Design and application of a spherical aberration free continuous zoom liquid-filled micro-cylindrical lenses system

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Abstract. A zoom system named liquid-filled micro-cylindrical lenses integrating a capillary and a biconvex micro-cylindrical lens in a polydimethylsiloxane (PDMS) substrate is designed, which can achieve continuous changed focal length from 2.675 to 9.012 mm by injecting variable concentrations of glycerol aqueous solution into the capillary. The aberration fan diagrams, root mean square spot radius, peak-to-valley wavefront aberration, and modulation transfer function curves at different zoom configurations of the liquid-filled micro-cylindrical lenses are all analyzed, showing good imaging quality over the whole focal length range. And an anamorphic zoom system for uniform collimated beam is designed putting such two PDMS substrates similar as the Kepler telescope structure. The anamorphic system can realize the zoom shaping function with 1:2 to 2:1 aspect ratio beam conversion without machine movement along optical axis and with no significant aberration. The introduced zoom system is characterized by high image quality, small volume, and a simple and stable structure, providing a design idea for cylindrical lenses with variable focal length. © The Authors. Published by SPIE under a Creative Commons Attribution 4.0 International License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI. [DOI: 10.1117/1.OE.61.8.085102]

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1 Introduction

Lenses are the basic and important element in the field of optics. Zoom lenses, more adaptive compared with the fixed ones, have a wide application in various practical areas, for example, in imaging systems, especially in microscopy or cameras, 1-3 in optical communication, 4 in aerospace or military operations, 5 and other optical fields. 6,7 With the rapid development of modern optical technology, the traditional mechanical zoom lenses are gradually unable to meet the increasing needs of the micro and integrated. Thus, the zoom system based on the liquid lens having a tunable liquid interface (e.g., change by the pressure in fluids, by electrical stress, or by wetting conditions) 8,9 or tunable refractive index (RI) of the lens medium (e.g., liquid crystal), 10 allows its focal length changing without any moving parts, drawing more attention in recent years. At present, spherical zoom systems, whether zooming based on traditional mechanical or based on liquid lens, have been extensively studied.

Beside the spherical lens, optical devices with no circular symmetry around the optical axis, such as cylindrical lens, are irreplaceable in some applications. At the most basic level, cylindrical lens can be used to focus the incoming light onto a line and can be used to correct astigmatism. Furthermore, two orthogonal cylindrical lenses possess function in imaging and spot shaping, and cylindrical lens array plays a role in three-dimensional displays and beam uniformity. In general, cylindrical lens can be used for various tasks in beam manipulation, which have been exploited in a range of sectors. 11-20 If the cylindrical lens is designed as variable focal

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length, more choices and more freedom can be brought into above applications. Meanwhile, the research on rotationally asymmetric zoom lenses is still limited compared with the spherical ones.

Very recently, a few studies reported the variable focus cylindrical lens systems using mechanical moving\textsuperscript{21,22} or making use of liquid zoom method.\textsuperscript{23,24} The mechanical zoom lens makes the optical system bulky and costly. The liquid methods, being almost the same stimuli as the zoom spherical lenses, always require high voltage control, and furthermore the surface shape of liquid interface or flexible membrane and the RI distribution of liquid crystal are more difficult to precise control compared with the liquid spherical lens. In recent publications, zoom systems based on rotating toroidal or cylindrical lenses have been suggested.\textsuperscript{25,26} Although the rotating lenses allow much easier for miniaturization, but the manipulation of angle is much harder than distance. In our previous study, the liquid-filled cylindrical zoom lenses changing the focal length by the variety of liquid RI were designed, characterized by high imaging quality over a wide focal length.\textsuperscript{27} But due to the main aim of this zoom system is to measure the liquid diffusion coefficient, the designed lens is large volume and inconvenient integration.

Considering the above problems, a spherical aberration free zoom liquid-filled micro-cylindrical system based on a polydimethylsiloxane (PDMS) substrate is designed using the optical design software ZEMAX in this paper. In the following sections, a detailed description of the lens structure is presented, and then, numerical analyses of the zoom ability, imaging quality, and application examples are successively introduced.

