoV for the DKIST calibration beam at $\frac{F}{13}$.

Figure 14(b) shows the same four MgF$_2$ crystal retarder but with a crystal optic axis perturbation. I take one crystal and plunge the crystal optic axis out of the optic surface plane by 1.5 deg. This represents and error roughly three times larger than the RMS of our axis measurements and slightly larger than the worst case measurement of 1.3 deg. There are slight asymmetries even near the center of each graphic showing the impact of a crystal optic axis misalignment.
I can also show the importance of this kind of modeling by creating a more realistic simulation of spectral retardance variation. I include clocking misalignments between individual compound retarder pairs and crystal optic axis misalignments away from the surface on all four MgF2 crystals. I do this at non-zero incidence angle as the optic is clocked (spun) through a full 360 deg rotation. I simulate a 0.2 deg clocking error between individual crystal fast axes of the first crystal pair. I use a 0.15 deg clocking error on the second crystal pair. I perturb the tilt of all four of the crystal optic axes by 0.5 deg out of the surface plane. Each crystal has the optic axis plunging away from the surface at clocking orientations of 0 deg, 60 deg, 45 deg, and 135 deg chosen arbitrarily to be diverse and variable.

I show in Fig. 15(a) the elliptical parameters for this four crystal retarder design but now at an incidence angle of 3 deg and over a full 360 deg rotation of this optic. Figure 15(b) shows the difference between the theoretically perfect optic elliptical retardance spectra and the optic retardance spectra including just the crystal optic axis misalignments against the surface normal. The retardance parameters vary by a few degrees with significant spectral dependence. Given that crystals are typically cut to a \( \pm 1 \) deg optic axis specification, this represents a very common type of error than using thicker crystal retarders.

Our simulation deliberately does not have symmetry. The crystal optic axes and also clocking oscillations couple in to create non-zero errors even for a perfectly aligned on-axis beam as the optic rotates. A beam at 0 deg incidence, and with perfect optical surface normal tilt alignment, will still see variable elliptical retardance upon rotation of the optic. This represents a fundamental limitation as all astronomical telescopes I am aware of assume one retardance for a rotating calibration optic without accounting for variable properties as the optic is clocked.

Substantial modeling efforts using POLARIS-M ray tracing in section 4 of Ref. 15 shows the ellipticity as a function of incidence angle through the ViSP SAR in figure 4.18 of Ref. 15. In section 4.4 of Ref. 15, the incidence angle dependence was averaged over simulated beam footprints to show how the optic design linear retardance turned elliptical as a function of
DKIST field angle. In section 5 of Ref. 15, the footprint averaged Mueller matrices were assessed for errors in fitting only linear retardance and also fully elliptical retardance. Decompositions were done using Lu–Chipman113 and symmetric114 methods. Depolarization terms were shown to be of order 1% at 633 nm wavelength in Ref. 15. I extend the work here and show depolarization terms over 10% at 397 nm wavelength. These degrees of freedom included in the future DKIST calibration algorithms if necessary. I note that a similar spatial averages over footprints cause smaller depolarization artifacts. See for instance depolarization from a non-uniform mirror coating (see Ref. 115), a non-uniform incidence angle across a powered mirror,116 and a non-uniform incidence angle from a converging beam reflecting off a flat mirror.116

5.4.3 Berreman spectral model cases: variation with rotation and incidence angle

The retardance of a realistic compound retarder optic is not accurately represented by a coordinate rotation of the retardance parameters. I show the difference between the nominal on-axis retardance and the Berreman model retardance at incidence angles of 0 deg, 1 deg, 2 deg in Fig. 16. Figure 16(a) shows the four MgF$_2$ crystal stack with nominal perfect alignment. For the 0 deg incidence angle in the upper left-hand graphic, there is no dependence of retardance on the clocking orientation. Figure 16(b) shows the four MgF$_2$ model but with one of the crystal optic axes perturbed 1.5 deg away from the surface normal. This provides an example of how imperfectly aligned crystals in an imperfectly aligned optic can be modeled to vary with rotation and non-zero incidence angle.

5.4.4 Berreman model and depolarization: compare two cases on axis at 653 nm

Currently I know of no astronomical telescope including depolarization degrees of freedom in the Mueller matrix model for their calibration retarder optics. In the calibration process, a thick crystal retarder produces significant depolarization due to the converging beam. Including this depolarization either via modeling and look-up table estimates or by direct fitting adds significant complexity to the telescope calibration process. I use our new Berreman simulations to estimate the depolarization magnitude and character for these thick crystal retarders. I follow section 6.11...
I implement Eq. (6.80) also listed here in Eq. (1). The Berreman calculus is used to propagate the $F/13$ beam over the full range of incidence angles. I compute the average degree of polarization (AvgDoP) for all possible fully polarized input Stokes vectors. This is effectively an average over the Poincaré sphere.

$$\text{AvgDoP} = \frac{1}{2\pi} \int_{0}^{\pi} \int_{-\pi/2}^{\pi/2} \text{DoP} [\text{MS} (\theta, \eta)] \cos(\eta) \, d\eta \, d\theta.$$ (1)

I note that any Stokes vector propagating through the Mueller matrix at any single spatial point in our model would have the degree of polarization (DoP) preserved at 100%. However, when I average the Stokes vectors across the beam footprint and then compute the AvgDoP, I see depolarization in the range of roughly some fraction of a percent to a few percent.

I show in Fig. 17 the DoP for every possible input Stokes component at a wavelength of 653 nm as propagated through the retarder when averaging over the $F/13$ range of incidence angles (±2.2 deg). In Fig. 17(a), I show the DoP for our nominal four MgF$_2$ crystal stack design with perfect theoretical alignment. Figure 17(b) shows the same calculation but now with the optic axis of one of the four crystals perturbed away from the surface plane by 1.5 deg. The converging beam creates depolarization for $U$ and $V$ inputs, but less depolarization for $V$ inputs. The perturbation breaks the symmetry, making $+Q$ inputs (horizontal axis positions of 0 deg and 180 deg) behave slightly different than $-Q$ inputs (horizontal axis position 90 deg).
In Ref. 15, the six crystal quartz based super achromatic calibration retarder (SAR) was modeled using the POLARIS-M software package. This 12.6 mm thick optic was designed to use calibrating DKIST with the ViSP and VTF instruments covering the 380 to 1000 nm bandpass. The new MgF$_2$ elliptical calibrator at 11.4 mm thickness is somewhat similar to the final 12.6 mm thickness of the quartz-based SAR per Ref. 110. Modeling was presented at 633.4 nm wavelength in the $F/13$ converging beam near DKIST GF in Ref. 15. Equation (2) shows the six elements of the depolarizer fit to the footprint-averaged beam at $F/13$ passing through the simulated ViSP SAR at a wavelength of 633.4 nm from the Lu–Chipman decomposition in Eq. (5.4) of Ref. 15. This decomposition was done for the central field point with a chief ray at zero incidence angle and without crystal optic axis manufacturing errors as added here in this article. The three diagonal depolarization terms $(a, b, c)$ are within 0.26% to 0.58% of 1. Similarly, the three phase depolarization variables $(d, e, f)$ are in the range 0.12% to 0.27%.

This quartz based six crystal retarder at this wavelength does depolarize the solar light collected by DKIST, but by <1% of the incoming signal. I show in the next section that including non-normal crystal optic axes per typical ±1 deg manufacturing tolerances can make these terms several times larger. I note that in the Sueoka decomposition, the retarder-then-depolarizer decomposition was able to completely fit the footprint-averaged optic Mueller matrix for the beam simulated, just like the symmetric decomposition of Ref. 114.

Crystal quartz is optically active, rotating the plane of linear polarization. Crystal SiO$_2$ is a circular retarder along the optic axis in addition to being a linear retarder for beams perpendicular to the crystal optic axis. Typical crystal quartz windows are $C$ plane cut, also called $Z$-plane cut, where the crystal optic axis is normal to the surface. A quartz crystal cut as a linear retarder has the optic axis parallel to the surface and is typically described as having an A plane cut. The complexity of even single compound SiO$_2$ retarders involves modeling this optical activity when simulating a beam propagating off-axis through an imperfectly aligned SiO$_2$ retarder. I show an example of an elliptical retardance fit to the measured Mueller matrix of a 1 mm thick crystal.
quartz window (C-plane cut) in Fig. 18(b). In October, 2022, measurement, I had six spectrographs in our NDSP6 setup covering the 0.37 to 2.5 μm bandpass. I followed our typical alignment procedure where the surface reflection is used to adjust the optic surface normal to the incident beam. The optic is over 120 deg circular retardance at wavelengths near 390 nm.

The beam is propagating through this window at a non-zero angle, and the crystal optic axis is also very likely cut some fraction of a degree off of the surface normal. Some 10 deg of linear retardance mostly in the second component is measured at the shortest wavelengths of our lab metrology system. Figure 18(a) shows a polariscope image of a 5 mm thick 120 mm diameter crystal quartz window. Circular retardance dependence on incidence angle and wavelength creates colored rings at integer values of circular retardance. See, for example, Chapter 19 of Ref. 73 showing details of optical activity and also part 2 of the MHW textbook on chiral media and Chapter 9 on handed media.85 I also show spatial retardance mapping of this 5 mm thick crystal quartz window in Sec. 15.3. Crystalline MgF2 has no such optical activity, simplifying the modeling of MgF2 type compound retarders.

5.6 DKIST MgF2 F/13 Calibration Retarder: Decomposition of Berreman Model

I show example calculations by averaging over our Berreman models as a simple method to estimate the MgF2 elliptical calibration retarder using six of the nine depolarization degrees of freedom in the Mueller matrix. I wrote modules to average the Mueller matrix computed using the Berreman calculus over a footprint covering a particular AOI range. I choose ±2.2 deg to represent the F/13 beam incident on the DKIST calibration retarders. This beam-averaged Mueller matrix can then be decomposed using whatever technique is desired. I have done a Lu–Chipman113 (polar) decompositions in both orientations of retarder-then-depolarizer and also depolarizer-then-retarder. More complex symmetric decompositions are straightforward to implement, but I highlight only the depolarizer-then-retarder model here. I note that the elliptical retardance parameters for an on-axis beam with no crystal tilt axis are recovered to numerical accuracies with our modules even when adding the six additional degrees of freedom. I only show here the additional depolarization degrees of freedom variables, as the retardance components found in this new decomposition remain unchanged compared to non-depolarizing Mueller matrix fits presented here.
Figure 19 shows the six depolarization degrees of freedom for the Lu–Chipman decomposition. Figure 19(a) shows the nominal four MgF₂ retarder design with perfect crystal optic axis alignment to the surface plane as well as perfect alignment of the optic surface normal to the incoming beam. Terms are below 2% at 633 nm wavelength, roughly the same order of magnitude as in the 12.6 mm thick six quartz crystal decomposition of Ref. 15. Figure 19(b) also shows the depolarization terms, but now including a 1.5 deg MgF₂ crystal optic axis misalignment of the first crystal axis away from the surface plane. The diagonal depolarization terms increase substantially for this perturbed crystal case to levels of 7% at 633 nm and up to 15% at 393 nm wavelength. Spectral oscillations between depolarization terms come from the unique orientation of these two MgF₂ retarder crystals, one modeled at 0 deg orientation and the other at 45 deg orientation, exactly aligned with QU inputs in the simulation.

I highlight this model as the depolarization degrees of freedom remain largely unexplored and are not included in typical astronomical system calibrations. I show in Appendix B several more detailed examples of depolarization fitting for both the diagonal and the phase...
depolarization degrees of freedom. I use our NDSP lab system to make Mueller matrix measurements of multiple kinds of depolarizers with different footprint sizes and spectral resolving power in a collimated beam. Our modules successfully fit diagonal depolarization for spatially patterned LCP type depolarizers. I also successfully fit both diagonal and phase depolarization for a quartz retarder wedged pair type depolarizer. The preliminary estimates here suggest depolarization terms could be over 10% at the shortest wavelengths. Future work is planned to both model and measure these effects for the DKIST calibration retarders. By including some simple manufacturing tolerances and doing the decomposition at a wide range of wavelengths, I show that the depolarization terms can be at large amplitudes.

6 Retarder Performance: Minimizing Clocking Oscillations

The spectral clocking oscillations represent a major limitation to retarder stability in spectral instruments capable of resolving the oscillations. Temperature changes of the optic cause the oscillations to drift in wavelength, even though designs may nominally be athermal.

I worked together with Meadowlark to improve the clocking accuracy of their optical contact bonding procedures. Special tooling was developed and refined during the multiple optical contact attempts funded by this project. I show here final performance of both retarder assemblies. Specifically, the specification for DKIST retarders has improved from a nominal ±1 deg clocking error to be consistently 5× to 10× improved in the range of 0.1 deg to 0.2 deg errors. Details of individual crystal clocking assessments are in Appendix D for the elliptical calibrator and Appendix E for the CN modulator.

6.1 Properties of MgF$_2$ Crystal Pairs for Spectral Clocking Oscillations

The spectral clocking oscillation period goes as $\lambda^2/db$ where $d$ is the thickness of the optic and $b$ is the birefringence of the material. For crystal MgF$_2$, the birefringence is roughly 1% as a mild function of wavelength. Figure 20(b) shows the spectral clocking oscillation periods expected for our crystal pairs. The individual crystals decrease in retardance quite substantially over the wide bandpass as shown in Fig. 20(b). The individual crystals are nominally around 100 to 200 waves net retardance at blue wavelengths falling to <20 waves at 5 μm wavelength.

6.2 MgF$_2$ Clocking Oscillation Improvement Example: CN2:CN4

The spectral clocking oscillations are greatly reduced by the 0.1 deg clocking error for this CN2:CN4 crystal pair. This MgF$_2$ pair has 1.5 waves net linear retardance at a wavelength of 542.6 nm. The 2.2 mm thick individual MgF$_2$ crystals have a net retardance of 48.0 waves at this particular wavelength. The spectral period for the fast axis oscillations computed as $\lambda^2/db$ is 11.4 nm where $d$ is the thickness and $b$ is the birefringence (~0.0119 at this wavelength).

