Error compensation for long-distance measurements with a photoelectric autocollimator

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Abstract. Although autocollimators enable noncontact measurement, their performance is limited in large-scale and long-distance applications because of errors caused by nonideal point light sources. Therefore, we analyze this type of measurement error, including the error source and the equations used to describe the irradiance distribution of the light spot. A two-dimensional exponential approximation formula is used to express the light irradiance distribution and compensate the spot imaging errors. Experimental results show that the measurement error could be reduced by sixfold. Therefore, the proposed compensation algorithm can be applied to autocollimator measurements over distances.

Key words: photoelectric autocollimator; long-distance measurement; error compensation.

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1 Introduction
The autocollimator is an important angle measuring instrument1–6 that enables noncontact measurement with high accuracy and high resolution. However, the performance of this instrument is limited in large-scale and long-distance applications because of the autocollimator’s physical characteristics and operating principle, and this limitation prevents use in many prominent areas, such as ship and aircraft manufacturing and generator and radar monitoring.7 Because the measurement range of the photoelectric autocollimator is closely related to the measurement distance,8 long-distance and wide-range autocollimator has become a significant research focus in recent years.9–13

Chen et al.12 proposed an angle measurement method based on optical frequency domain and realized a measurement range of 21,600 arc sec. Li et al.13 employed a cube corner as a reflector to extend the angle measurement range from 1.2 deg to 12 deg with an accuracy better than 35 arc sec. Zhu et al.13 proposed a common-path design criterion for laser-datum based on measurement of small angle deviations, and this approach realized high-precision measurement at long distances. However, these methods have large measurement errors when used for far ranging. The main cause of these errors is that the optical beam is not uniform due to the nonideal point source.14

Therefore, this paper analyzes the measurement error caused by an uneven light beam under long working distance conditions. An algorithm is proposed to compensate the error and thereby increase the measurement accuracy. The algorithm is verified by experiments that demonstrate a sixfold reduction in the error under long-distance conditions.

2 Model Principle and Analytical Methods
A schematic of a typical photoelectric autocollimator is shown in Fig. 1. A light emission diode is used as the source of radiation source, and the autocollimator projects a collimated beam onto a planar reflector that is mounted on a target at distance L. The autocollimator unit measures the tilt angle of the target by detecting the deviation of the reflected beam with respect to the projected beam’s axis. The beam is deflected by a beam splitter (BS), collimated by a collimating lens (CL), and then projected onto the reflector. The reflected beam from the reflector is focused by CL, passed through the BS and propagates to the photoelectric sensor, which is a complementary metal-oxide-semiconductor (CMOS) matrix used to measure the displacement between the source and reflection of the focused laser light spot.

The photoelectric autocollimator is a precision instrument for small-angle measurement, and the tilting of the target will change the displacement of the reflected light spot on the CMOS. Although this measurement principle is based on a point light source, the light source is not actually a point light source, and this discrepancy causes measurement errors over long distances, as follows.

A schematic of the radiation component of the autocollimator is shown in Fig. 2.

The radius of the light source is a. A Lambertian approximation can be used for the condition f ≫ a, where f is the focal length of the CL. The beams from every point of the light source are collected by the CL at an angle β with the main optical axis CL. The largest value of β is a divergence angle βm, which can be expressed as

$$\beta_m = \arctg(a/f).$$

The whole optical path can be divided into two regions, the internal region I and the external region II. This area is determined by beams B, b and C, c, which radiate from the bottom and top of the point source, respectively.

Each point in region I has the same irradiance, and thus region I can be defined as the uniform irradiance region, within which lies the optimal position of the reflector. The length of the nearest part of region I from the point...
Light source

\[
P \text{ to the CL is } L_{fr}, \text{ which is defined as the formation distance of the beams. } L_{fr} \text{ can be calculated by}
\]

\[
L_{fr} = \frac{D_1}{2 \cdot \tan(\theta_m)} = \frac{D_1 \cdot f}{2 \cdot a},
\]

where \(D_1\) is the diameter of the CL.

In a conventional autocollimator, the reflector is located at the formation distance \(L_1\) not more than formation distance \(L_{fr}\), the illumination of the entire image is uniform, and beams radiated from all point sources will be imaged on the CMOS. As shown in Fig. 1, the reflector is indicated by the dotted line.