### 2 Liquid-Filled Zoom Micro-Cylindrical Lenses Structure

The zoom lenses design process has been introduced in Ref.\textsuperscript{27} in detail, mainly including the creation of the initial structure, optimization, and imaging quality evaluation. The design idea is similar in this paper, hence, only the choice of initial structure and the completed finial structure are highlighted in this section, and the details of the optimization process are omitted.

Capillary is the simplest micro-cylindrical lens, whose zoom function can be realized easily by changing the RI of the liquid injected into it. However, the capillary is easy to be damaged, difficult to install, and inconvenient to put into use. Therefore, we propose to embed the capillary into the PDMS substrate and design the substrate as a cuboid, which can greatly improve the stability of the device. Considering a single simple capillary and cuboid device usually has a large aberration, it is necessary to place an aberration elimination system in front or behind the capillary. The simplest is to add another cylindrical lens to the PDMS substrate, as shown in Fig. 1. If the ideal imaging quality can be achieved in a long zoom range after optimization using ZEMAX software, the designed initial structure is feasible. In the event of the ideal imaging quality not being gained throughout after repeated optimization, the single cylindrical lens for aberration elimination can be replaced by double glued cylindrical lens or other more complex lenses structure, and then the optimization process should be repeated.

Luckily, an acceptable liquid-filled zoom micro-cylindrical lenses structure is obtained after repeated optimization based on the compact initial structure. The structure of the final designed result is shown in Fig. 1, consisting of a capillary and a biconvex cylindrical lens embedded in a PDMS substrate, which overall size is only 5.0 mm × 2.0 mm × 4.6 mm. A square aperture (0.8 mm × 0.8 mm) is placed next to the left wall of the PDMS (not pasted together), and the light will pass through eight refracting surfaces after the aperture. The specific parameters of the eight refracting surfaces, include the surface type, radius of curvature, the axial distance between two adjacent surfaces (i.e., thickness), the medium after every surface (i.e., glass), the RI of the medium at $\lambda = 587.6$ nm, and the semidiameter of every surface, are listed in detail in Table 1. The length of the PDMS substrate is determined by the total thickness of the refracting medium, namely the sum of the fourth column of Table 1 (5.0 mm), and the PDMS substrate width should be equal to or slightly greater than twice the maximum of the semidiameter of all surfaces (1.0 mm, listed in the last column of Table 1). The PDMS substrate height is depending on the height of the cylindrical, which is decided by the zoom mechanism willing be introduced in Sec. 3.
The back focal length $f_B$ is easily measured and is convenient for setting up zoom system, while the effective focal length $f$ is generally used to reflect the imaging performance. Both those two parameters are analyzed in this paper to prove the zoom ability of the liquid-filled micro-cylindrical lenses. For a cylindrical lens, only the radial focusing ability needs to be considered, and its axial diopter is sure to be 0, and therefore the focal length mentioned in this paper refers in particular to the value in the $y$ direction (as the coordinate axes given in Fig. 1). In the case that one knows the specific parameters listed in Table 1, the back focal length $f_B$ of the lens system can be obtained by the following recursive equation:

$$ f_B = s_B^i, $$  

$$ \frac{n_i - n_i}{s_i} = \frac{n_i - n_i}{R_i} (i = 1, 2, \ldots, 8), $$  

$$ s_i = \infty. $$

### Table 1  Structure of the liquid-filled zoom micro-cylindrical lenses based on a PDMS substrate.

<table>
<thead>
<tr>
<th>No.</th>
<th>Surface type</th>
<th>Radius (mm)</th>
<th>Thickness (mm)</th>
<th>Glass</th>
<th>RI value at $\lambda = 587.6$ nm</th>
<th>Semidiameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Standard</td>
<td>Infinity</td>
<td>0.3</td>
<td>PDMS</td>
<td>1.4115</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>Toroidal</td>
<td>1.0</td>
<td>0.3</td>
<td>K9</td>
<td>1.5168</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>Toroidal</td>
<td>0.7</td>
<td>1.4</td>
<td>Liquid</td>
<td>1.3325 to 1.4730</td>
<td>0.7</td>
</tr>
<tr>
<td>4</td>
<td>Toroidal</td>
<td>−0.7</td>
<td>0.3</td>
<td>K9</td>
<td>1.5168</td>
<td>0.7</td>
</tr>
<tr>
<td>5</td>
<td>Toroidal</td>
<td>−1.0</td>
<td>0</td>
<td>PDMS</td>
<td>1.4115</td>
<td>1.0</td>
</tr>
<tr>
<td>6</td>
<td>Toroidal</td>
<td>1.4</td>
<td>1.1</td>
<td>F2</td>
<td>1.62004</td>
<td>1.0</td>
</tr>
<tr>
<td>7</td>
<td>Toroidal</td>
<td>−1.1</td>
<td>1.6</td>
<td>PDMS</td>
<td>1.4115</td>
<td>1.0</td>
</tr>
<tr>
<td>8</td>
<td>Standard</td>
<td>Infinity</td>
<td>—</td>
<td>Atmosphere</td>
<td>1.0000</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### 3 Analysis of the Zoom Ability and Imaging Quality

The back focal length $f_B$ is easily measured and is convenient for setting up zoom system, while the effective focal length $f$ is generally used to reflect the imaging performance. Both those two parameters are analyzed in this paper to prove the zoom ability of the liquid-filled micro-cylindrical lenses. For a cylindrical lens, only the radial focusing ability needs to be considered, and its axial diopter is sure to be 0, and therefore the focal length mentioned in this paper refers in particular to the value in the $y$ direction (as the coordinate axes given in Fig. 1). In the case that one knows the specific parameters listed in Table 1, the back focal length $f_B$ of the lens system can be obtained by the following recursive equation:

$$ f_B = s_B^i, $$  

$$ \frac{n_i - n_i}{s_i} = \frac{n_i - n_i}{R_i} (i = 1, 2, \ldots, 8), $$  

$$ s_i = \infty. $$
where \( s_i \) and \( s'_i \) are the intercepts in object and image space of the \( i \)'th surface of the optical system, respectively; \( n_i \) and \( n'_i \) are the RI of the medium in the front and behind of the \( i \)'th surface, respectively; \( R_i \) is the curvature radius of the \( i \)'th surface; and \( d_i \) is the axial distances between adjacent optical surfaces. Meanwhile, the effective focal length \( f \) is expressed for the paraxial optical system as

\[
f = \frac{D}{2}, \quad \text{(2a)}
\]

\[
u'_i = \frac{s_i u_i}{s'_i} \quad (i = 2, 3, \ldots, 8), \quad \text{(2b)}
\]

\[
u_{i+1} = \nu'_i \quad (i = 2, 3, \ldots, 7), \quad \text{(2c)}
\]

\[
u'_1 = \nu_1 + i_1 - i'_1, \quad \text{(2d)}
\]

\[
u_1 = 0, \quad i_1 = \frac{D}{2 R_1}, \quad i'_1 = \frac{n_i}{n'_1} i_1, \quad \text{(2e)}
\]

where \( D = 0.8 \text{ mm} \) is the aperture width in \( y \)-direction; \( \nu_i \) and \( \nu'_i \) are the aperture angles in object and image space of the \( i \)'th surface, respectively; and \( i_1 \) and \( i'_1 \) are the incident and refraction angles at the first surface, respectively.