I performed spatial Mueller matrix mapping with the NLSP metrology system. With NLSP having an optical resolution near 0.35 nm, I sample the spectral retardance clocking oscillations.
with more than 30 resolution elements. Figure 21 shows the circular retardance in Fig. 21(a) and the fast axis of linear retardance in Fig. 21(b) for the aperture center spectrum for a bandpass centered about 545 nm wavelength near the 1.5 waves net linear retardance bandpass. The dashed lines show a range of imperfectly clocked crystal pair models using 2.2 mm thick crystals with a 68.7 μm thickness difference. Clocking misalignment models are created in steps of 0.05 deg.

### 6.3 CN Modulator Spectral Clocking Oscillations
Spectral clocking oscillations are expected from the 0.2 deg offset in the thicker crystals CH3: CH4. An additional contribution will be seen at a slower spectral period from the 0.1 deg offset estimated for the thinner C1a:C1b crystal pair. Figure 22 shows the spectral clocking oscillations detected for the assembled modulator (all four crystals) as a spatial average over the first three
rings in the rosette spatial pattern. These three rings represent 37 measurements covering a circular aperture of 39 mm diameter. A parabolic fit to the bandpass has been applied to remove the nominal design retardance and highlight the spectral retardance clocking oscillations. I selected a bandpass using the short IR channel of NDSP4 covering 948 to 1065 nm. This spectrograph has a nominal 0.15 nm spectral resolving power (profile FWHM) with 0.03 nm per pixel spectral sampling. For a wavelength near 1 μm, the thicker CH3:CH4 crystals have a retardance spectral oscillation period near 21 nm with a larger expected magnitude. The thinner C1a:C1b crystals have a period near 12 nm. Circular retardance oscillates by up to 1 deg while the fast axis of linear retardance oscillates by roughly 0.2 deg. These oscillations are thermally sensitive and represent a calibration accuracy limitation through any thermal perturbations.

6.4 Elliptical Calibrator: Clocking Errors and Spectral Oscillations
Clocking oscillations are also similar to the two-crystal optical contact retarder and misalignments within other six-crystal retarders. Figure 23 shows the clocking oscillations in two select bandpasses for a 5.5 mm diameter beam illuminating the center of the elliptical calibrator aperture. Figure 23(a) shows a bandpass near 860 nm wavelength while Fig. 23(b) shows most of our NIR spectrograph covering 1180 to 1550 nm. In our final Mueller matrix metrology data set, I recorded the aperture center at the start of all 16 individual maps. I thus get 16 repeated Mueller matrix spectra separated by 1 to 3 days distributed over the roughly 25 day mapping campaign.

6.5 Elliptical Calibrator: Thermal Perturbation with Clocking Errors
Computing the thermal drifts and spectral clocking oscillations of an elliptical four crystal retarder is straightforward and similar to other multi-crystal retarders. Clocking oscillations are also similar to the DKIST quartz crystal optical contacted retarder pair shown in figures 6, 20, and 21 of Ref. 87, as well as from misalignments within oiled six-crystal retarder stacks as shown in Refs. 86 and 110. See Appendix I for the material properties assumed in our thermal modeling here.

I show in Fig. 24(a) an example elliptical retardance fit to a thermally perturbed four crystal optical model. The four individual crystals are perturbed in their relative rotational alignment (clocking) by −1.0 deg and −1.3 deg for the first pair and 1.0 deg and 1.5 deg for the second pair. This perturbation changes the design net retardance and introduces spectral clocking oscillations. This behavior is similar to the two-crystal optical contact retarder and misalignments within other six-crystal retarders. See Refs. 86 and 87. I apply a 20 deg bulk temperature change from the nominal 22 deg assembly temperature. The clocking oscillations are largest at short wavelengths where the net retardance of each individual crystal can be 100 waves.

Fig. 23 The spectral retardance oscillations from clocking errors. (a) A 40 nm spectral bandpass measured with our red optimized spectrograph. There are 1680 independent spectral measurements shown for 16 repeated data sets recorded at aperture center over roughly 25 days. Each spectrum was separated by 24 to 36 h as well as using a new system calibration. The small scale retardance oscillations are very stable. (b) A 500 nm bandpass measured with our NIR spectrograph.
The compound retarder mostly compensates for any changes in the net retardance as deviations from nominal are <1 deg, much smaller than the clocking oscillation magnitudes of 3 deg. Figure 24(b) shows the perturbation of a single crystal, the fourth crystal in the stack, by 0.5 deg rotation. This simulation shows there are net design changes as well as the creation of spectral oscillations of a fraction of a degree magnitude by a single crystal being rotated away from design.

The DKIST team measured temperatures for three of the DKIST retarders as well as the polarizers directly using thermocouples and remotely using IR diode sensors while observing the Sun (see Ref. 111; Appendix B in particular Figs. 28–32). They measured a 1.5°C temperature rise for the six MgF$_2$ crystal Cryo SAR retarder over 30 min of 300 W solar illumination (figure 31 of Ref. 111). In this configuration, the retarder was not protected by the calibration polarizer upstream, so infrared flux was present and absorbed in the retarder by both the refractive index matching oil and the MgF$_2$ crystal itself. All current DKIST calibration sequences never allow the retarder to be in the beam without the polarizer upstream. The polarizer reduces the flux by more than 65% and also blocks all infrared wavelengths absorbable by the MgF$_2$ beyond 6000 nm. I anticipate nearly negligible thermal forcing of this four-MgF$_2$-crystal optically contacted calibration retarder during the expected <15 min of a typical calibration sequence. I note that clocking oscillations were observed with DKIST calibrating on-Sun using an existing SiO$_2$ optically contacted compound retarder in figure 24 of Ref. 44 and adjacent text. These oscillations were near expectations and were also roughly stable given the optic had been illuminated in the beam and thermally stabilizing for several hours. I show explicitly in Secs. 11.4.3 and 11.4.4 examples of how spatial variation and spectral variation in our optically contacted MgF$_2$ changes in response to temperature variation. Minimizing the spectral clocking oscillations substantially improves the temporal stability of these MgF$_2$ optics in a changing thermal environment.

7 Summary
I have shown here the successful development of very large clear aperture optically contacted MgF$_2$ retarder crystal pairs and their application to both calibration and modulation. DKIST funded and collaborated on developing several new manufacturing processes to allow use of the softer, more difficult MgF$_2$ material. Optical contact of 120 mm diameter retarders with clean apertures passing 60/40 scratch dig specifications removes the need for oils, epoxies, the associated IR absorption, and any temporal degradation from UV exposure. The 300 W DKIST beam is used over 380 to 5000 nm wavelength, with substantial flux down below 320 nm. This high flux beam and wide wavelength requirements create challenging retarder performance requirements.
Improvement in deterministic FJP driven by retardance spatial variation metrology, not TWE nor crystal thickness, results in substantially more uniform retardance. The retardance uniformity ultimately improves telescope polarization calibration accuracy both on-axis and across a wider FoV. New active alignment procedures repeatably deliver optically contacted crystal subtraction pairs with fast axis clocking alignment better than 0.2 deg. For some of our crystals, clocking alignment was better than 0.1 deg. This fast axis clocking alignment of the individual compound subtraction pairs greatly improves spectral retardance behavior of the assembly by reducing the spectral retardance oscillations. This in turn improves temporal stability and retardance spatial uniformity as the spectral oscillations are both thermally sensitive and spatially variable. Calibration accuracy is significantly improved when accurate clocking techniques are combined with the uniform retardance delivered by the retardance-mapping-driven FJP.

I created an elliptical calibration retarder design using only 2 MgF₂ crystal pairs that, when combined with a wire grid polarizer, can calibrate the entire 380 to 5000 nm bandpass strictly simultaneously with our standard 10-state sequence. I also designed a new modulator for the Cryo-NIRSP instrument using only two crystal MgF₂ retarder pairs. With these manufacturing improvements, I show reduction in several error sources and anticipate successful calibration down to 380 nm wavelengths. Better than ±2 deg retardance uniformity was achieved for the elliptical calibrator over the full 105 mm clear aperture at 633 nm wavelength. For comparison, the six MgF₂ crystal calibrator called the Cryo-SAR had a ±10 deg retardance measured at 589 nm over a reduced 90 mm aperture as shown in Ref. 86 (Appendix B1 and figure 31).

I develop new modeling tools to show the polarization accuracy limitations caused by several types of error sources. I demonstrate Berreman-calculus based modeling to show the behavior of these retarders in depolarization and retardance with converging beams, FoV dependence, and crystal optic axis misalignments. I model the depolarization and elliptical retardance variation caused by the spatial average over the converging F/13 DKIST beam on the calibration optics. I find fitting six of the nine depolarization variables using a polar (Lu–Chipman) decomposition is successful when covering ±2.2 deg incidence angle range over the full 380 to 5000 nm bandpass. Appendix B shows a demonstration of measuring depolarizing commercial optics with our lab Mueller matrix spectral metrology system along with various depolarizing Mueller matrix decomposition fits as examples. I also show the impact of imperfect crystal cutting where the crystal optic axis is misaligned with the surface of each optic. I model typical manufacturing specs of ±1.5 deg and show direct measurements in this range for a set of MgF₂ retarder pairs used in DKIST. These tools should prove useful in the future for comparing multiple optical design strategies, informing tolerances, and stating limitations on calibration procedures given known error sources (e.g., FoV limits versus wavelength).

The suppression of interference fringes over such a wide wavelength range is a challenge. Berreman modeling tools were applied to DKIST use cases in Refs. 86 and 87 leading to this development of optical contact of thicker MgF₂ substrates. The uniformity enabled by the FJP allows us to use substantially thicker crystal pairs, hopefully suppressing fringes by at least a factor of 5 at the longest current CN wavelength of 4651 nm (the CO band). I use an air gap between the two MgF₂ crystal pairs given the waves-per-degree surface flatness sensitivity to temperature changes from Appendix G.

With this successful development, I anticipate the availability and use of MgF₂ contacted retarders for challenging astronomical applications. MgF₂ has greatly reduced absorption in a solar beam. Optical contact bond removes the need for oils or epoxies that do not absorb, heat, or degrade in UV irradiation. The crystal fast axis alignment improvements suppress spectral oscillations and their associated thermal sensitivity, increasing calibration accuracy for spectral instruments. Deterministic polishing driven by retardance metrology delivers spatially uniform retardance, reducing calibration errors and modulation efficiency losses at shorter wavelengths.

8 Appendix A: Bandpass and Heat Loads at Solar Telescopes
I compare some retarders and calibration optic locations in a variety of large solar telescopes to illustrate the trade-offs made at other facilities. Different choices can be made between materials, heat rejection filters, wavelength coverage, optic size, etc. I outline some of the optical choices at solar telescopes below. Some additional details are outlined in Ref. 110.
### Table 3  Solar telescopes and polarization calibration optic properties.

<table>
<thead>
<tr>
<th>Name</th>
<th>Diam (m)</th>
<th>FoV (')</th>
<th>Bandpass (nm)</th>
<th>Blocking (UV/IR?)</th>
<th>Power (W)</th>
<th>Cir Ap (cm)</th>
<th>Irrad (W cm⁻²)</th>
<th>Optics upstream</th>
</tr>
</thead>
<tbody>
<tr>
<td>DKIST</td>
<td>4.0</td>
<td>5.0</td>
<td>380 to 5000</td>
<td>None</td>
<td>13</td>
<td>300</td>
<td>10.5</td>
<td>M1, M2</td>
</tr>
<tr>
<td>GREGOR</td>
<td>1.5</td>
<td>2.5</td>
<td>380 to 1800</td>
<td>Filters</td>
<td>6</td>
<td>8</td>
<td>0.7</td>
<td>M1, M2, filters</td>
</tr>
<tr>
<td>GST</td>
<td>1.6</td>
<td>2.0</td>
<td>390 to 1600</td>
<td>None</td>
<td>52</td>
<td>8</td>
<td>14.0</td>
<td>M1, M2</td>
</tr>
<tr>
<td>DST</td>
<td>0.76</td>
<td>2.8</td>
<td>400 to 1600</td>
<td>Win.</td>
<td>72</td>
<td>3</td>
<td>4.3</td>
<td>Win, Trt, Win, M1</td>
</tr>
<tr>
<td>CLST</td>
<td>1.8</td>
<td>3.0</td>
<td>380 to 2500</td>
<td>—</td>
<td>23</td>
<td>—</td>
<td>—</td>
<td>M1, M2</td>
</tr>
<tr>
<td>EST*</td>
<td>4.2*</td>
<td>2.1</td>
<td>380 to 2300</td>
<td>—</td>
<td>12</td>
<td>53</td>
<td>—</td>
<td>M1, M2</td>
</tr>
</tbody>
</table>

The 1.5 m GREGOR solar telescope has had several calibration retarders located near a focal plane after their secondary mirror servicing multiple polarimeters.⁶⁹,¹¹⁷-¹²⁵ Recent GREGOR optical upgrades are shown in Ref. ⁶⁹. Another useful comparison is The Big Bear Solar Observatory (BBSO) which operates the Goode Solar Telescope (GST) formerly named the New Solar Telescope (NST). The GST has an off-axis Gregorian design with a 1.6 m clear aperture primary mirror. An alternate case study with calibration performed only after several optics is the spectro-polarimeter for infrared and optical regions (SPINOR) spectropolarimeter at the 0.76 m Dunn Solar Telescope (DST) in Sacramento Peak, New Mexico.⁸⁰,¹²⁶ I outline the details of the optical power, location of polarization calibration optics, and some relevant optical details in Table 3.

I describe each column of Table 3 first and then provide detailed calculations and citations for each telescope in the paragraphs below. The first column of Table 3 shows the telescope name acronym. The second column shows the primary mirror diameter in meters. The third column shows the FoV in arc minutes passed through the heat stop as seen by the polarization calibration optic. I note that increasing the field increases the beam power by the square of the field, scaling heat the same way increases in the primary mirror diameter. For comparison, the DKIST sees 39× the optical power with a 4 m primary and a 5 arc minute field compared to the GST at 1.6 m primary and a 2 arc minute field. The polarization calibration optics see a beam with 6.25 times larger collecting area primary mirror and also a 6.25 times larger area field passed through the heat stop.

The fourth column of Table 3 shows the bandpass of the instruments working at the telescope. I note that GREGOR, GST, and the DST do not presently use wavelengths beyond 2000 nm, meaning that more common calibration retarder materials such as crystal quartz or polycarbonate are options. The DKIST must pass 5 μm wavelengths restricting allowable materials for polarization calibration optics. The fifth column of Table 3 shows wavelength bandpass rejection ahead of the polarization calibration optic. The GREGOR uses heat rejection filters. The DST has glass windows that absorb significant amounts at infrared wavelengths.

