Fast image-formation algorithm for ultrahigh-resolution airborne squint spotlight synthetic aperture radar based on adaptive sliding receive-window technique

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Abstract. Adaptive sliding receive-window (ASRW) technique was usually introduced in airborne squint synthetic aperture radar (SAR) systems. Airborne squint spotlight SAR varies its receive-window starting time pulse-by-pulse as a function of range-walk, namely, the linear term of range cell migration (RCM). As a result, a huge data volume of the highly squint spotlight SAR echo signal can be significantly reduced. Because the ASRW technique changes the echo-receive starting time and Doppler history, the conventional image algorithm cannot be employed to directly focus airborne squint spotlight ASRW-SAR data. Therefore, a fast image-formation algorithm, based on the principle of the wave number domain algorithm (WDA) and azimuth deramping processing, was proposed for accurately and efficiently focusing the squint spotlight ASRW-SAR data. Azimuth deramping preprocessing was implemented for eliminating azimuth spectrum aliasing. Moreover, bulk compression and modified Stolt mapping were utilized for high-precision focusing. Additionally, geometric correction was employed for compensating the image distortion resulting from the ASRW technique. The proposed algorithm was verified by evaluating the image performance of point targets in different squint angles. In addition, a detailed analysis of computation loads in the appendix indicates that the processing efficiency can be greatly improved, e.g., the processing efficiency could be improved by 17 times in the 70-deg squint angle by applying the proposed image algorithm to the squint spotlight ASRW-SAR data. © The Authors. Published by SPIE under a Creative Commons Attribution 3.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI. [DOI: 10.1117/1.JRS.8.083620]

Keywords: squint spotlight synthetic aperture radar; adaptive sliding receive-window technique; modified Stolt mapping; azimuth deramping.

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1 Introduction

Synthetic aperture radar (SAR) plays a significant role in remote sensing.1–3 Recently, airborne squint spotlight SAR has become one of the most important topics in microwave remote sensing, as it has the advantages of illuminating a particular area within a single pass of the platform by foreword/backward-squinted antenna and providing higher azimuth resolution by operating at a spotlight mode.4,5 Therefore, squint spotlight SAR currently has significant applications in intelligence surveillance, security monitoring, target detection, etc.6,7 However, the state-of-the-art ultrahigh-resolution airborne squint spotlight SAR has to deal with an extremely huge volume of raw data due to the large range cell migration (RCM) that results from the highly squinted angle and high-signal sampling rate corresponding to a high-range resolution. As a result, it leads to higher computation loads and lower efficiency in SAR image formation processing. In this article, the adaptive sliding receive-window (ASRW) technique was introduced for effectively reducing RCM in a -squinted airborne spotlight SAR system. In this system, its receive-window starting time varies pulse-by-pulse as a function of range-walk, namely, the linear term of RCM. Consequently, both data volume and
computation loads can be significantly reduced. As a consequence the squint SAR system makes real-time processing feasible, especially for the intelligence reconnaissance airborne SAR system. However, those classical squint SAR data image algorithms, such as the nonlinear chirp-scaling algorithm, extended nonlinear chirp-scaling algorithm, and wave number domain algorithm (WDA)

\(^8\)–\(^11\) cannot be employed to focus squint spotlight ASRW-SAR data directly for not considering the sliding receive-window effects.

In this article, a fast image-formation algorithm for ultrahigh-resolution airborne squint spotlight ASRW-SAR was proposed based on the principle of WDA and the azimuth deramping technique. Azimuth deramping preprocessing was implemented for eliminating azimuth spectrum aliasing.\(^12\) Moreover, WDA is a good method for focusing squint spotlight SAR data because of its high accuracy and efficiency.\(^1,13\) In the WDA part, bulk compression and modified Stolt mapping were utilized for implementing high-precision focusing. Geometric correction was also employed for compensating the geometric distortion that results from the ASRW technique.

In this article, Sec. 2 analyzes the airborne squint spotlight SAR in detail using the ASRW technique. Subsequently, a fast image-formation algorithm for airborne squint spotlight ASRW-SAR was proposed in Sec. 3. Then, the point targets simulation experiments were carried out in Sec. 4 to prove the validity of the proposed algorithm, and it was verified to have a high-processing accuracy and efficiency for squint spotlight ASRW-SAR data. Finally, conclusions are summarized in Sec. 5.

2 Airborne Squint Spotlight Synthetic Aperture Radar Using Adaptive Sliding Receive-Window Technique

Figure 1 shows the acquisition geometry of squint spotlight SAR. We define the rectangular coordinate system \(OXYZ\): the origin is the nadir at azimuth beginning time, the \(x\)-axis is parallel to the flight path and points along the azimuth direction, the \(y\)-axis is perpendicular to the \(x\)-axis on the ground and points along the range direction, and the \(z\)-axis provides a right-hand Cartesian coordinate system. As shown in Fig. 1, radar \(S\) is at \((x, 0, H)\), moving at speed \(v\). \(\varphi\) is the squint angle, \(r\) is the reference range, and \(x\) is the function of the azimuth time \(t\) which indicates the position of the airplane along the azimuth direction. \(C\) is the scene center and \(A\) is a certain target in the scene. Also, the azimuth distance between the scene center \(C\) and target \(A\) is \(x_A\), where \(x_A = vt_A\). \(R(t)\) denotes the instantaneous slant-range distance from the antenna phase center to a certain target \(A\).