According to Eq. (1b), the image distance of the third surface varies depending on the RI of the liquid \( n \) injected in the capillary, and then the object and image distances of the subsequently surfaces are affected, hence both the back focal length \( f_B \) and the effective focal length \( f \) are the function of \( n \) based on Eqs. (1) and (2). We adjust the liquid RI by changing the concentration (mass fraction) of glycerol aqueous solution \( C \). At 298.15 K and \( \lambda = 587.6 \text{ nm} \), the RI of pure water is 1.3325, and the RI of glycerol is 1.4730. Glycerol and water can be soluble with each other at any proportion, therefore the RI of glycerol aqueous solution can be continuous variation between 1.3325 and 1.4730. Putting the lens parameters listed in Table 1 into Eqs. (1) and (2), the \( y \)-direction \( f_B \) and \( f \) changing curves can be obtained as shown in Fig. 2, which illustrates that the \( f_B \) decreases from 10.798 to 0.974 mm continuously and

![Fig. 2 Y-direction focal length curves of the designed micro-cylindrical lenses varied with the concentration (mass fraction) and RI of the glycerol aqueous solution filled in the capillary. The black solid curve indicates the back focal length \( f_B \), and the red dotted curve indicates the effective focal length \( f \).](https://www.spiedigitallibrary.org/journals/Optical-Engineering on 26 Oct 2022)

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smoothly when the $C$ increased from 0% to 100% and the RI changed from 1.3325 to 1.4730, and the $f$ correspondingly decreases from 9.012 to 2.675 mm continuously and smoothly. The zoom ratio is close to 3.5.

For the experiment, the height of the capillary is longer than the PDMS substrate, the bottom of the capillary is sticking out and put a rubber tube on it tightly, and a clamp is used for sealing up the liquid. The glycerol aqueous solution with different concentrations was preconfigured. To change the liquid concentration, we need to let the original liquid outflow, clean and dry the

![Fig. 3](https://www.spiedigitallibrary.org/journals/Optical-Engineering)

**Fig. 3** ZEMAX simulation results of the designed micro-cylindrical lenses filled with variable $C$ and RI of the glycerol aqueous solution when the width of the incident light (entrance pupil) along $y$-direction is 0.8 mm. (a)–(d) The ray tracing drawings in the $y$–$z$ plane of the cylindrical lenses. (a′)–(d′) The corresponding $y$ aberration fan diagrams along tangential. (a) and (a′) $C = 0$, $n = 1.3325$, $f = 9.012$ mm; (b) and (b′) $C = 30\%$, $n = 1.3705$, $f = 5.337$ mm; (c) and (c′) $C = 60\%$, $n = 1.4119$, $f = 3.760$ mm; (d) and (d′) $C = 100\%$, $n = 1.4730$, $f = 2.675$ mm. (e) The ray tracing drawing in the $x$–$z$ plane of the cylindrical lenses filled with whatever kind of liquid showing no focusing ability along $x$-direction.
capillary, and then inject into another concentration liquid. Obviously, this mechanism of how to vary the liquid concentration is troublesome and cannot work in real optical equipment. Therefore, a product convenient to use is designed. We inject five different concentrations liquid (corresponding to five RI shown in Fig. 1) into the capillary separately, being separated by the PDMS reagent. The ends of the capillary are also sealed up by PDMS, forming a stable device. The thickness of each liquid layer is 0.8 mm, and the PDMS layer thickness is 0.1 mm, so the total height of the capillary or PDMS substrate is 4.6 mm. The focal length of the zoom micro-cylindrical lenses can be changed easily by keeping the aperture fixed and overall moving the PDMS substrate along x-axis, as shown in Fig. 1, with moving precision of ±0.1 mm, much lower requirements than mechanical zoom system. But this zoom mechanism sacrifices the continuity of the focal length.

Simulation results of the designed micro-cylindrical lenses for four different focal length structures based on the optical design software ZEMAX are shown in Fig. 3. Figures 3(a)–3(d) show the ray tracing drawings in the y–z plane of the cylindrical lenses filled with variable concentration (mass fraction) and RI of the glycerol aqueous solution when the width of the incident light (entrance pupil) along y-direction is 0.8 mm, presenting the lens’ zoom function visually. And Figs. 3(a′)–3(d′) show the corresponding y aberration fan diagrams along tangential, which show that the y-direction geometric distortion of the diffuse focal line is always <16 μm over the whole focal length range. The ray tracing drawing in the x–z plane of the cylindrical lenses filled with whatever kind of liquid is all same, as shown in Fig. 3(e), indicating cylindrical lenses have no ability to focus along the axial direction.