The beam focal ratio ($F/)$ and beam optical power in Watts at the polarization calibration optical station are shown in the sixth and seventh columns. The eighth column shows the illuminated clear aperture diameter for the polarization calibration optic station in centimeters. I note that DKIST and the GST chose large optics at >10 cm diameter presenting certain challenges while GREGOR chose optics <1 cm presenting other challenges. Column nine shows the irradiance in Watts per square centimeter averaged over the beam footprint at the polarization calibration optic station. I note that the GREGOR has 8 W of collected power but has concentrated this power in a <0.7 cm diameter aperture.

The 11th column of Table 3 lists the optical elements ahead of the calibration optic. In the DST case, multiple windows (Win) and a turret mirror pair (Trt) are used ahead of their primary mirror (M1). The three other telescopes put polarization calibration optics after both the primary and secondary mirrors (M1, M2). The GREGOR has heat rejection filters ahead of their small but high irradiance calibration optic station. These four solar telescopes present a range of possible design choices for polarization calibration optics that each present unique challenges.

At GREGOR, the second focus ($F/2$) is near the calibration optics and is stated to receive about 8 W of power and a 2.5 arc minute FoV.⁶⁹,¹²⁴ This can be compared to 300 W at DKIST covering 5 arc minutes FoV. I estimate 10 W when scaling 300 W by $(1.5/4)^2$ and $(2.5/5)^2$, close to the stated value. The GREGOR beam at the second focus is $F/6$ with an image scale of...
23 arc seconds field angle per millimeter across the calibration retarder.\textsuperscript{124} The power density was stated as about 24 W/cm\textsuperscript{2} and with the GREGOR plate scale with their second focus at only 6.5 mm in diameter. The DKIST optics were 105 mm clear aperture and 120 mm physical diameter and see a similar irradiance of roughly 3.4 W/cm\textsuperscript{2}. From Ref. 118, the GREGOR polarization calibration unit has two super-achromatic retarders. Each retarder consists of five individual layers each at true zero-order. The polymethyl-methacrylate retarder layers were designed by Astropribor with an angular acceptance of \( \pm 7 \) deg, a temperature range of \(-20^\circ\text{C} - 50^\circ\text{C}\), and a damage threshold of 500 W/cm\textsuperscript{2}. These two separate retarders cover 380 to 800 nm wavelength range for the first and 750 to 1800 nm for the second. These bandpasses are roughly similar to the DKIST calibration retarders for ViSP and DL-NIRSP, respectively. I note the DKIST team has measured uniformity and done several custom designs for these kind of multi-layer achromatic retarders and also elliptical modulators in Ref. 87 and the appendices of Ref. 110. In addition to the nominal GREGOR polarimetric calibration unit (PCU, Ref. 118), a second PCU was installed.\textsuperscript{117}

At the GST, the first generation instrument with polarimetric capability was the infrared imaging magnetograph (IRIM).\textsuperscript{127-129} The calibration unit is similar to DKIST in that both units contain a linear polarizer ahead of a \( \sim \) quarter wave linear retarder mounted near GF. The polarization modulator for IRIM is also mounted up stream of the GST tertiary mirror, after GF. I estimate the flux delivered at the GST by comparing with the DKIST flux budget. The focal ratio of the GST aluminum coated M1 is \( F/2.4 \) with a protected silver coated M2 delivering an \( F/52 \) beam at GF. The 2 arc minute circular heat stop defines the flux passed to the polarization calibration optical station. I calculated 300 W for a 5 arc minute DKIST field where I have included bare aluminum coating on the primary and an enhanced protected silver coatings on the secondary. The Haleakala atmospheric transparency calculations were done with MODTRAN software version 6, the U.S. Air Force (USAF) standard moderate spectral resolution atmospheric radiative transfer model maintained by Spectral Sciences, Inc. For a 1.6 m aperture and 2 arc minute field I would see 6.25\(x \) less flux collected by M1 and the same factor for the reduced field. This gives roughly 8 W of heat at their GF with a plate scale is 2.48 arc seconds per mm. The modulator is located away from focus with a 140 mm clear aperture. I compute an irradiance of <0.06 W/cm\textsuperscript{2} if the full aperture is illuminated compared with 24 W/cm\textsuperscript{2} for GREGOR and 3.4 W/cm\textsuperscript{2} for DKIST. The UV flux load would be highly dependent on the protection and properties of the silver coating on the GST M2.

The SPINOR instrument at the DST covers from 400 to 1600 nm and can be operated with up to four separate cameras, each covering nm-scale bandpasses of interest. For SPINOR, there are three retarders for the instrument. There is a 3/8 wave sapphire and quartz bi-crystalline achromat that is used as the modulator. Others have done a polarization fringe analysis, placing this modulator in the \( F/36 \) beam near the slit.\textsuperscript{99,100} As the SPINOR slit is scanned across the FoV, the instrument samples different spatial regions of the retarder, introducing noticeable changes in system performance.\textsuperscript{99} The modulator nominal location is in a collimated beam in the fore-optics near a pupil plane. Spatial non-uniformity of the two crystals only mildly depolarizes the beam. The angular dependence across the field introduces field variation. SPINOR optical and heating calculations are shown in the appendix of Refs. 110 and 100.

The EST project in design phase at 4.2 m diameter is also listed as the last row in Table 3. An overview of the current EST designs is in Ref. 37. In 2022, there were several EST optical updates published: an overview in Refs. 38 and 39 focusing on the adaptive optics, Ref. 40 outlining the Coudé light path, and Ref. 36 showing a Zemax analysis of the EST polarization.

The EST has 4.2 m diameter on-axis primary with a 1.1 m central obscuration. The collecting area is thus roughly 12.9 m\textsuperscript{2}, slightly larger than the 12.6 m\textsuperscript{2} area of DKIST. The FoV for the EST is listed at 2.1 arc minutes compared to the 5.0 arc minutes of DKIST. To make a power estimate, I scale down the beam optical power from DKIST by the FoV area ratio of 5.7\(x \) to 53 W in Table 3.

Figure 8 of Ref. 34 shows a 2010 conceptual design for the calibration station of the EST with polarizers and retarders a few hundred to several hundred millimeters ahead of their second focus, after the EST secondary mirror. For DKIST, the GOS polarization optics are 350 to 550 mm ahead of an \( F/13 \) focus giving rise to the >100 mm diameter requirements. Both EST and DKIST polarization optics are near a focal plane, and with EST passing a \( \sim 2.4x \) smaller field, the calibration retarder size requirements will be accordingly reduced. Figure 2 of Ref. 34 also shows polarization error estimates from imperfect matching of coatings, in particular Eq. (2).
They conclude that a thickness difference of 0.5 nm in a protected silver mirror coating layer already exceeds the polarization error budget for the V to U cross-talk component for the EST. In any real optical system with non-identical coatings on multiple mirrors, polarization calibration will remain a critical observatory task with polarizers and retarders mounted in a calibration unit.

The beam after the EST M2 is listed as $F/12.5$ in Ref. 40 and $F/11.8$ in figure 1 of Ref. 39. Regardless of the final actual focal ratio, the beam on the EST calibration optics seems slightly faster than the DKIST $F/13$ beam, making our retarder modeling here reasonably applicable. I list $F/12$ in Table 3 for the EST. If I use figure 8 of Ref. 34 as a guide to optical locations for possible EST polarization optics, an individual footprint from an $F/12$ beam at 940 nm from focus would be 78 mm in diameter for any individual field angle. The optic clear aperture of course would need to be larger to accommodate the full 125 arc second FoV. If any revisions to the EST design mounts polarization optics closer to focus, the optic size would be reduced, relaxing the uniformity requirements but also increasing the irradiance. If IR absorptive materials such as crystal quartz are used, they would experience a similar heating problem to the DKIST polarizers and retarders described here. I leave the clear apertures, bandpass blocking filter options, and optic irradiance for EST as unspecified in Table 3.

The Chinese Large Solar Telescope (CLST) is an on-axis Gregorian stated to cover 380 to 2500 nm with a 1.8 m diameter primary mirror. First light is announced in Ref. 130. Optical parameters are outlined in Refs. 131 and 132 including a 3 arc minute FoV, the focal ratio of 1.7 for the primary, and details of the AO system. A prototype was developed by Rao et al.133 where the focal ratio after the secondary was stated as 10.9. I assume the polarization calibration optics would be at a similar focal ratio. Instrumental polarization of the CLST is in Ref. 130. It was unclear from the references that I found the details about any particular polarization calibration strategy, thus I leave the optic-related entries for CLST in Table 3 empty. Assuming a similar optical path between GREGOR and the CLST, I scale the power by the increased primary collecting area as $(1.8/1.6)^2$ for a 1.6× increase, as well as the larger FoV as $(3/2)^2$ for 2.25× increase. This gives us an estimate of 23 W power at the secondary focus for polarization calibration optics.130 An 8 m diameter ring-type telescope called the Chinese Giant Solar Telescope is outlined in Ref. 134.

Night-time astronomical observatories do not produce significant heat loads on their optics through the beam. However, polarization calibration optics often must be mounted in converging beams near focal planes and operate in outside summit environments. The polarization calibration concerns are very similar. Some calibration impacts of fringes in a cemented super achromatic MgF$_2$ retarder using the Keck 10 m telescope and the low resolution imaging spectograph in polarimetric mode (LRISp) are shown in Refs. 86 and 96. The calibration is somewhat similar to DKIST in that it LRISp also has a calibration polarizer and achromatic retarder made of six MgF$_2$ crystals working in a converging beam near a focal plane. However, LRISp has no independent retarder optics ahead of Cassegrain focus in front of the instrument modulator.86,96,135–139 The nominal secondary mirror gives an $F/15$ beam but after accounting for the outer edges of the hexagonal mirror segments, the beam is $F/13.7$.140 This instrument sees a circularly symmetric primary and secondary mirror except for a slight off-boresight mounting location of roughly 10 arc minutes equivalent field angle.135–139 The typical LRISp beam would have a beam footprint of a few millimeters on the retarder.136 The induced polarization is low but I also found that the typical alignments and the recommended demodulation procedures can leave several percent residual cross-talk as found in the daytime sky calibrations of LRISp in Ref. 96.

At other observatories, a polarization sensitive instrument may not be allowed to access upstream foci for calibration purposes. This was the case for the Air Force 4.0 meter AEOS telescope and the HiVIS spectropolarimeter as well as for optics after primary and secondary mirrors in typical symmetric telescope designs.141–148 Other systems such as the European Extremely Large Telescope (E-ELT) have a 39 m diameter primary and correspondingly large beams that do not provide convenient access to calibration before a Nasmyth focus, introducing time dependent polarization from the fold mirror.46,49,149

In this context, the team pursued developing low absorption, wide wavelength range, 120 mm diameter calibration retarders using MgF$_2$ crystals. For calibration stability, I avoided any absorptive substrate materials, oils, cements, adhesives, etc. I note that polarizers were also developed and deployed using sapphire substrates as shown in Ref. 41. Another polarizer upgrade for DKIST using thin fused silica substrates and nano-imprint lithography was shown in Ref. 44.
9 Appendix B: Laboratory Measurements of Depolarizers

Depolarization is commonly caused by an average of variation over time, spatial location (aperture/beam footprint), or wavelength. These degrees of freedom represent Mueller matrix elements at levels above 0.1 for our calibration retarder, important for accurate calibration of a polarimetric system. I demonstrate straightforward measuring and modeling these degrees of freedom using our lab metrology equipment and commercially available optics. I demonstrate depolarization properties of both spatially variable retarders and retarders with strong spectral oscillations using our laboratory Mueller matrix spectral metrology tools. Two types of depolarizer are tested: thin film of LCP patterned depolarizers and crystal quartz retarder wedge pair type depolarizers. Both types are commercial parts purchased from Thor Labs. I also demonstrate here decomposition of these measured depolarizing retarder type optics showing I can reproduce the measured depolarizing Mueller matrix using combinations of depolarizer and retarder Mueller matrices following standard procedures.

9.1 Liquid Crystal Polymer Depolarizers: Diagonal Depolarizers

Each LCP type optic is a polymer retarder layer sandwiched between two glass plates. The catalog parts come with particular bandpass optimized anti-reflection coatings. A pattern of varying retardation and fast-axis orientation is imprinted in the LCP material. For this depolarizer, the angle of the linear retardance fast axis is increased by 2 deg across consecutive 25 μm wide strips. The A type LCP retarder has a periodic variation in magnitude between 250 and 300 nm phase retardance. The B type has a periodic variation between 380 and 430 nm phase. The C type has a periodic variation between 720 and 770 nm phase. The A type anti-reflection coating is only optimized for 350 to 700 nm, the B type is optimized for 650 to 105 nm, and the C type is optimized for the 1050 to 1700 nm bandpass. Transmission oscillations out of band can create up to 70% transmission losses. Figure 25 shows a polariscope inspection image of the C type LCP depolarizer. When placed between crossed polarizers, the spatially variable sinusoidal pattern of

![Fig. 25](image_url) A polariscope inspection image for a 25 mm diameter C type LCP depolarizer taken with a cell phone camera. Placing this LCP between crossed linear polarizers in a polariscope reveals the sinusoidal pattern in retardance fast axis orientation along with retardance magnitude changes. Spatially averaging over a beam having a 5 mm diameter footprint on this optic leads to substantial depolarization with strong spectral variation.
retardance leads to oscillating transmission easily visible by eye. This particular LCP pattern is insensitive to the incoming orientation of linear polarization. Polariscope inspection gives effectively the same pattern as Fig. 25 for all orientations of the optic between crossed linear polarizers. This spatial pattern corresponds to the spatially variable retardance in both fast axis orientation and magnitude. Our Mueller matrix measurement system sees depolarization when this pattern gets averaged across the probe beam footprint.

I measured the Mueller matrix for several arrangements of these LCP depolarizers in our NDSP5 spatial mapping lab setup. Figure 26 shows the Mueller matrix measured at the center of each type of depolarizer (A, B, C) using a 5.5 mm diameter beam footprint. I also show a measurement of a stack of four LCP depolarizers in C–A–B–A ordering to illustrate a strongly depolarized measurement. The spatial oscillations in retardance occur over ∼1 mm distance meaning our systems 5.5 mm probe beam averages over roughly five spatial cycles of the retardance pattern. I note that the slight variation from zero in the first row and first column (depolarization, polarizance) of the measured Mueller matrix are system alignment errors present even when no sample is mounted. I ignore these small systematic errors for this LCP data analysis discussion in this section. I note that the anti-reflection coatings on each LCP depolarizer have very substantial reflection out-of-band. The $II$ element of Fig. 26 Mueller matrices show the unpolarized throughput. All other elements have been normalized by the $II$ term to clearly show polarization dependence, separate from unpolarized intensity effects.