The working distance \(L_1\) can be expressed as

\[
L_1 = \frac{D_2 - D_3}{2 \cdot \tan(\theta_m)} = \frac{(D_2 - D_3) \cdot f}{2 \cdot a},
\]

where \(D_3\) is the diameter of reflector 3. In general, a conventional photoelectric autocollimator has \(D_1 = 50\ \text{mm}, D_3 = 30\ \text{mm}, f = 500\ \text{mm},\) and \(a = 0.25\ \text{mm}.\) The reflector 3 is the entrance purple of the optical system path. Applying these specifications to Eq. (3), the working distance of a typical photoelectric autocollimator is 10 m, which significantly limits its application.

To expand the working distance of the photoelectric autocollimator, we place the entrance purple of the optical system in region I behind the point \(P_{fr}\), namely, \(L_2 > L_{fr}\). The equivalent optical path diagram in which the CL2' is the image of the lens 2 by the reflector 3 as shown in Fig. 3.

When the diameter of the reflector is \(D_3 \geq D_1\), CL2' is the entrance purple of the optical system path. As shown in Fig. 3, the working distance \(L_2\) of the autocollimator can be expressed as

\[
L_2 = 2L_{fr}.
\]

Equation (4) shows that the maximum range of the autocollimator can reach 100 m.

2.1 Measurement Error Caused by Beam Limitation

The irradiance distribution on the imaging surface must be analyzed when the reflector is located behind the beam formation distance to determine to what extent the angle measurement error is caused by the beam limitation. To study the imaging of a beam on the CMOS under this condition, the diameter of \(D_3\) is assumed to be larger than that of the CL, \(D_1\) and \(D_2\), as in Fig. 4: \(D_3 > D_1\) and \(D_3 > D_2\). A diagram of the optical path with then trance purple of the optical system located behind \(L_{fr}\), is shown in Fig. 4.

As can be seen from Fig. 4, the beams from the light source are collimated by CL2 and projected onto the reflector \(R\), and then the reflected beams are focused by CL2' onto the CMOS4, with most of the energy focused on the point O. As seen in Fig. 5, a radiant point at a distance \(r\) from the center O of the light source generates the radiation beam that is inclined at an angle of \(\beta\) to the main optical axis. At this
point, some off-axis beams passing through CL2, with a diameter of \( D_1 \), will be irradiated on the equivalent CL2’.

The displacement of the facula relative to the equivalent objective lens is \( r’ \), which can be expressed as

\[
    r’ = 2 \cdot L \cdot \tan(\beta),
\]

(5)

where \( \beta \) is the divergence angle between the off-axial beam and the main optical axis.

The irradiance value on the imaging plane of the off-axis beams is proportional to the area of intersection between the aperture of CL2’ and the off-axis beam. This intersection area is smaller than the aperture size of the on-axis beam. Therefore, the irradiance of the facula formed by the off-axial beam on the imaging plane is less than that at the point of \( O \).

The optical path in Fig. 4 shows that the irradiance \( E(r) \) on the CMOS plane is distributed from the maximum point \( E(0) \) to the minimum point \( g \), attenuating according to a certain law. The distance from \( g \) to the central coordinate of the CMOS plane is \( r = R_m \).

The final imaging on the CMOS is shown in Fig. 6. The area circumscribed by radius \( a \) is not the sum of the imaging of all the radiant points on the source; rather, it is the sum of the imaging of the radiant points \( R_m \) from the center point of the source. The irradiance of point \( g \) on the imaging plane corresponding to the point \( g \) in Fig. 4 is \( O \).

According to the light path of the radiant point \( g \) from the light source, the corresponding point of \( g \) on the imaging plane can be found, and the following relation can be obtained:

\[
    R_m = \frac{D \cdot f}{2 \cdot L}.
\]

(6)

The irradiance of the corresponding image surface can be calculated as

\[
    E(r) = \frac{2}{\pi} \left\{ \arccos \left[ \frac{\gamma(r)}{R_m} \right] - \sqrt{1 - \left[ \frac{\gamma(r)}{R_m} \right]^2} \cdot \gamma(r) \right\},
\]

(7)

\[
    \gamma(r) = \frac{r}{R_m}, \quad 0 < r < R_m.
\]

(8)

By calculating the \( E(r) \) function, a two-dimensional imaging pattern of \( E(r) \) can be obtained as shown in Fig. 7.