The root mean square (RMS) y radius of the focal line, Airy spot radius, and the peak-to-valley wavefront aberrations in y-direction at focal plane for the zoom cylindrical lenses over a focal length range from 2.675 to 9.012 mm is shown in Fig. 4, when the width of the aperture along y-direction is 0.8 mm. The higher value between the RMS radius and the Airy spot radius is always <8 μm, the same order of magnitude as the pixel size of commonly used image receiving devices. The peak-to-valley wavefront aberration in y-direction remains lower than λ/4, marked by the horizontal dotted line in Fig. 4, when the focal length ranges from 3.287 to 4.408 mm and from 6.725 to 9.012 mm, revealing relatively completed wavefront.

A set of modulation transfer function (MTF) curves for the cylindrical lenses with different zooming status are shown in Fig. 5. The MTF values in tangential direction for all the four zoom configurations are always higher than 0.95 at low spatial frequency (10 lp/mm), meaning high contrast of lenses. To match the resolution of usual charge coupled device or complementary metal oxide semiconductor, usually about 5μm/pixel, we set the high frequency considered at

![Fig. 4 RMS radius, Airy spot radius, and the peak-to-valley wavefront aberration in y-direction at focal plane are as functions of the effective focal length. The λ/4 wavefront aberration limit is denoted by the horizontal dotted line.](https://www.spiedigitallibrary.org/journals/Optical-Engineering)
100 lp/mm according to the Nyquist sampling law. The MTF values = 0.45, 0.50, 0.77, and 0.33 at high spatial frequency, when $f = 9.012$, 5.337, 3.760, and 2.675 mm, respectively. Figures 3–5 demonstrate that the image on the focal plane of the designed micro-cylindrical lenses maintains high quality for the complete zoom range.

4 Application of the Zoom Cylindrical Lenses

Laser using square fiber optics can generate excellent uniform beam for laser machining and plastic welding. Enabling it to match a zoom lens in the ability to change the aspect ratio and achieve variable anamorphic magnification properties can improve its applicability significantly. As shown in Figs. 6(a)–6(e), an anamorphic zoom system for uniform collimated beam is designed using two previously introduced liquid-filled zoom micro-cylindrical lenses structures setting as Kepler telescope, separated by the air. Assume $n_F$ and $n_R$ being the RI of the liquid filled in the front and rear PDMS structures, respectively. The respective corresponding change of $n_F$ and $n_R$ changes the power of the two structures in the Kepler system resulting in a change in aspect ratio. Setting a detector at 1.0 mm behind of the last surface of the rear PDMS structure, an image simulation of a uniform 0.6 mm square beam is shown for various anamorphic zoom ratios from 2:1 to 1:2 in Figs. 6(a’–6(e’). The anamorphic zoom system can realize the zoom shaping function without machine movement along optical axis, and the simulated images show sharp corners and edges, indicating there is no significant aberration of the system.
5 Conclusion

This paper introduced a spherical aberration free zoom liquid-filled micro-cylindrical lenses system based on a PDMS substrate (5.0 mm × 2.0 mm × 4.6 mm) in detail. The focal length of the optical system can be changed continuously and smoothly from 9.012 to 2.675 mm when variable concentration of glycerol aqueous solution with RI change from 1.3325 to 1.4730 injected.
into the capillary, and high imaging quality is verified in the whole zoom range. Two those zoom cylindrical lenses are combined to form an anamorphic zoom system, which can achieve various anamorphic zoom ratios from 2:1 to 1:2 for uniform beam. Although the mechanism of varying liquid concentration is inconvenient in experiment or the liquid concentration cannot be changed continuously using the layered product we designed, the zoom optical system has the advantages of small volume, good stability, and high imaging quality.

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**References**


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