![Fig. 26](image-url) The NDSP5 measured Mueller matrix for various Thor Labs patterned liquid crystal depolarizers. (a) The A type LCP. (b) The C type. (c) The B type LCP. (d) A stack of four LCP depolarizer optics in C–A–B–A ordering. The [0,0] element shows the unpolarized throughput (transmission). All other matrix elements have been normalized by this [0,0] element for clarity. I note that this [0,0] element represents the particular anti-reflection coatings applied. In the case of the four LCP stack in the lower right, the four AR coating flavors combine to suppress for all but wavelengths around 700 nm.
9.2 Depolarization Index Comparison: LCP Types A, B, and C

The Mueller matrix for this kind of LCP depolarizer is very well represented by a diagonal depolarizer as in Eq. (3). The $QQ$, $UU$, and $VV$ elements of the Mueller matrix are the only substantial non-zero elements in the lower $3 \times 3$ sub-matrix. These diagonal elements are represented with variables $a$, $b$, and $c$. The total transmission is shown with the variable $T$. There is no substantial rotation of coordinates (retardance) nor any diattenuation in Eq. (3). The ideal depolarizer (ID) has $a = b = c = 0$.

The depolarization index (DI) is a common metric for the overall amount of depolarization in a particular Mueller matrix. See Chapter 6 of Ref. 73, along with several other references. The DI is defined as the Euclidian distance of the Mueller matrix under test from the ID. For these calculations, all tested Mueller matrices are assumed to be intensity normalized where the [0,0] element has been divided out as overall transmission can be removed from this geometrical metric. The Mueller matrix for the ID has the [0,0] element for transmission equal to 1 with all other elements equal to 0:

$$\text{DI} = \frac{1}{\sqrt{3}} \sqrt{\sum_{ij} M_{ij}^2} - 1 = \|M - \text{ID}\|.$$  

(4)

Given a normalized Mueller matrix under test ($M$) where $M[0,0] = 1$, I can define the calculation for DI as in Eq. (4). For polarization preserving optics, the DI is 1. An ID has a DI of 0. The DI is a quick scalar assessment related to how well an optic with a particular Mueller matrix preserves polarization.

The top row of Fig. 27 compares depolarization indices for A, B, and C type LCP depolarizer optics. The bottom row shows the absolute value of the diagonal elements of the Mueller matrix ($QQ$, $UU$, and $VV$) for reference. Figure 27(a) shows the A type LCP depolarizer with 250 to 300 nm phase retardance. I see that the DI is minimum near 420 and 800 nm wavelength where the overall magnitude of the diagonal elements is lowest. The Mueller matrix for the A type LCP depolarizer approaches the identity matrix at short wavelengths, where the DI approaches 90%. Figure 27(b) shows the B type LCP depolarizer with the identity matrix being preserved around 470 nm wavelength. Circular polarization ($VV$) is also preserved around 800 and 370 nm wavelength.

Figure 27(c) for the C type LCP depolarizer shows two wavelengths where polarization is preserved (DI = 1) as the optic Mueller matrix approaches the identity matrix. At 440 and 710 nm wavelength, the bottom right-hand graphic of Fig. 27 shows all diagonal elements approaching a value of 1. Given the phase retardance for the polymer is stated to be oscillating between 720 and 370 nm phase (at 633 nm wavelength), this would correspond to the wavelength where the retarder has exactly 1 wave of phase, preserving input polarization. As the phase retardance of polymers is wavelength dependent and typically increases toward short wavelengths, the identity matrix being observed again near 410 nm wavelength would suggest two waves of phase or 820 nm of phase retardance at this blue wavelength, a mild increase of only 10% from the nominal 633 nm HeNe specification wavelength. The DI for this C type LCP depolarizer does not go below about 0.35 because there is no wavelength where circular and linear polarization states are all simultaneously driven to zero. The minima in $QQ$ and $UU$ elements occurs near 540 and 1400 nm wavelength, but these correspond to wavelengths where circular polarization is preserved, i.e., where $VV = 1$.

9.2.1 Depolarization: polariscope images for combinations of types A, B, and C

These patterned LC depolarizers can be combined using various flavors and orientations to increase the depolarization of more states over a wider bandpass. By combining these optics
at a range of orientations, the spatial scale for the retardance variation shrinks. I show example polariscope images for a range of 2, 3, and 4 LCP depolarizer combinations in Fig. 28. The sinusoidal pattern across the aperture interacts as a function of wavelength and spatial position. These polariscope images indicate the kind of spatial non-uniformity that can be expected across a beam footprint when using these kind of optics. Our lab metrology system works as an analogy to the DKIST calibration retarders. A spatially non-uniform beam footprint is present but is condensed into a detector element. For DKIST, this would be a pixel of a science instrument. For our lab metrology system, it is the fiber feeding the spectrograph entrance slit.

9.2.2 Depolarization: Mueller matrix diagonal elements for A, B, and C combinations

Combining more of these LCP depolarizers leads to a better approximation of an ideal elliptical diagonal depolarizer where \( QQ = UU = VV = 0 \). Figure 29 shows the diagonal elements for the Mueller matrix for a selection of LCP depolarizer combinations. The beam footprint was roughly 5 mm in our lab metrology system, averaging spatially over the patterns shown in Fig. 28. These diagonal elements have all been normalized by the unpolarized transmission (\( II \)) term. I note that the NDSP5 measurements show that the first row, first column, and off-diagonal (retardance) elements are effectively zero within systematic error limits. The top left graphic of Fig. 28 shows just two A-type LCP depolarizers. There are a few wavelengths where either linear or circular polarization is still preserved (small depolarization). However, in the lower right-hand graphic of Fig. 28 where four of these LCP depolarizers are combined in C–A–B–A ordering, the depolarization is almost ideal for both \( Q \) and \( U \) inputs with <30% preservation (\( VV \)) for circular polarization.

Fig. 27 The DI is shown in the top row for A, B, and C type LCP depolarizers as measured in NDSP5 with a 5.5 mm diameter beam footprint. An ID has DI = 1. (a) The A-type LCP depolarizer with retardance magnitudes oscillating between 250 and 300 nm phase. (b) The B-type LCP depolarizer with magnitudes between 380 and 430 nm phase. (c) The C-type LCP depolarizer with retardance magnitude oscillations between 720 and 770 nm. The bottom row shows the absolute value of the measured diagonal elements of the Mueller matrix \( QQ, UU, \) and \( VV \) in blue, green, and red, respectively. These diagonal elements correspond to the variables \((a, b, c)\) in the decomposition. The A-type LCP depolarizes linear states \((Q, U)\) at 550 nm wavelength as the blue and green curves go to zero, but circular polarization is completely preserved as \( VV = 1 \). The B-type LCP in the lower middle graphic has no significant depolarization at all for a wavelength near 480 nm as the absolute value of all Mueller matrix diagonal elements is measured at \( \sim 1 \). The C-type LCP in the lower right-hand graphic has a similar lack of depolarization for any state for wavelengths of 420 and 750 nm.
Fig. 28 Polaroscope images of combinations of 2, 3, and 4 LCP depolarizers of 25 mm round diameter at different relative orientations. A cell phone camera is used to look at the stack of LCP depolarizers placed between crossed polarizers using white light illumination. (a) Two A type LCP depolarizers combined with a 45 deg rotation between the two optic axes. (b) A B and C type LCP depolarizer combined with their axes at 90 deg relative rotation. (c) B and C type LCPs also, but now with their axes at 30 deg relative orientation. (d) An A and C type LCP optic together. (e) Three LCP depolarizers combined in ordering A–C–B. (f) Four LCP depolarizer optics combined in ordering C–A–B–A. I note the C-type LCP has an anti-reflection coating that is infrared optimized with low transmission at short wavelengths, giving rise to a yellowish color in the cell phone camera images. I note that our ~5 mm beam footprint covers many of the spatial patterns seen in the 25 mm aperture. The spatial average over this pattern gives rise to the depolarization.

Fig. 29 The diagonal Mueller matrix elements (QQ, UU, and VV) for various combinations of LCP depolarizers as measured by our lab metrology system (NDSP5). (a) The combination of A and C types. (b) Again the A and C combination but now with C rotated. (c) The combination of A and B. (d) Two of the A types combined with a 30 deg angle between LCP depolarizer axes. (e) The stack of three LCP depolarizers in type ordering A:C:B. (f) A stack of four LCP depolarizers in type ordering of C:A:B:A.
9.2.3 Depolarization indices for A, B, and C combinations

The DI for these combinations of multiple LCP depolarizers decreases significantly over a wider wavelength range. Figure 30 shows the DI for a collection of LCP depolarization combinations. The DI is near 0.2 to 0.6 for the two A type LCP depolarizers in the upper left-hand graphic of Fig. 30. When four of the LCP depolarizers are combined in C–A–B–A ordering in the lower right-hand graphic of Fig. 30, the DI is always below 0.2 and is well below 0.1 for much of the lab metrology system bandpass.

9.3 Quartz Wedge Pair: Combined Phase and Diagonal Depolarization

More complex depolarizer fits can be demonstrated in the lab using a combination of two quartz retarders cut with very substantial wedge to create spatially and spectrally variable retardance. I successfully fit the three diagonal and three phase degrees of freedom along with three elliptical retardance variables using our modules on data collected with our lab metrology system. Thor Labs’ quartz-wedge achromatic depolarizers are specified to convert a polarized beam of light into what is called a pseudo-random polarized beam of light. The beam transmitted through the optic has a large and complex spatial and spectral change in retardance when the two wedges are combined. This kind of optic can be useful to imprint a spatial variation on a monochromatic source (laser) or to effectively depolarize low spectral resolving power instruments through rapid spectral changes in retardance (combined with spatial variation).

The drawing included with the part shows a 2 deg wedge angle, a total crystal thickness of 7.4 mm, and the thicker of the two crystals of thickness of 5.4 mm at maximum extent (thick side of the wedge). The assembly is held together by epoxy separated by a 0.2 mm edge spacer. The orientation angle between the optic axes of the two quartz crystal wedges is at 45 deg relative angle. This design of the quartz-wedge depolarizers gives low sensitivity to incoming linear polarization orientation and obviates the need to orient the optic axes of the depolarizer at any specific angle (Fig. 31).

I measured one of these quartz depolarizers with a 3 mm beam footprint in our NDSP5 Mueller matrix measurement system. Our system has sufficient spectral resolving power to characterize residual spectral retardance variations from the two crystal wedges. I note that a...
5.4 mm thick quartz crystal retarder has over 140 waves of retardance at our shortest measurement wavelength of 370 nm but only 26.5 waves at the longest wavelength of 1670 nm. Given the wedge angle cut and a total 7.4 mm assembly thickness, the second quartz retarder would have 2.0 mm thickness at this aperture location (minimum thickness due to wedge). A 2 mm thick quartz retarder has 53 waves retardance at 370 nm wavelength falling to 10 waves at 1670 nm. The depolarizer design is effectively a linear retarder at 0 deg fast axis and another linear retarder at 45 deg fast axis.

Figure 32 shows the DI for every wavelength measured in NDSP5. This particular design does not depolarize a circularly polarized beam and as such never sees a DI below 0.3. I note that NDSP5 has five separate spectrographs all run simultaneously. Each spectrograph has different bandpasses, spectral resolving power, and sampling. Discontinuities in the DI curve around 1 μm wavelength show a decrease in depolarization as I spectrally resolve oscillations in the Mueller matrix for the particularly high resolving power, well sampled spectrograph I use in our metrology system covering this bandpass. At longer wavelengths, the influence of spectral resolving power and sampling begin to show up in the DI trending upward toward a non-depolarizing Mueller matrix.

I demonstrate the depolarizing nature of this particular optic by fitting a pure elliptical retarder model. Particular narrow bandpasses are chosen to highlight the impact of spectral resolving power and the influence of the spatially variable retardance across our 3 mm diameter beam footprint. I show three particular bandpasses in Fig. 33. In each case, I fit a pure elliptical retarder Mueller matrix to each measured Mueller matrix. As a retarder has zero depolarization (DI = 1), this model will fail to match the Mueller matrix, but it will characterize the residual retardance of this particular optic in this bandpass.
Fig. 32 The DI for the NDSP5 spectral measurements of the quartz wedge depolarizer Mueller matrix. I note the discontinuities represent separate spectrographs used in our system with different spectral sampling and resolving power. Around 1 μm wavelength, I have a particularly high resolution spectrograph in our metrology system which resolves the spectral retardance oscillations, reducing the measured depolarization. At longer wavelengths, the depolarizing effect begins to fade.

Fig. 33 (a) The measured Mueller matrix spectra in blue and the best fit elliptical retarder Mueller matrix model in green. The top row shows a 15 nm bandpass beginning at 400 nm wavelength. The bottom row shows a 50 nm bandpass beginning at 800 nm. (b) The best fit elliptical retarder model to the measured Mueller matrix. The three components of retardance are shown as blue, green, and red colors. The total elliptical retardance magnitude is shown in black. These best fit retarder models capture the large spectral oscillations of both linear and circular retardance, but fail to reproduce a depolarizing Mueller matrix. I note a DI of 0.38 for 400 nm bandpass and 0.40 for the 800 nm bandpass.
9.4 Polar (Lu–Chipman) Decomposition: Depol-Then-Ret and Ret-Then-Dopol

\[
M = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & a & d & e \\
0 & d & b & f \\
0 & e & f & c
\end{pmatrix}.
\tag{5}
\]

Many techniques exist for characterizing the magnitude and character of depolarization in Mueller matrices. For this particular wedged quartz retarder pair type depolarizer, I find using a six element depolarizer in a Lu–Chipman\textsuperscript{113} type polar decomposition to be relatively straightforward to demonstrate. The depolarization caused by optic includes the three terms \((a, b, c)\) from the diagonal depolarizer as in Eq. (3) with three additional terms \((d, e, f)\) corresponding to the phase depolarization within the \(3 \times 3\) retardance sub-matrix. A six element depolarizer with both diagonal depolarization \((a, b, c)\) and phase depolarization \((d, e, f)\) is shown in Eq. (5). The amplitude depolarization terms for the first row and first column of the Mueller matrix can be ignored for this optic as our measurements show the first row and column to have no significant terms above the systematic error limits:

\[
M_{\text{optic}} = M_{\text{depol}}M_{\text{retarder}},
\tag{6}
\]

\[
M_{\text{optic}} = M_{\text{retarder}}M_{\text{depol}}^{-1}.
\tag{7}
\]

I decompose the measured Mueller matrix for this quartz wedge pair depolarizer into a two component model following Lu–Chipman.\textsuperscript{113} I model the measured matrix in two ways. First, I consider a model with an ideal six element depolarizer per Eq. (5) followed by an ideal elliptical retarder as in Eq. (6). Second, I consider a model with an elliptical retarder followed by the ideal six element depolarizer as in Eq. (7). As Mueller matrices do not commute, these two models will have different parameters. However, I show that both models reproduce the measured Mueller matrix equally well.