2.2 Calculation of the Measurement Error Caused by the Different Tilt Angles of the Reflector Plane

When the reflector plane deflects a tilt angle of \( \theta \) in relation to the main optical axis, the displacement of the imaging circle can be expressed as

\[
    x = f \cdot \tan(\theta).
\]

(9)

However, the reflected beam will be deflected from the main optical axis at an angle of \( 2\theta \) after the collimating beams have been reflected by a deflected reflector. In this case, the displacement of the aperture of the radiant point at the image edge on the image plane can be expressed as

\[
    \bar{x} = f \cdot \tan(2 \cdot \theta) \approx 2 \cdot x.
\]

(10)

In Fig. 8, when the displacement of the light source boundary (circle 2) with respect to the image boundary (circle 1) is greater than \( l_0 \), a part of the image is eliminated, and the irradiance distribution of the image is no longer symmetrical. The value of \( l_0 \) can be expressed by the following equation:

\[
    l_0 = \bar{x} - x = a - R_m.
\]

(11)

For a tilt angle \( \theta_0 \), the corresponding irradiance displacement can be found by identifying the root of the following equation:

![Fig. 6 Image plane when reflector is at rest: the number 1 represents the image boundary of the light source on the CMOS, and the number 2 represents the corresponding circumference of the light source boundary.](Image)

![Fig. 7 Three-dimensional form of image irradiance in different fields of view when the reflector plane is at rest - (a) Tilt 40 deg, Twist 125 deg, (b) Tilt 80 deg, Twist 160 deg.](Image)
where $R_m$ is the radius of the image in Eq. (6), and $k = 0.61$ can be found by the two-dimensional exponential approximation $E(x, y)$ and the least squares method. The image surface irradiance distribution obtained by Eq. (16) is shown in Fig. 10.

### 2.3 Measurement Error Analysis after Using Exponential Function

The image plane model should appropriately change when calculating the irradiance distribution using the exponential approximation formula $E_e(x, y)$. Figure 11 shows a simulation of the image irradiance contour boundary obtained by the exponential approximation function. Here, the image boundary and the corresponding light source boundary are approximately squares.

In Fig. 11, $l$, which represents the displacement offset between the image and the edge of the light source, can be expressed as

$$l = x - (R_m - a).$$

Thus the measurement error of the image’s energy center will occur only in the case of $l > 0$. 

---

Fig. 8 Image contour of the light source with a tilted reflector plane: the number 1 represents the image boundary of the light source on the CMOS, and the number 2 represents the corresponding circumference of the light source boundary.

Fig. 9 Relative position of the light source boundary 2 and the image boundary 1.

expresses the light source boundary conjugate circle, and $E(x, y)$ expresses the two-dimensional irradiance distribution of the image, yielding the result of Eq. (7) in the case of $r = \sqrt{x^2 + y^2}$. However, because of the complexity of the integrand in Eq. (15), $E(x, y)$ is not suitable for the study of the measurement error relationship.

Therefore, an approximation of the CMOS analyzer based on Eq. (7) is suggested by the two-dimensional exponential formula $E_e(x, y)$ with a separable variable based on the exponential function:

$$E_e = \exp\left[\frac{(x + y)}{k \cdot R_m}\right],$$

In Eq. (15), $b_1$ and $b_2$ are the points on the OX coordinate axis, $F_1(x, y)$ expresses the image boundary, $F_2(x, y)$ expresses the light source boundary conjugate circle, and $E(x, y)$ expresses the two-dimensional irradiance distribution of the image, yielding the result of Eq. (7) in the case of $r = \sqrt{x^2 + y^2}$. However, because of the complexity of the integrand in Eq. (15), $E(x, y)$ is not suitable for the study of the measurement error relationship.

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$$E_e = \exp\left[\frac{(x + y)}{k \cdot R_m}\right].$$

In order to simplify the calculation of the error value, the image boundary can be considered to have a fixed location and only the light source boundary is displaced by a distance $x$ from the center. Figure 9 shows the relative position of the light source boundary and the image boundary.

In this case, the value of the energy center of the constricted image is equal to the measurement error of the image displacement, which can be expressed as

$$\hat{x}_c = x_c - x.$$  

(14)

In fact, the coordinates of the energy center of the constricted image are equal to the difference in Eq. (13).