Figure 34 shows the six element depolarizer model fit terms for both decompositions. In both cases, the Mueller matrix produced by the fits are equally good representations of the measured Mueller matrix. Given that the Mueller matrices do not commute, the six depolarization variables take on different amplitudes and spectral oscillations in order to suppress the pure retarder terms appropriately.

The diagonal depolarization terms correspond to Mueller matrix elements \((QQ, UU, VV)\) directly transferring incoming Stokes parameters. The off-diagonal phase depolarization variables \((d, e, f)\) are symmetric about the diagonal, and each correspond to a pair of Mueller matrix elements \((QU, UQ, QV, VQ, UV, and VU)\), each transferring (mixing) particular states.

10 Appendix C: Retarder Fringes: Converging Beams and Suppression

I update the fringe model using the as built final thicknesses along with our decision to not apply any anti-reflection coatings or refractive index matching oils to the MgF\(_2\) crystals. Figure 35(a) shows the fringe periods for the pair of 3.5 mm thick MgF\(_2\) crystals in blue. The 2.1 to 2.2 mm thick MgF\(_2\) crystals are shown in green. I note that an instrument with spectral resolving power of \(R = 50 \text{ k} \) is quite capable of resolving these fringes at all but the shortest wavelengths as denoted by the Resolution 50k line in Fig. 35.

Figure 35(b) shows the estimated fringe amplitudes in percent for the \(F/18\) CN beam and the \(F/13\) GOS beam compared to a collimated beam. I note that there will also be a very small fringe caused by the refractive index mismatch between extraordinary and ordinary rays at the boundary between contacted crystals. The immersed reflection is near 0.002\% using the MgF\(_2\) refractive index equations. The first order equation for fringes from window surfaces is \(4\sqrt{R_1^2 + R_2^2}\). Thus, the interference of this immersed reflection is roughly 50 times lower than that from the air–crystal–air interface fringe estimates plotted in Fig. 35. I also note that the Berreman calculus formalism includes all the immersed reflections.
Fig. 34 The variables fit in the ideal six element depolarizer component of the Lu–Chipman decomposition. (a) The depolarizer first, retarder second model. (b) The retarder first, depolarizer second model. I note that in both decomposition orderings, I recover the same retarder-only fit variables to a fraction of a degree or better. All models recover the same retardance, but the depolarization terms vary substantially. Both decompositions reproduce the measured Mueller matrix equally well. As Mueller matrices do not commute, different variable amplitudes are expected for the different orderings in the two decompositions. The top row here shows the 15 nm bandpass starting at 400 nm wavelength. The middle row shows the 50 nm bandpass beginning at 800 nm. The bottom row shows the 170 nm bandpass beginning at 1500 nm. The six ID variables are shown in the legend with their corresponding Mueller matrix transfer coefficient names. Variables $(a, b, c)$ are the diagonal depolarization terms corresponding to $QQ$, $UU$, and $VV$ elements of the Mueller matrix, shown as solid lines. The off-diagonal variables $(d, e, f)$ are shown as dashed lines.

Fig. 35 (a) Fringe periods for the two thicknesses of our optically contacted MgF$_2$ crystal pairs. Blue shows a pair of 3.5 mm thick crystals. Blue shows a pair of 2.2 mm crystals. Magenta shows a spectral sampling of one part in 50,000. (b) The estimated fringe amplitude using the simple interference formula of 4R accounting for the $r^{-2}$ scaling estimate derived in Ref. 86. The solid black lines show the fringes near 10% for a collimated beam. The $F/18$ beam of the CN modulator is generally a factor of a few worse than the $F/13$ beam of GOS. At long wavelengths, I anticipate little fringe suppression in the converging beam.
I use the fringe magnitude suppression from the spatial average over an aperture as shown in Ref. 86. The $r^{-2}$ envelope follows marginal ray path length. The 3.5 mm crystals at 380 nm wavelength converging at $F/13$ see 20 waves of path difference across the aperture leading to very substantial spectral fringe reduction through the spatial average over fringes. For the 2.2 mm crystals at 1 $\mu$m wavelength converging at $F/18$, there may only be a few waves of marginal ray path. By 5 $\mu$m wavelength, the marginal ray sees the same optical path within one wave.

10.1 Flatness and Figure: Finite Element Modeling and Impact on Fringes
Fringes are impacted by the flatness of the interfering surface over the beam aperture. In both our elliptical calibrator design and the CN modulator design, there is an air gap of 5 mm between parallel crystal pair exit faces. I show here that the surface flatness is not only an issue for an optical contact bond during fabrication but is also substantially impacted by the thermal environment. I show here that the flatness of the individual crystal pairs takes on a saddle shape with sensitivity approaching a micron of deformation per degree Celsius temperature change from the nominal flat bonding temperature. Finite element modeling of bonded MgF2 retarder pairs suggests that the surface deformation is a fraction of a micron P-V per degree Celsius. I show details of the thermal finite element modeling in Appendix G. The dependence on the thickness of the crystal substrates gives these rough scaling relations:

- 0.8 $\mu$m P-V of deformation per degree Celsius in a pair of contacted crystals, 2 mm thick each.
- 0.4 $\mu$m P-V of deformation per degree Celsius in a pair of contacted crystals, 3.8 mm thick each.

10.2 Impact of Flatness for Fringes from Air Gap Between Pairs
There is a 5.3 mm thick air gap between these two retarder pairs set by a machined PEEK vented spacer ring. This is a machined part with flatness claimed by the vendor to be better than $\pm 8 \mu$m. For the calculation of fringes, I need to consider both the diverging $F/13$ and $F/18$ beam paths. The temporally variable flatness of the individual surfaces as well as the diverging spatial footprints will both conspire to reduce fringe magnitudes. For now, I note that a fringe period corresponding to a 5.3 mm air gap at magnitudes up to those predicted by the marginal ray path scaling and 3.5% per surface reflections is entirely possible given the rigid, permanently bonded nature of this spacer ring inside the retarder mounting cell.

The distance between the CN modulator and the CN slit increased from 630 mm in the 2013 optical design. The appendix B.2 of Ref. 110 had the footprints at 39.2 mm diameter consistent with the 630 mm slit-to-modulator distance. The updated design at 920 mm distance from slit to modulator puts the footprint diameter up to 51.3 mm. I note that vignetting now exists within the 3 arc minute tall slit. There is no vignetting inside a 2.6 arc minute field. The vignette fraction is 1% at 2.8 arc minute field diameter. The vignetted fraction is 3.7% at a 1.5 arc minute radius for the full 3 arc minute slit height.

Even with this increased footprint size to 51.3 mm diameter, the tables in Ref. 86 show that at 3.9 and 4.6 $\mu$m wavelength, a 5.3 mm air gap is not more than a wave of marginal ray path difference at $F/18$. The flatness of the crystals at microns per degree Celsius PV can contribute a fraction of a wave worth of change in path length for interference calculations.

I used a laser reflection near aperture center propagating over a 2.4 m distance to assess parallelism of the two optically contacted MgF2 crystal pairs. Both Fresnel surface returns were within a few millimeters of each other showing parallelism at levels of a few arc minutes between contacted crystal pair faces. A 5.3 mm thick air gap is only 1150 waves at 4.6 $\mu$m wavelength, and a few arc minutes of parallelism between crystal faces will not produce a significantly different optical path. The spatial offset between the back reflected footprint and the incoming beam for a 2.5 arc minute tilted surface is only about 8 $\mu$m. The optical path difference is $<20$ nm. Fringes in the 4.6 $\mu$m bandpass will likely be significant.

11 Appendix D: Elliptical Calibrator: Retardance Metrology
This appendix details the new deterministic FJP driven by retardance spatial mapping of the individual MgF2 crystal pairs (CN2:CN4 and CH5:CH6) as well as the final elliptical calibrator assembly. During the FJP of the CH5:CH6 named MgF2 pair, the optical contact bond failed. Another optically contacted crystal pair catastrophically cracked and failed. I detail the
retardance metrology for this new elliptical calibrator as it intended to provide the calibration of the DKIST system for all instruments at all wavelengths covering all fields of view.

11.1 Elliptical Calibrator: Mueller Matrix Final Metrology
Over more than 3 weeks, 5378 spatial locations were sampled with our NDSP5 Mueller matrix measuring lab system. A sequence of 15 independent maps covering a 110 mm aperture were compiled.

11.1.1 Elliptical Calibrator: Spatial Maps of Retardance Parameters
The centering and symmetry about aperture center of the spatial pattern of retardance impacts calibration accuracy through changing retardance as the optic spins and through depolarization (see Ref. 41). Figure 36 shows the spatial variation of retardance for the elliptical calibrator for all 5378 spatial locations at selected wavelengths. The elliptical design and the differing spatial uniformity from the two individual contacted crystal pairs causes certain components of retardance to show relatively larger or smaller errors as a function of wavelength and aperture. The measurements at 396 nm wavelength were shown n Fig. 8.

Fig. 36 The spatial variation of retardance components when the NDSP5 measured Mueller matrix for the elliptical calibrator is fit by an elliptical retarder model. Several spatial maps recorded over more than 3 weeks have been combined to produce these graphics. (a) 655.8 nm wavelength. (b), (c) 1085.1 and 1562.4 nm, respectively. The left column shows the elliptical retardance magnitude. The middle column shows the fast axis of linear retardance orientation variation. The right-hand column shows circular retardance. I note each graphic is on a separate color scale. In some cases, the temporal and thermal variation of the crystals combined with systematic error limits of the NDSP5 is seen in the high spatial frequency patterns (e.g., 1562.4 nm fast axis at ±0.1 deg magnitudes and 1085.1 nm circular retardance at ±0.3 deg). The NDSP5 beam was re-calibrated every 1 or 2 days. Several systematic drifts (e.g., input beam partial polarization) can cause slight differences in the recorded parameters.
11.1.2 Elliptical Calibrator: Spectral Variation in Retardance from Aperture Center

Figure 37 shows the spectral variation of the elliptical retardance parameters from aperture center. Each spectrum shows the difference between the parameter at the aperture center and the 5378 individual spatial locations measured. (a) The elliptical retardance magnitude in black. (b) The linear retardance magnitude in magenta. (c) The fast axis of linear retardance in green. (d) The circular retardance component in red.

11.1.3 Elliptical Calibrator: Fitting for Pair Orientation with Refractive Index Model

There are small mismatches in the refractive index model that assumes perfect crystal optic axis orientation and also alignment of the optical assembly within the metrology equipment. This is consistent with the crystal optic axes being at ±1 deg non-normal to the polished optical surface along with other alignment imperfections. I show here how I derive that the two optically contacted MgF2 crystal pairs of the elliptical calibrator were bonded into the tip/tilt cell assembly within ±1 deg relative fast axis orientation. I compute an ensemble of elliptical retardance predictions with the MgF2 crystal pairs at range of relative fast axis clocking orientations. I then minimize the difference between our retardance measurements derived from Mueller matrix spectra and this ensemble of models. Figure 38 shows the difference between this range of models and the actual measured retardance of the optic after final bonding in the cell. Though
there are clear and measurably significant differences between the measurements and model, the difference is minimized at the correct 45 deg clocking orientation at ±1 deg tolerance.

11.2 Elliptical Calibrator First Contacted Pair—CN2:CN4

This section presents the metrology on the first optically contacted crystal pair, CN2:CN4 before and after the FJP procedure. The Boulder-based NLSP measurements were recorded over 58 h from July 5th to 7th, 2021. A rosette spatial pattern covers a 100.8 mm diameter aperture with 271 Mueller matrix measurements. Each measurement takes roughly 12 min and includes a lamp brightness spectral calibration. The Maui based NDSP4 measurements were recorded in January, 2022, with 918 independent spatial samples.

11.2.1 Mapping CN2:CN4 Contacted Pair: Elliptical Magnitude with FJP

I compare the retardance uniformity before and after deterministic FJP based on two of our custom lab metrology equipment setups. Figure 39 compares spatial maps of elliptical retardance magnitude for the CN2:CN4 crystal pair at wavelengths near 396 nm. There is a 0.6 nm

Fig. 38 The difference between the MgF₂ elliptical calibrator measurements and a range of refractive index based models. (a) Elliptical retardance magnitude differences. (b) The circular retardance component differences. The solid blue line shows the difference between data and the nominal design at 45.0 deg clocking angle between the two MgF₂ crystal pairs. The green lines show clocking errors of ±1 deg. These retardance difference curves do not go to zero at all wavelengths due to several common errors including: incidence angle being non-zero, crystal optic axes being non-normal to the reflecting surface, individual thickness estimate errors, and refractive index model errors.

Fig. 39 The CN2:CN4 elliptical retardance magnitude spatial maps before and after FJP. (a) 394.4 nm wavelength measured in our Boulder based NLSP before FJP on a color scale from −13 deg to +25 deg. (b) 395 nm wavelength measured in the Maui based NDSP4 after FJP −4 deg to +3.5 deg. This represents a ~5× reduction in spatial variation from 38 deg P-V down to 7.5 deg P-V.
difference in wavelength (394.4 and 395.0 nm) due to differences in the spectrograph configuration. The P-V change in retardance across the aperture is improved by roughly a factor of 5 using the FJP. There is some residual wedge-like behavior from top to bottom, and some smaller spatial scale features are visible in the retardance map.

11.2.2 NLSP Mapping CN2:CN4 Contacted Retarder: Clocking Oscillation Detail

The spectral retardance oscillations are easily visible in all our spectral Mueller matrix data sets. This optic is 1.5 waves net retardance at a wavelength of 542.6 nm. A 2.2 mm thick MgF₂ crystal has a net retardance of 48.0 waves at this wavelength. Figure 40 shows the circular retardance in Fig. 40(a) and the fast axis of linear retardance in Fig. 40(b) for a bandpass centered about 545 nm wavelength measured in NLSP. The spectral period for the fast axis oscillations ($\lambda^2/\Delta\lambda$) is 11.4 nm. With NLSP having an optical resolution near 0.35 nm at this wavelength, I sample the oscillation with more than 30 resolution elements.

11.3 Elliptical Calibrator Second Contacted Pair—CH5:CH6

This section summarizes the metrology of the second individual contacted pair used in the elliptical calibrator: CH5:CH6. The clocking error for the March, 2022 contact bond achieved an estimated 0.1 deg orientation. The surface quality assessment showed a 105 mm clear aperture free of internal defects, bubbles, patches of non-contact, or scratches larger than number 60. The transmitted wavefront error (TFE) (circular polarization) shows 0.39 waves PV with 0.06 waves power. The RMS was 0.06 waves. The TWE map has a 30 mm circular feature dominating the TWE map. This also correlates with a feature of low retardance in the same spatial region.

The surface figure was measured after the post FJP optical contact measured March, 2022. The CH5 surface had 8.85 waves PV of surface figure looking mostly like cylinder/astigmatism. The CH6 surface had almost the same shape with the opposite sign at 8.18 waves PV. As I note below in the thermal analysis section (Appendix G), these flatness values are very strong functions of temperature.

The CH5–CH6 pair had an estimated beam deviation of 1.9 arc seconds. Offset aperture wedge measurements were done at four orientations of the CH5–CH6 pair using a 100 mm diameter interferometer beam. The component of wedge that rotated with the optic was extracted using vector addition. With this technique, systematic errors from particular components of TWE can influence the measurement at arc second levels. I conclude that the beam deflection is below a few arc seconds.

11.4 Improvement Limited By Contact Bond Failure in FJP: MgF₂ CH5:CH6

FJP was also done on the second elliptical calibrator pair with thicknesses near 3.5 mm—CH5:CH6. The thicker substrates are much stiffer giving rise to a much higher likelihood of optical
contact failure. Unfortunately, the optical contact failed during the first round of FJP likely due to fluid penetration. Additional partial failures were noted during parts of the procedure requiring heating of the contacted pair. There was a substantial improvement in the retardance uniformity just from this first round of polishing. The retardance uniformity was converted to material removal for a comparison in microns required for future FJP. Figure 41 compares removal maps on the same color scale.

11.4.1 NDSP5 mapping of CH5:CH6—elliptical retardance spectra at aperture center

The elliptical spectra for the CH5:CH6 contacted pair follow the expected retardance curves predicted by the refractive index model to within a small fraction of a micron physical thickness difference over this bandpass. The specification of 0.27 waves ±0.01 waves net retardance at 633 nm wavelength was achieved. This optic is <0.45 waves at 396 nm wavelength falling to near 0.1 waves by 16 μm wavelength. I extract the oscillations of circular retardance (red) and second component of linear retardance (green) in Fig. 42. The measurements have been rotated to have a

![Figure 41](https://www.spiedigitallibrary.org/journals/Journal-of-Astronomical-Telescopes,-Instruments,-and-Systems on 08 Oct 2023)

**Fig. 41** The material removal maps derived from MLO measured spatial retardance maps for the optically contacted crystal pair named CH5:CH6. (a) The removal map used during the FJP process using a 25 point measurement. (b) A 41 point map measured after FJP, deriving the residual removal error March 9, 2022, on the same scale from 0 to 1.5 μm.

![Figure 42](https://www.spiedigitallibrary.org/journals/Journal-of-Astronomical-Telescopes,-Instruments,-and-Systems on 08 Oct 2023)

**Fig. 42** The clocking oscillations seen in elliptical retardance fits to the aperture center Mueller matrix measurement on CH5:CH6 using NDSP5. The red curves show circular retardance. The green curves show the second component of linear retardance after de-rotating the optic to have the bulk of the retardance contained in the first component.
fast axis of linear retardance near 0 deg to isolate clocking oscillations in the second linear retardance component. I note the primary oscillations at roughly 0.2 deg peak to peak magnitudes with a period near 20 nm. I chose this spectrum because secondary oscillations at roughly an order of magnitude smaller magnitude and an order of magnitude faster spectral period are expected from the crystal axis misalignments at ±1 deg as discussed in Sec. 5.2.

11.4.2 NDSP5 mapping of CH5:CH6—elliptical retardance magnitude variation

The spatial pattern of retardance is repeated across the aperture at all wavelengths. I show the elliptical retardance magnitude maps for the CH5:CH6 crystal pair in Fig. 43. The color scale shows PV variation. The range changes by a factor of a few between wavelengths, but the pattern of spatial variation remains nearly identical. As this optic only had one round of FJP before the optical contact bond failed, a second round was not done to improve the obvious and notable feature slightly off the aperture center. This variation of less than ±4 deg at 382 nm wavelength is already quite an improvement.

11.4.3 NDSP5 mapping of CH5:CH6—retardance spectra variation across the aperture

The spectral clocking oscillations have a variable phase across the aperture of an optic that gives rise to large and obvious changes in retardance spectral variation as a function of wavelength. Some wavelengths have minimal oscillation magnitude across the aperture. Figure 44 highlights the variation of the spectral clocking oscillations across the aperture in the 1180 to 1430 nm bandpass. The oscillations can be seen with maximal and minimal spatial variation of retardance shown as the envelope over all curves.

11.4.4 NDSP5 mapping of CH5:CH6—retardance thermal variation between maps 1 and 4

The thermal sensitivity of the contacted MgF₂ crystal pair can be seen by comparing two maps recorded at two different times. I collected and processed multiple Mueller matrix spectral maps separately in the Maui based laboratory at the DKIST Science and Support Center (DSSC). The temperature stability of the lab is only ±2 deg, typically of an office building and not tightly controlled. The number of spatial points in the was limited in order to keep the total map run time shorter. Figure 45 compares the retardance variation between maps 1 and 4 in a series of multiple repeated maps. Figure 45(a) highlights a relatively obvious change in the fast axis of linear retardance at levels of roughly 0.05 deg. The circular retardance in Fig. 45(b) shows less obvious changes as the aperture is less uniform.

Fig. 43 The elliptical retardance magnitude spatial variation across the aperture. (a) A wavelength of 382 nm and (b) 1550 nm. The color scales change between graphics to show P-V variation per the color scale on the right of each graphic.
11.5 Polariscope Images: AOI and Clocking Dependence for Two Aligned EliCal Pairs

I collected a series of polariscope images before final bonding of the two pairs in the elliptical calibrator. Here, the fast axes of the two pairs (CN2:CN4 and CH5:CH6) were aligned to add in net retardance. Figure 46 shows a series of polariscope images as the four crystal stack is rotated.

Fig. 44 The aperture variation of the spectral retardance fits to the Mueller matrix measurements at many spatial locations on CH5:CH6. (a) The circular component of retardance in red. (b) The fast axis orientation of linear retardance in green. I chose the 1180 to 1430 nm bandpass to highlight that some wavelengths show more or less spatial variation due to the phase of the spectral oscillations.

Fig. 45 Retardance was fit to the Mueller matrix measurements at many spatial locations on the optically contacted MgF$_2$ crystal pair named CH5:CH6 using NDSP5 at the Maui based DSSC labs. (a) The spatial variation for the fast axis of linear retardance across the aperture. (b) The circular retardance. Map 1 is shown in the top row. Map 4 is shown in the bottom row. The lab temperature stability is measured near ±2°C, typical of an office building.

11.5 Polariscope Images: AOI and Clocking Dependence for Two Aligned EliCal Pairs

I collected a series of polariscope images before final bonding of the two pairs in the elliptical calibrator. Here, the fast axes of the two pairs (CN2:CN4 and CH5:CH6) were aligned to add in net retardance. Figure 46 shows a series of polariscope images as the four crystal stack is rotated.
through 90 deg. The combined AOI dependence from adding the two individual compound retarders creates a pattern when the fast axis is not aligned with the two linear sheet polarizers of the polariscope. The images go dark as no light is transmitted through the polariscope at 0° incidence when the fast axis of the assembly is aligned roughly horizontally and vertically in these images.

12 Appendix E: CN Modulator Performance and Metrology

I outline here the optical performance and metrology on the two optically contacted MgF₂ crystal pairs that make up the CN modulator installed in the instrument July, 2022. These two MgF₂ pairs were not fluid jet polished and were completed before the elliptical calibrator due to construction and commissioning schedule constraints. I detail the metrology here as it impacts the CN instrument performance and expectations about the modulation matrix spatial properties.

12.1 CN Modulator: Individual Crystal Pair Retardance Uniformity

MLO used their spatial mapping system to measure the linear retardance magnitude of the individual contacted retarder pairs at 640 nm wavelength before final assembly. Figure 47(a) shows the C1a:C1b crystal pair while Fig. 47(b) shows the thicker CH3:CH4 crystal pair. The color scale shows PV variation with a wedge-type pattern dominating the spatial behavior.
10.2 CN Modulator: Individual Pair Clocking Errors: MLO Data and Model

The clocking errors of the contacted crystals are assessed here with retardance spectra. Measurements of the linear retardance fast axis seem to be the most robust and repeatable at MLO and compare well with our fiber fed spectrograph based system. Figure 48 shows the MLO fast axis data along with three of our refractive index models. The C1a:C1b pair seems to have clocking error near 0.08 deg with temporal drifts in the MLO equipment limiting the measurement at these small magnitudes. The best fit is near 0.08 deg clocking error. I note the MLO spectral data are a scanning monochromator as opposed to our fiber fed spectrograph based systems. I note that similar measurements from MLO on the CH3:CH4 pair are consistent with a 0.2 deg clocking error.

10.3 CN Modulator: Spectral Retardance Variation of Elliptical Parameters

The CN modulator is highly elliptical with very non-uniform spectral variation in elliptical parameters due to the uncorrected residual wedge type variation in the two optically contacted MgF2 crystal pairs. For modulators, this leads to a need for spatially variable demodulation on a per-state basis to calibrate. Figure 49 shows the spectral variation of elliptical retardance parameters as the difference in each component from the value at aperture center. I note that this optic is...
almost entirely a circular retarder near 680 nm wavelength. At some wavelengths, such as 700 nm, there is nearly zero spatial variation in the magnitude of retardance but substantial variation of the components (e.g., fast axis of linear retardance).

12.4 CN Modulator: Efficiency Change with 12 Deg Clocking Error
I show here a comparison of modulation efficiency models when the two MgF₂ crystal pairs are bonded with a 12 deg rotational (clocking) change. The clocking of the individual compound retarder pairs is critical at ±0.3 deg, but the mounting orientation between the pairs is greatly relaxed. I was initially surprised at the mismatch between our refractive index based elliptical retardance model and the CN modulator lab measurements. A model search revealed a very good fit to the data if the two MgF₂ crystal pairs were rotated −12 deg from their nominal specified orientation. I note that this 12 deg error is consistent with a mistake in identifying the fast axis as the slow axis. Discussion with Meadowlark (Tom Baur) confirmed this would have been possible and is the likely explanation given how well our model fits with exchange of fast and slow axes. Figure 50 compares the modulation efficiencies. The $I$, $Q$, and $U$ efficiencies are higher by more than 150% around 1.4 μm wavelength. For the 3.9 μm and 4.6 μm lines, the $I$, $Q$, and $U$ efficiencies are reduced to levels of 70% to 80% of the nominal design. As an example, the $QU$ efficiency was nominally 21% but is now 18% for the 4.6 μm line.

12.5 CN Modulator: Retardance Spatial Uniformity Maps
The retardance spatial uniformity follows a pattern resembling wedge. The spatial variation is mostly contained within the variation among elliptical parameters, much less so in overall changes in magnitude. The elliptical magnitude, circular retardance, and linear retardance fast axis variation are shown in Fig. 51. I note that the CN modulator maps for 1083 nm wavelength were already shown in Fig. 10. The wedge-shaped thickness variation of both crystals in the two
**Fig. 50**  (a) The difference between the nominal CN modulator design as dashed lines and the best fit as-built design as the solid lines. The thicker lines show the NDSP4 measurements on the final assembled CN modulator. (b) The ratio of modulation efficiencies between the design and the as built optic computed as (actual/design). A constant 100% would represent no change from design. There is a 70% boost in $I$ and a 50% $Q/U$ efficiency for the 1.5 μm wavelength observations with CN but with a corresponding ~25% loss for 4.6 μm wavelength CN observations.

**Fig. 51** The elliptical retardance spatial maps of the new CN modulator derived from the Mueller matrix measured by NDSP4. I select relevant wavelengths for each row. (a) 650.0 nm, (b) 857.2 nm, and (c) 1430.0 nm wavelength. The elliptical magnitude varies spatially in the left-hand column. The middle column shows the circular component of retardance. The right-hand column shows the fast axis orientation for linear retardance.
compound pairs combine to make a wavelength dependent pattern that can rotate with wavelength. For instance, the left side has higher retardance at 650 nm wavelength while the right side is higher at 857 nm, and the bottom is higher at 1430 nm. The circular retardance varies by 12 deg PV at 650 nm wavelength but varies by over 22 deg at 857 nm even though spatial variation generally decreases at longer wavelengths.

12.6 AOI for CN Modulator: Two MgF$_2$ Compound Zero Order Retarder Crystals

Visual inspection images for these compound retarders between crossed polarizers illustrate the strong incidence angle dependence. Figure 52 shows a series of polariscope images as the 4 MgF$_2$ crystal CN modulator is rotated through 90 deg. The combined AOI dependence of the two individual compound retarders creates an asymmetric pattern especially when the linear polarization states of the polariscope are not aligned with the crystal fast or slow axes within either of the two compound retarder pairs.