Referencing the coordinate axis in Fig. 9, the coordinates of the energy center in Eq. (14) can be expressed as

$$\hat{x}_c = \frac{\int_{-b_1}^{b_1} \int_{-b_2}^{b_2} x \cdot E(x, y) dx dy}{\int_{-b_1}^{b_1} \int_{-b_2}^{b_2} E(x, y) dx dy}$$

$$+ \frac{\int_{-b_1}^{b_1} \int_x^{b_2} x \cdot E(x, y) dx dy}{\int_{-b_1}^{b_1} \int_{-b_2}^{b_2} E(x, y) dx dy}$$

$$+ \frac{\int_{x}^{b_2} \int_{-b_1}^{b_1} x \cdot E(x, y) dx dy}{\int_{-b_1}^{b_1} \int_{-b_2}^{b_2} E(x, y) dx dy}.$$

(15)

In Eq. (15), $b_1$ and $b_2$ are the points on the OX coordinate axis, $F_1(x, y)$ expresses the image boundary, $F_2(x, y)$ expresses the light source boundary conjugate circle, and $E(x, y)$ expresses the two-dimensional irradiance distribution of the image, yielding the result of Eq. (7) in the case of $r = \sqrt{x^2 + y^2}$. However, because of the complexity of the integrand in Eq. (15), $E(x, y)$ is not suitable for the study of the measurement error relationship.
The error of the image coordinates at \( l < R_m \) is given by

\[
\hat{x}_1 = \frac{\int_0^{R_m} \int_{(l-R_m)}^0 x \cdot E_e(x,y) \, dx \, dy}{\int_0^{R_m} \int_{(l-R_m)}^0 E_e(x,y) \, dx \, dy}
\]

When \( l > R_m \), the error is given by

\[
\hat{x}_2 = \frac{\int_0^{R_m} \int_{(l-R_m)}^0 x \cdot E_e(x,y) \, dx \, dy}{\int_0^{R_m} \int_{(l-R_m)}^0 E_e(x,y) \, dx \, dy}
\]

We can obtain the expected expression of the measurement error of the image coordinates after substituting Eq. (16) into Eqs. (17)–(19). The measurement error is normalized using the parameter \( R_m \), which is the radius of the image on the photo sensor to compute the formula:

\[
\sigma_x = \frac{\hat{x}_c}{R_m}.
\]

When \( l < R_m \), this value can be obtained from Eq. (18):

\[
\sigma_1 = \frac{e^{\frac{q}{k}} \cdot (q - k - 1) + (k + 1)}{\left(e^{\frac{q}{k}} - 2\right) \cdot e^{\frac{q}{k}} + 1}.
\]

When \( l > R_m \), the value can be obtained from Eq. (19):

\[
\sigma_2 = \frac{e^{\frac{q}{k}} \cdot e^{\frac{k+q}{k}}(k + q - 1) - (k + 1)}{e^{\frac{q}{k}} \cdot e^{\frac{k+q}{k}} - 1}.
\]

Here \( k = 0.61 \) is the parameter of the approximation \( E_e(x,y) \) in Eq. (16), and \( q \) is the measured displacement \( l \) of the image after normalization by \( R_m \), which can be expressed as

\[
q = \frac{l}{R_m}.
\]

The normalized function graph of \( \sigma_1 \) and \( \sigma_2 \) is shown in Fig. 13. The computer simulations indicated that the difference between the irradiance distributions obtained by the suggested approximation and by the exact formula did not
exceed 5% few percent, demonstrating the reliability of the proposed model.

3 Results of Experimental Comparison
The aberrations of the objective lens CL 2 and the uneven refractive index of the optical components, air turbulence, and temperature variations all lead to measurement errors of the photoelectric autocollimator. However, since a high-quality objective lens is used in the photoelectric autocollimator, the aberration is extremely small compared to the size of the image on the CMOS and can be ignored. The measurement error caused by the uneven refractive index of the optical component is smaller because the measurement beam is always completely filled with the entrance purple of the objective lens when measured by the photoelectric autocollimator. In the process of measurement, if the stability of the ambient temperature can be maintained and the nonturbulent airway is ensured, the influence of the external environmental test will be reduced. Therefore, the main cause of the measurement error of the photoelectric autocollimator of this experiment is the measurement error caused by the vignetting phenomenon of the beam caused by the aperture of the optical component.