13 Appendix F: Compensation of Beam Deviation with Optic Tilt

An alignment technique used for all the DKIST modulators is to apply some tip-tilt of the modulator optic in order to compensate for the residual beam deviation caused from imperfect optic
flatness (wedge). This tilt increases the retarder crystal optic axis alignment errors and also the retarder field angle dependence. The optic is tilted away from normal to the incoming beam, increasing angular dependent issues. I show here this tilting of the retarder optic is a small effect for the arc second level beam deflection in the optically contacted MgF₂ modulator for Cryo-NIRSP. Minimizing image motion at the spectrograph entrance slit is far more important for polarization accuracy than a small residual retardance error from the beam being non-normal to the MgF₂ crystals. This follows closely an internal ViSP instrument team Technical Note 5263-TN-9202 led by Casini.\(^{150}\)

An imperfectly parallel optic has some beam deflection for the beam exiting the optic. For our oiled retarders, this is in the range of a few to a few tens of arc seconds. For optically contacted and spatially uniform retarders, this deflection is often <3 arc seconds. The beam deflection angle (\(\theta_{\text{defl}}\)) in a wedged optic with wedge angle \(\alpha\) is estimated as \((n - 1) \alpha\) where \(n\) is the refractive index using the small angle approximation. For a tilted plane parallel optic (window) of finite thickness denoted \((t)\), there is a beam translation (displacement, parallel to the crystal optic axis) through that optic, denoted \((h)\). The beam displacement \((h)\) goes like the thickness of the optic \((t)\) multiplied by the incidence angle \((\theta_{\text{AOI}})\) and a factor related to the refractive index: \(h = t \theta_{\text{AOI}} (1 - 1/n)\). I denote the angle of propagation for the refracted beam in the medium (e.g., MgF₂ crystal) as \(\theta_{\text{disp}} \sim \theta_{\text{AOI}} (1 - 1/n)\) when using a small angle approximation of Snell’s law.

These two effects can be combined and controlled to compensate each other and return a deflected and displaced beam to the optical axis at some specified distance from the optic. The beam translation (displacement) through a tilted optic \((h)\) is set equal to the height propagated at a particular beam deflection angle \((\theta_{\text{defl}})\) over a particular distance \((D)\) to the location desired for minimal beam translation. This sets the beam translation \((h)\) equal to \(D \tan(\theta_{\text{defl}})\). In the case of a DKIST instrument, this distance is the distance from the modulator optic to the image plane. In the case of CN, this is the spectrograph entrance slit focal plane. Figure 53 shows the ray trace diagram for the combined incidence angle and deflection angle.

For the CN modulator, the total thickness is near \(t = 11.4\) mm at a refractive index near \(n = 1.38\). The distance between the modulator and the spectrograph entrance slit is \(D = 920\) mm. I measured a beam deflection \((\theta_{\text{defl}})\) near 2.5 arc seconds using a 630 nm laser in our labs giving a wedge angle (\(\alpha\) near 6.5 arc seconds). This gives a runout of roughly 1.9 \(\mu\)m over a 110 mm aperture. With these values, I compute a sensitivity of 5 arc minutes of physical tilt as required to compensate 1 arc second of beam deflection. The prediction is that this optic must be tilted by 12.4 arc minutes physical angle (0.2 deg) in order to compensate for the measured 2.5 arc seconds of beam deflection. The beam will translate by 0.9 \(\mu\)m for every arc minute of physical tilt. A total translation near 11 \(\mu\)m is required to compensate the 2.5 arc seconds of deflection by the slit station. I note this 0.2 deg physical tilt is small compared to the ±1.3 deg

\[\text{Fig. 53} \quad \text{The ray diagram for tilting the optic to compensate wedge-based image motion.}\]
crystal optic axis alignment uncertainty as well as the ±1.6 deg marginal ray angles of the incoming CN F/18 beam at the modulator station.

14 Appendix G: Thermal FEM: Flatness, Bond Stress, and Fringes

These optically contacted retarders need to operate in a mountain environment without failure from temperatures from −5°C to +40°C. Additional stresses can be put on the optical contact bond during several of the polishing steps where heat may be applied. Heating of the crystals from absorption can create spatial temperature variations, inducing retardance spatial variation, degrading the accuracy of a polarization calibration (see Ref. 86 for additional examples). The differential expansion of the retarder crystals along ordinary and extraordinary axes is coupled with the 90 deg clocking between axes of each compound retarder component, a worst-case orientation. DKIST funded Hofstadter Analytical LLC to perform detailed finite element modeling to understand the behavior of the bond stress as well as the surface deformation for our optically contacted MgF2 crystals. Interference fringes are a concern between parallel interfaces such as within the air gap between the two contacted retarder pairs. The order of magnitude of the flatness changes is in the microns PV surface figure per degree Celsius suggests strong dependence of the interior fringes with temperature. I show details of those calculations here.

I show a worst-case thermal forcing of a MgF2 retarder pair by exposure to the full 300 W DKIST solar beam without any protection from an upstream polarizer in Fig. 54. I note that the oiled six MgF2 crystal retarder (Cryo-SAR) at DKIST was subjected to exactly this test as shown in appendix B and figure 31 of Ref. 111 showing a 1.5°C temperature rise over 30 min full 300 W beam exposure. The CN modulator also sees up to 230 W with substantial infrared power due to the all-reflective optical path of DKIST. The crystal surface deflection shown in Fig. 54(b) is roughly 26 μm P-V on a contacted MgF2 crystal pair. The current DKIST calibration schemes reduce the flux on the MgF2 retarders to levels well below ambient temperature change by always using a fused silica substrate polarizer upstream of the retarder. Thus, I expect the actual temperature dependence of the DKIST calibration retarder to track the ambient temperature as it is controlled in the GOS enclosure. However, calibration sequences that use the retarder alone in the beam are possible and provide improved efficiency options should the thermal issues be mitigated by this new optically contacted MgF2 design.

DKIST has two different thicknesses of MgF2 retarder stock. I estimate the surface deformation for both 2.1 mm thickness and 3.8 mm thickness (though the final thickness was closer to 3.5 mm) in Fig. 55. I note the illumination pattern (beam footprint) from any individual field angle is roughly a 30 mm diameter patch on the optic. Different focal plane locations will correspond to only a 30 mm patch of the 120 mm retarder optic, giving some field dependence to the flatness of these retarders.

Fig. 54 Thermal FEM calculations from Hofstadter Analytical LLC. Temperature distributions can be non-uniform in response to thermal loading. (a) A worst case temperature distribution for the crystal pair fully exposed to the 300 W of solar irradiation near GF without a polarizer upstream. The crystal pair is bonded in the rotary stage metal cell with the RTV108 bond line as an insulator. The metal cell is held at a constant −10°C. The depth dependent absorption of solar incident flux is applied over the 105 mm clear aperture following appendix B of Ref. 86 adjusted for the appropriate MgF2 crystal thickness. (b) Surface figure deformation based on temperature change with the color scale showing deflection in meters. The scale runs from −44.4 to −18.8 μm.
14.1 Lab Verification of Contacted MgF$_2$ Surface Figure with Temperature:
CN2:CN4

DKSIT funded MLO to perform an experimental validation of these surface flatness predictions by heating an optically contacted pair while making measurements with an interferometer. MLO measured the optically contacted CN2:CN4 crystal pair with the room air temperature perturbed by 3°C over a few hours time. The Zernike astigmatism term in the reflective surface figure changed strongly as a function of temperature, as predicted. Figure 56 shows two interferometric measurements of the front reflected surface shape for CN2:CN4.

Table 4 collects the Meadowlark 4D interferometry results from 2021 on the CN2:CN4 optically contacted retarder pair. The first column shows the 2021 measurement date. The PV TWE in waves at 632.8 nm wavelength is shown in the second column. The pair is consistently near 0.8 to 0.9 waves PV in transmission even though FJP took place between the September and November measurement. The third and fourth columns show microns P-V surface figure on the CN2 and CN4 surfaces, respectively. The March and September measurements vary by roughly 5 μm. The variation between September and November measurements is much smaller, even though FJP of 5 μm of material in the shape of vertical wedge was removed from the CN4 crystal face.

Fig. 55 Thermal FEM calculations from Hofstadter Analytical LLC. The optically contacted MgF$_2$ retarder surface shape change at amplitudes of several microns PV in response to thermal forcing. (a) An optically contacted pair of 2.1 mm MgF$_2$ retarders. (b) Surface deflection for an optically contacted pair of 3.8 mm thick MgF$_2$ retarders. The 30 mm diameter circles correspond to the beam footprint of a single field angle in the converging F/13 beam at the DKIST calibration retarder optical station. The color scales are different to capture the peak deformation range in each contacted crystal pair.

Fig. 56 The surface figure map from the optically contacted CN2:CN4 MgF$_2$ crystal pair using the MLO 4D AccuFiz interferometer. (a) A ±6.5 wave surface wavefront error (WFE) with the optic at 20°C. (b) A ±2.8 wave WFE color scale at 22°C.
14.2 Measured Astigmatism for Thermally Perturbed Contacted Pair CN2:CN4
I performed an experimental validation of these surface flatness predictions by heating a MgF₂ optically contacted pair while making measurements with an interferometer. MLO measured the optically contacted CN2:CN4 crystal pair with the room air temperature perturbed by 3°C over a few hours time. The Zernike astigmatism term in the reflective surface figure changed strongly as a function of temperature, as predicted.

I can decompose the five surface figure WFE maps into low order Zernike polynomial modes to show that astigmatism is the dominant term and is the temporally variable term. Figure 57(a) shows the P-V reflected WFE is over 14 waves at 20°C dropping to the range 7 to 8 waves P-V at 23°C. The two points at the same temperature represent a repeated measurement after waiting several minutes. Figure 57(b) shows the magnitude of the Zernike polynomials fit to the optic surface reflected WFE. Focus, coma (X and Y), and spherical all remained essentially constant with temperature. Astigmatism in Y (green curve) decreased from 2.5 waves to 1 wave as the temperature changed 3°C. The vertical astigmatism (X) was at a 5× lower overall magnitude but also dropped as the temperature increased 3°C.

15 Appendix H: Alternate Vendor Performance
Other observatories might choose placement of calibration retarders that can accept smaller diameter designs. Some may use bandpass rejection filters or have much lower heat loads, such that other retarder options become available. Instruments at lower spectral resolving power average over fringes and spectral clocking oscillations, reducing their impact. I outline here performance considerations of smaller optically contacted MgF₂ pairs and a cemented athermal achromatic three-crystal retarder. The spectral clocking errors, interference fringes, and spatial variation of retardance are all present at expected levels in smaller parts from several vendors.
I also show the wedge-type limiting polishing error in a single crystal quartz window at 120 mm diameter from another vendor.

15.1 Optically Contacted MgF₂ Compound Zero-Order Retarder at $\phi$25 mm

Other vendors can provide optically contacted MgF₂ retarders at smaller aperture size and thinner plate bias thickness. As part of this project, I evaluated standard 25 mm diameter optically contacted compound retarders at 0.87 mm total thickness. I did specify a ±0.3 deg fast axis clocking tolerance on all parts along with a defect free clear aperture. I bought a batch of five optics for each of the two magnitudes specified for the elliptical calibrator: 0.27 waves and 1.33 waves, respectively, at 633 nm wavelength. I received and tested this batch of 10 retarders in our NDSP6 lab Mueller matrix spectral mapping system.

Figure 58 shows the circular retardance fit to the Mueller matrix spectra at aperture center. I include a nominal retardance model for two individual retarders at 0.435 mm thickness bias each with a 0.3 deg separation between fast axes. The appropriate thickness of MgF₂ was added to one of the two crystals in the model to create the correct net retardance. This is 14.5 $\mu$m for our 0.27 wave retarder and 71.6 $\mu$m for our 1.33 wave retarder. For upcoming astronomical instruments, off-the-shelf optically contacted MgF₂ retarders can be an option, provided the retarder size and specifications are chosen to account for current manufacturing limits.

Interference fringes in these 0.87 mm thick MgF₂ parts are resolved in some bandpasses of our lab metrology system. Figure 59(a) shows the spectral variation in elliptical retardance magnitude across the 0.37 to 2.4 $\mu$m bandpass well measured by our NDSP6 system. Figure 59(b) highlights the interference fringes in retardance measured by the highest spectral resolving power spectrograph in our system. The 4.5 nm bandpass around 993 nm wavelength has good signal and clearly shows fringes above ±2 deg retardance magnitude. I note the interference fringe period is roughly 0.41 nm for both e- and o-beams with refractive indices (1.385 and 1.374) and the period scaling as $\frac{\lambda^2}{(2dn)}$.

The spatial maps of aperture variation in elliptical retardance magnitude have low-order patterns. I show four of these parts measured across the wavelength extremes of our NDSP6 lab metrology system in Fig. 60. Figure 60(a) shows one of the 0.27 wave magnitude compound MgF₂ retarders. Figures 60(b)–60(d) show 1.33 wave magnitude retarders. The spatial pattern is not consistent part to part. The PV amplitude of spatial retardance variation can be a factor of almost 3× different. As an example, the three optics at 1.33 waves net magnitude shows PV variations from as low as 1.2 deg and up to 3.2 deg at 395.5 nm wavelength.

**Fig. 58** The circular retardance spectra in an appropriate bandpass as measured by our Mueller matrix spectral mapping system. Black curves show the circular retardance spectra at aperture center. The blue curve in each graphic shows a retardance oscillation model derived from a pair of crystals with a 0.3 deg fast axis clocking misalignment. I note that this model was not adjusted to match the specific thickness of each individual crystal, creating some phase offset (wavelength shifts) between the modeled and measured clocking oscillations. (a) The five contacted MgF₂ crystal pairs at 0.27 waves net retardance at 633 nm wavelength. (b) The five pairs at 1.33 waves net magnitude.
15.2 Cemented Al₂O₃ - MgF₂ - SiO₂ Athermal Achromat

Another type of athermal achromatic retarder is a combination of sapphire (Al₂O₃), MgF₂ and quartz (SiO₂) as outlined in Ref. 151. I have tested two independent builds of a custom design using 370 μm of Al₂O₃ followed by 206 μm of MgF₂ rotated by 90 deg and ending with 580 μm of SiO₂. This design makes a quarter wave linear retarder with very low thermal coefficients. Though this design has quite thin crystals, and was cemented instead of optically contacted, I still see clocking oscillations. I also detect interference fringes with our lab metrology system as thin crystals make for relatively longer fringe periods detectable with R > 3000 spectrographs. Figure 61 shows example retardance in Fig. 61(a), spectral clocking oscillations in Fig. 61(b), and interference fringes seen in linear retardance in Fig. 61(c) for this cemented three-crystal athermal-achromat made of Al₂O₃ - MgF₂ - SiO₂.