3.1 Experimental Study on Compensation for Errors Caused by Beam Limitation
The experimental platform consisted of a tested photoelectric autocollimator, high-precision calibrated standard photoelectric autocollimator TriAngle TA 1000-140 produced from Germany Trioptics GMBH, corresponding two reflectors, Laser TrackerAPI Radian 80 and corresponding cube-corner reflector.

As shown in Fig. 14, the tested photoelectric autocollimator 1 and laser tracker 3 are placed in rigid flat platform 4. The reflectors 6 and 7 of autocollimators 1 and 2 are perpendicular to each other and they are rigidly connected with rotary table 5. The rotary table 5, cube-corner reflector 8 of the laser tracker, and high-precision standard autocollimator 2 are fixed with rigid base 9. The rigid platform 4 and the rigid base 9 are installed on the concrete walls of two adjacent rooms, respectively. The distance between the rigid platform 4 and rigid base 9 is 20.5 m, and the distance from the objective lens of the photoelectric autocollimator 2 to the reflector 7 is 0.2 m. The cube-corner reflector 8 is always kept static to monitor rigid base 9 no linear displacement.

The linear position stability of the rigid base 9 is measured by a laser tracker with an error of 0.05 mm. The rotation angle of the rotary table 5 is measured by the autocollimator 2, and the measurement error is 0.2 arc sec. The stability of the experimental environment including air humidity, temperature, and air pressure is monitored by laser tracker 3. The air temperature is $21 \pm 0.2^\circ C$ in the experimental time.

This experiment was tested several times from 1 to 4 a.m. In order to eliminate the influence of environmental factors on the measurement results, we repeat the measurement process when using a laser tracker to monitor environmental changes. Finally, we compared the environmental changes during several measurements and selected a set of measurements in the most stable environment. The CMOS of the photoelectric autocollimators 1 and 2 measures the imaging spot of each measuring angle 10 times and then takes the average value of the data as the final measured value of the angle. This approach avoids the effects of air interference on measurement errors. In the first stage of the experiment, we gave the artificial tilting angle of the reflector 6 and 7 along the $Y$ axis through the rotary table 5. The tilting angle is measured by autocollimators 1 and 2 at the same time.

In the second stage of the experiment, we established an image processing algorithm based on the proposed model and repeated the first stage of the experiment to obtain the error results. The correctness of the compensation method was verified by the comparative analysis of the two results.

The distance between the photoelectric autocollimator 1 and the reflector 6 is larger than the beam formation distance $L_f$ in Fig. 2. The measurement distance of the experimental photoelectric autocollimator needs to reach 20.5 m, and the
measurement range needs to reach 4.5 arc min with an accuracy error requirement of not more than 2 arc sec. The objective lens of the autocollimator that we used had a 400-mm focal length, a 55-mm caliber, and a circle of confusion of 0.02-mm in diameter. The autocollimator has a circle of confusion of 0.02-mm in diameter and a 0.6-mm light source radius. The light source was a semiconductor diode SFH 485 P with a power of 10 mw, the photoelectric sensor was a OV05620 CMOS QSXGA with a $2592 \times 1944$ resolution and a pixel size of $2.2 \times 2.2 \, \mu m^2$, and the reflector 6 had a size $75 \times 100 \, mm^2$ (Fig. 15).

### 3.2 Experimental Results

In the experiments, we gave the rotary table 5 at tilting angle $\theta$, ranging from 0 to 4 arc min 30 arc sec, in intervals of 10 arc sec. Figure 16 shows the performance of the transducer function of autocollimator $\theta_i = \Phi(\theta)$. On the horizontal axis, $\theta$ is the tilting angle of the reflector. On the vertical axis, $\theta_i$ is the angle measured by the autocollimator.

The error of a measurement $\sigma\theta$ was the difference between the tilting angle $\theta$ of high-precision photoelectric autocollimator 2 and the tested photoelectric autocollimator 1 measured angle $\theta_i$:

$$\sigma\theta = \theta - \theta_i.$$  \hspace{1cm} (24)

In Fig. 16, $\theta$ is shown by the red cube, and the error $\sigma\theta$ is shown by the black circles.

Figure 17 shows the imaging spot of the light source on the CMOS when the reflector was static and when the
reflector was rotated by 200 arc sec. The above images verify the error generation mechanism proposed in this paper.