The interference fringes are large and easily detectable for these thin crystals with significant refractive index mismatch. For instance, fringe periods are near 0.4 to 1.3 nm around 850 nm wavelength following Fig. 61(a). The e- and o-beam for the SiO₂ crystal fringes near 0.40 nm period. The Al₂O₃ fringes near 0.56 nm period. The MgF₂ fringes around 1.27 nm period. There is a roughly 1% difference between e- and o-beam refractive indices giving rise to slightly different periods and wavelength dependent beating of fringes.

I show the spatial variation in measured elliptical retardance magnitude for a 61 point spatial map in Fig. 62(a). The spectra show variation of roughly 3 deg PV but with substantial interference fringes at levels of at least 1 deg. Depending on the wavelength observed, the fringes can cancel out at particular individual wavelengths. Figure 62(b) shows a few select spatial points measured around 770 nm wavelength. Some spatial locations have relatively small fringes compared to others.

Figure 63 shows two example spatial maps derived from the elliptical retardance magnitude spectra measured with our NDSP5 system. I only show two wavelengths here as all wavelengths display a very similar same spatial pattern, with the magnitude largely scaled with wavelength. I apply a ±11 spectral pixel averaging at each wavelength to minimize impact of the fringes.

15.3 Optical Activity and Circular Retardance in Crystal SiO₂ Windows

Quartz crystals are optically active and commonly manufactured to be right-handed. In addition to linear retardance, there is circular retardance along the crystal optic axis. A crystal quartz window is also a circular retarder (also called a polarization rotator). This optical activity is easily visible in polariscope images of crystal quartz windows, typically denoted as c-cut or z-cut. I showed above in Sec. 5.5 and Fig. 18 that the spectral retardance for a 5 mm thick crystal quartz window along side a polariscope image of this crystal quartz window. The spatial uniformity of retardance for this window also follows a wedge-type shape in circular retardance.
similar to the MgF₂ retarders presented in this article. Figure 64 shows maps of the elliptical retardance parameters across the aperture for wavelengths of 407.0 nm in Fig. 64(a) and 852.6 nm in Fig. 64(b).

The retardance uniformity from typical polishing errors can be highlighted with a Zernike decomposition of the retardance spatial map. I show maps of the circular retardance across the aperture in Fig. 65 for a wavelength of 416.7 nm. The retardance uniformity is dominated by a wedge shape of roughly 0.6 deg. Removal of this wedge shape reveals a bar-type shape, similar to what was reported in the DKIST quartz-based A-plane cut retarders shown in Ref. 86.

Fig. 60 The elliptical retardance magnitude maps of selected optically contacted MgF₂ parts. The NDS6 was used to create a 127 point Mueller matrix map over an 18 mm aperture. (a) I show one of the 0.27 wave magnitude parts with serial number 3 denoted in the upper left corner as \(0.27, 3\). (b) A 1.33 wave magnitude part with serial number 1, denoted \(1.33, 1\) in the upper left-hand corner. (c), (d) 1.33 wave magnitude parts with serial numbers 4 and 5, respectively. The left-hand graphics show 395.5 nm wavelength. The middle graphics show 655.2 nm wavelength. The right-hand graphic shows 2117.6 nm wavelength. The color scale for retardance variation is roughly \(5\) × reduced at long wavelengths, and also ranges by more than \(3\) × depending on the part. All wavelengths have a similar spatial pattern for each compound retarder, with the color scale reducing with wavelength.

similar to the MgF₂ retarders presented in this article. Figure 64 shows maps of the elliptical retardance parameters across the aperture for wavelengths of 407.0 nm in Fig. 64(a) and 852.6 nm in Fig. 64(b).

The retardance uniformity from typical polishing errors can be highlighted with a Zernike decomposition of the retardance spatial map. I show maps of the circular retardance across the aperture in Fig. 65 for a wavelength of 416.7 nm. The retardance uniformity is dominated by a wedge shape of roughly 0.6 deg. Removal of this wedge shape reveals a bar-type shape, similar to what was reported in the DKIST quartz-based A-plane cut retarders shown in Ref. 86.
The spectral retardance properties for a very thin crystal athermal achromat made of Al₂O₃ - MgF₂ - SiO₂. (a) The total elliptical retardance magnitude for samples number 1 and 4. (b) Spectral clocking oscillations for the linear retardance fast axis in green and the circular retardance in red. Samples number 1 and 4 have dashed and solid lines, respectively. The fast axis was offset vertically for clarity. (c) Interference fringes seen in retardance. Fringes are confined to the linear retardance magnitude shown in blue. Sample 4 fringes are shown in a darker shade than sample 1 fringes. I note the fast axis of linear retardance in green and the circular retardance in red have both been vertically offset for clarity.

(a) Variation in elliptical retardance magnitude over 61 points in an NDSP5 Mueller matrix map. Interference fringes are seen beating across wavelengths. I note our infrared wavelengths have more than 20× lower spectral resolving power giving rise to the discontinuity in fringe detection near 1.1 μm wavelength. (b) Retardance spectra at five select spatial regions around 770 nm wavelength to illustrate fringe behavior.

The elliptical retardance magnitude maps for this three crystal athermal achromat measured at the two extreme wavelengths of our NDSP5 lab system. A 61 point spatial map was made to cover a 20 mm aperture in NDS5. (a) 396.9 nm wavelength. (b) 1562.4 nm wavelength. I used a 21 pixel spectral averaging to remove the influence of interference fringes on the bulk spatial variation of retardance. (a) is roughly 3.2 deg PV while (b) is 0.64 deg PV.
Thermo-optical properties of common retarder crystals must be considered for their impact on the accuracy of a polarization calibration. Thermal drifts in retardance both from ambient temperature change and absorption from absorbing a solar beam can drive retardance changes. Higher refractive index retarder materials such as crystal sapphire can have very large interference fringes, corrupting and complicating calibration. Higher heat conductivity of materials can smooth out temperature gradients within a part, reducing temperature induced retardance non-uniformities (which corrupt calibrations across a wide FoV per Ref. 86). The conductivity of the crystal can mitigate spatially non-uniform temperature dependence by reducing temperature gradients and responding better to active cooling. There are three contributing factors to retardance dependence on temperature. First is physical expansion through the coefficient of thermal expansion, CTE.

\[
OP = d(1 + a\Delta T) \ n(1 + TOC\Delta T). \tag{8}
\]

Second is the change in refractive index with temperature through the thermo-optic coefficient \((dn/dT, TOC)\). Third is the dependence of crystal birefringence on temperature \((d(n1 - n2)/dT)\). All three parameters cause the fringes and spectral retarder properties to change per Eq. (8).

Fig. 64 The circular retardance parameter maps for the crystal quartz window at 5 mm thickness and 120 mm diameter. (a) 407.0 nm wavelength and (b) 852.6 nm. Each panel has the aperture center value printed in the lower right corner. For instance, the aperture center circular retardance magnitude was \(-512.0\) deg at 396.2 nm wavelength as seen in (a).

Fig. 65 The circular retardance parameter maps for the 5 mm thick Z-plane cut crystal quartz window at a wavelength of 416.7 nm. (a) The unfiltered retardance map. (b) Removal of tip/tilt Zernike terms of 0.53 deg in X and 0.27 deg in Y. The residual retardance parameter is cylinder shaped. (b) The retardance map with power removed in addition to tip/tilt.
I compare material properties for MgF$_2$ and SiO$_2$ in Table 5. Both crystals are positive uniaxial. For crystals cut as windows, the crystal optic axis is usually specified as perpendicular to the surface, also stated as parallel to the surface normal. This is often called z-cut as shorthand for zero degrees between the optic axis and surface normal cut. When windows of uniaxial materials are cut c-plane/z-plane, the beam traveling through the window will have the two orthogonal polarization states traveling through material of the same two ordinary refractive indices. For materials cut “with the C axis perpendicular to the surface normal, the C axis points along the direction of travel for the beam.” For a window, thermal conductivity on the “parallel to C axis” direction would be “parallel to the direction of beam travel” representing the “depth through the window” direction as well as the single extraordinary refractive index axis. Window conductivity radially along the surface (center to edge) would be the “perpendicular to the beam travel direction” with same two ordinary refractive indices.

I show crystal material quantities either averaged or computed from the extraordinary and ordinary axes in Table 6. The refractive index for SiO$_2$ is roughly 12% higher than MgF$_2$. The birefringence is the difference between refractive indices $\Delta n$ shown at a wavelength of 633 nm. The thermal dependence of the birefringence is $d(n_e - n_o)/dT$. Both materials have negative sign with the extraordinary refractive index changing more with temperature than the ordinary index. However, crystal MgF$_2$ changes 0.6x less with temperature making it a more stable material in thermally variable environments. The average conductivity of MgF$_2$ is also roughly 3.2x higher making thermal gradients in the part substantially smaller. Example impacts of radial and depth dependent temperature gradients were shown in Ref. 86. In addition, our optics are conductively and convectively cooled, giving MgF$_2$ additional advantages in mitigating heat. The single surface reflectivity is also roughly 0.6x for MgF$_2$ reducing the magnitude of interference fringes. The shortest wavelength at which absorption is detectable in a 2 mm thick substrate is shown as $\lambda_{\text{IR}}$. MgF$_2$ does not show substantial absorption until roughly 5.8 $\mu$m, a 1.7x longer wavelength than SiO$_2$. For solar applications, there is almost a 6x difference in heat due to absorption of wavelengths beyond 3.4 $\mu$m or 5.8 $\mu$m. Figures 21 and 22 of Ref. 86 showed that the absorption is dominated in the layers of the optic closest to the incident beam. Reference 111 showed that the (oiled) quartz retarders heated up by more than 12°C in 30 min while the MgF$_2$ retarders barely rose 1.5°C in the same timeframe.

I compute and example comparing 100 $\mu$m thick MgF$_2$ crystal to a 130 $\mu$m thickness of crystal SiO$_2$ corresponding to the same net retardance given the 1.3x birefringence difference.

### Table 5 Retarder crystal material properties.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MgF$_2$E</th>
<th>MgF$_2$O</th>
<th>SiO$_2$E</th>
<th>SiO$_2$O</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$ @633 nm</td>
<td>1.3887</td>
<td>1.3770</td>
<td>1.5517</td>
<td>1.5426</td>
</tr>
<tr>
<td>$dn/dT$ (10$^{-6}$/K)</td>
<td>2.3</td>
<td>1.7</td>
<td>-5.5</td>
<td>-6.5</td>
</tr>
<tr>
<td>$\alpha$ CTE (10$^{-6}$/K)</td>
<td>13.7</td>
<td>8.9</td>
<td>7.1</td>
<td>13.2</td>
</tr>
<tr>
<td>$xW$/mK</td>
<td>21</td>
<td>33.6</td>
<td>10.7</td>
<td>6.2</td>
</tr>
</tbody>
</table>

### Table 6 Average properties.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MgF$_2$</th>
<th>SiO$_2$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>1.3829</td>
<td>1.5472</td>
<td>1.12</td>
</tr>
<tr>
<td>$\Delta n$ @633 nm</td>
<td>0.01178</td>
<td>0.00907</td>
<td>1.30</td>
</tr>
<tr>
<td>$d(n_e - n_o)/dT$ (10$^{-6}$/K)</td>
<td>-0.6</td>
<td>-1.0</td>
<td>0.6</td>
</tr>
<tr>
<td>$xW$/mK</td>
<td>27.3</td>
<td>8.5</td>
<td>3.2</td>
</tr>
<tr>
<td>$R$ @633 nm</td>
<td>2.6%</td>
<td>4.6%</td>
<td>0.6</td>
</tr>
<tr>
<td>$\lambda_{\text{IR}}$ ($\mu$m)</td>
<td>5.8</td>
<td>3.4</td>
<td>1.7</td>
</tr>
</tbody>
</table>
between the two materials. In Fig. 66(a), I show how the difference between optical thickness of extraordinary and ordinary rays as the net retardance for both crystals. I have scaled the SiO$_2$ thickness to 130 μm to achieve the same net retardance as 100 μm of MgF$_2$. Ordinary and extraordinary rays are solid lines. MgF$_2$ is green while SiO$_2$ is blue. (b) The thermally perturbed optical path difference following Eq. (8) for both E and O beams for both SiO$_2$ and MgF$_2$.

In Fig. 66(b), I show a 1°C perturbation for two 100 μm thick crystals. I note a significant difference seen as MgF$_2$ as both thermo-optic coefficients ($dn/dT$) and thermal expansion coefficients ($\alpha$) positive. Thus, all terms of Eq. (8) are increasing. For SiO$_2$, $dn/dT$ is negative which acts against the expanding thickness ($\alpha$) to reduce the total waves optical path difference. However, the thermal dependence of the birefringence as $d(n_o - n_e)/dT$ is 0.6$x$ less for MgF$_2$. The absolute optical thickness of MgF$_2$ changes more but the net retardance changes less.

These two competing materials properties conspire to give the same thermally perturbed change in retardance for the same 100 μm of MgF$_2$ or SiO$_2$ crystal. For our applications, I need to compare crystals of the same net retardance magnitude. The 1.3$x$ birefringence difference between MgF$_2$ and SiO$_2$ is the driving factor in the thermal sensitivity of the crystal retarders.

In Fig. 67(a), I show the waves net retardance change for a 1°C thermal perturbation in the two crystals of the same net retardance magnitude (100 μm of MgF$_2$ and 130 μm of SiO$_2$). In Fig. 67(b), I show the ratio of these thermal perturbations. The magnitude and sign differences of the various thermal perturbations effectively cancel between 100 μm of MgF$_2$ and 130 μm of SiO$_2$ leaving only the 1.3$x$ ratio across most of the bandpass. This corresponds to the nearly achromatic 75% relative retardance change seen in Fig. 67(b).

![Fig. 66](https://www.spiedigitallibrary.org/journals/Journal-of-Astronomical-Telescopes,-Instruments,-and-Systems on 08 Oct 2023)

![Fig. 67](https://www.spiedigitallibrary.org/journals/Journal-of-Astronomical-Telescopes,-Instruments,-and-Systems on 08 Oct 2023)
Code Data Materials Availability Statement
All data in support of the findings of this paper are available within the article. Any additional information can be requested from the author at dharrington@nso.edu.

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References

Harrington: Large aperture optically contacted MgF2 retarders for calibration and...


David M. Harrington is a polarimetry scientist at the National Solar Observatory.