The measurement error is made up of both systematic and random errors. The systematic error of the image can be approximated as $\sigma \theta = \Phi(\theta)$. The systematic error of the photoelectric autocollimator is almost zero when the tilt angle of the reflector is not more than $\theta_0 = 30$ arc sec, as is the case when Eq. (11) does not exceed its limit. The initial experiment showed that the systematic error reaches 35% of the output value without the compensation algorithm.

Figure 18 shows the results of the second stage of the experiment, when the compensation formula was applied to the calculation results of Eqs. (21) and (22), representing the $\theta_t$ measured by the autocollimator. The performance of the transducer function $\theta_{\text{con}} = \Phi(\theta)$ of autocollimator is shown by the curved line.

In Fig. 18, the blue circles represent the measured angle $\theta$ of autocollimator 1 and the red cubes represent the measured angle $\theta$ of autocollimator 2. Compensated measurement error $\sigma \theta$ is shown by the pink cube, and the black circles show the error before the compensation.

According to Eq. (12), when the measured value $\theta$ exceeds the value $\theta_0$, the error compensation algorithm was used, and the $R_m$ parameter of the image on the CMOS was calculated according to Eq. (6). The compensation algorithm included the following steps.

1. The rotary table rotates $\theta$ and the measurement angle of the high-precision autocollimator is $\theta$. Then the measurement value of the autocollimator contains the measurement error $\sigma \theta$:
   \[ \hat{\theta} = \theta + \sigma \theta. \]  \hspace{1cm} (25)
   In the expression, the measured values $\theta$ and $\sigma \theta$ are independent of each other.

2. We can calculate the displacement value $\hat{x}$ of the image by $x = f \cdot \tan(\theta)$ according to the measurement angle $\hat{\theta}$. This value contains the measurement error value $\hat{x}_c$ of the image coordinates:
   \[ \hat{x} = x + \hat{x}_c. \]  \hspace{1cm} (26)
   In the expression, the measured values $x$ and $\hat{x}_c$ are independent of each other.

3. We can calculate the value $\hat{l}$ by $l = x - (R_m - a)$ according to the value $\hat{\theta}$. This value contains the measurement error value $\hat{x}_c$:
   \[ \hat{l} = l + \hat{x}_c. \]  \hspace{1cm} (27)
   In the expression, the measured values $l$ and $\hat{x}_c$ are independent of each other.

4. Equation (26) is divided by the obtained $R_m$, and an expression is obtained according to Eq. (23). The calculation of $\hat{q}$ is as follows:
   \[ \hat{q} = q + \sigma_x(q). \]  \hspace{1cm} (28)
   The parameters $\hat{q}$ are obtained from the measurement angle $\theta$ of the autocollimator, where the mathematical meaning of $\sigma_x(q)$ is expressed by Eqs. (21) and (22). Equation (27) is thus a nonlinear equation.

5. The value $q$ can be calculated by Eq. (27), and the value of $l$ can be calculated by Eq. (23).

6. Find the $l$ value and the $\hat{x}_c$ value by Eq. (26), and use them to calculate the value $x$ of the image according to Eq. (25).

7. Then through Eq. (9), the accurate measurement $\theta_t$ after the error compensation can be obtained.

From the experiment results, the systematic error after compensation is <6%. The error of measurement $\sigma \theta$ was linearly fitted as $\delta \theta$, and the difference $\sigma \theta - \delta \theta$ was considered the random error as shown in Fig. 19. The standard deviation of measurement error of is $\sim 2$ arc sec.

Although the random error remains the same as before the compensation, this experiment demonstrated that the proposed compensation algorithm can effectively compensate the measurement error caused by the beam limitation of the photoelectric autocollimator.

4 Conclusion

In this paper, we analyzed the measurement errors produced by the nonideal point light sources of photoelectric autocollimators used over long distances. Through the error
analysis, the influence of vignetting on the measurement was elucidated, and then the error was compensated using a known two-dimensional approximation formula. However, the complexity of the formula made the compensation impossible, and we, therefore, proposed a two-dimensional exponential approximate formula to analyze and compensate the spot imaging. Through experimental verification, this algorithm was shown to effectively reduce the error caused by the nonideal point source in the long-distance measurements of the photoelectric autocollimator.

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References

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