Combining all the geometry, the instantaneous slant-range distance \(R(t)\) can be written as\(^1,9\)

\[
R(t) = \sqrt{r^2 + v^2(t - t_A)^2 - 2rv(t - t_A)\sin \varphi} = r - v(t - t_A)\sin \varphi + \frac{v^2(t - t_A)^2\cos^2 \varphi}{2r} + \ldots
\]

\[
= r + \underbrace{\frac{\lambda}{2} f_d(t - t_A)}_{\text{range - walk}} + \underbrace{\frac{\lambda}{4} f_r(t - t_A)^2}_{\text{range - curve}} + \ldots
\]

(1)

Fig. 1 Acquisition geometry of airborne squint spotlight synthetic aperture radar (SAR) mode.
where \( \lambda \) is the signal wavelength and \( f_d = -2v \times \sin \phi / \lambda \) and \( f_r = 2v^2 \cos^2 \phi / (\lambda r) \) are referred to as the Doppler centroid and Doppler rate, respectively. Figure 2 shows range-walk and range-curve variations as a function of the squint angle using the SAR system parameters listed in Table 1 (in Sec. 4). As range-walk is the prominent part of RCM in squint SAR, the removal of range-walk can be considered as the law of the ASRW technique. Also, the receive-window starting time has been changed as follows:

\[
\Delta T(t) = 2 \cdot \frac{1}{c} f_d (t - t_A) = \frac{2v(t - t_A) \sin \phi}{c} = \frac{2(x - x_A) \sin \phi}{c},
\]

where \( c \) is the velocity of light and \( x = vt \).

Figure 3 shows the operation mechanism for airborne squint spotlight SAR using the ASRW technique. The geometry of the squint spotlight SAR operating mode is demonstrated in Fig. 3(a). Also, the echo from the target can be received by sliding the receive window as the red lines show in Fig. 3(b). However, if the receive window is fixed, as the green lines in Fig. 3(c) show, the RCM closes to the receive window and the receive window must be long enough to receive all the RCM. As the RCM increases with high resolution and squint angle, the RCM may be outside the receive window. Figure 3(d) shows both of the two modes’ SAR echoes in two-dimensional (2-D) signal processor memory, and the red lines illustrate that the RCM is reduced. As the range-walk is removed, the RCM is reduced significantly. This implies that the ASRW technique reduces the received echo data volume and saves the storage. Besides, most of the reduced echo data are invalid for the RCM, which would not affect the final imaging results. From the view of information entropy, the ASRW technique

\[\text{Fig. 2 Range cell migration (RCM) variation as a function of squint angle.}\]

<table>
<thead>
<tr>
<th>Table 1 Synthetic aperture radar (SAR) parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>( \lambda )</td>
</tr>
<tr>
<td>( v )</td>
</tr>
<tr>
<td>( f_s )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \tau )</td>
</tr>
</tbody>
</table>
Actually performs a lossless compression in the data acquisition stage. Therefore, the ASRW technique has no adverse effect on the final imaging.

However, the receive window should start at the specified range of the sampling time due to the radar hardware design principle. This implies that the receive-window starting time is discrete

\[
\Delta T_{\text{dig}}(t) = \left\lfloor \frac{\Delta T(t)}{1/f_s} \right\rfloor \cdot f_s, \tag{3}
\]

where \(\lfloor \cdot \rfloor\) is an integral function. Also, a time-shift of the starting time results from \(\Delta T_{\text{dig}}(t)\) is given by

\[
\Delta T_{\text{shift}}(t) = \Delta T(t) - \Delta T_{\text{dig}}(t). \tag{4}
\]

So, the phase error, caused by \(\Delta T_{\text{shift}}(t)\), can be compensated for by Eq. (5) before the imaging focusing process. Then, the continuous variable \(\Delta T(t)\) will still be used in the subsequent mathematical derivation

\[
\Omega_1(f_r, t) = \exp\{j2\pi f_r \cdot \Delta T_{\text{shift}}(t)\}, \tag{5}
\]

where \(f_r\) is range frequency.
3 Adaptive Sliding Receive-Window Synthetic Aperture Radar Fast Image-Formation Algorithm

Three components were proposed in this section: First, the expression of 2-D matched filtering function in the wave number domain was derived in detail. Then, based on the principle of WDA, a new Stolt-mapping relationship was illustrated according to the 2-D matched filtering function. Finally, a presentation of the process flow of the fast image-formation algorithm was given, which was proven to focus the ultrahigh-resolution airborne squint spotlight ASRW-SAR data accurately and efficiently.

3.1 Two-Dimensional Matched Filtering Function

In the subsequent analysis, the antenna beam pattern, backscatter coefficient, and other nonessential amplitude factors have been corrected. So, the point-target spectrum of the innovative mode can be given by

\[ S(\tau, x) = p \left[ \frac{2R(x)}{c} - \frac{2(x - x_A) \sin \varphi}{c} \right] \cdot \exp \left\{ -j2b \left[ \frac{2R(x)}{c} - \frac{2(x - x_A) \sin \varphi}{c} \right]^2 \right\} \cdot \exp \left\{ -j\frac{4\pi R(x)}{\lambda} \right\}, \]  

where \( p[\cdot] \) is the signal envelope, \( b \) is the range chirp FM rate, and \( \tau \) is the range time. Simply put, after range compression, the 2-D matched filtering function signal \( h_r(\tau, x) \) is given by adopting the impulse response function instead of the sinc function as follows:

\[ h_r(\tau, x) = \delta \left( \tau - \frac{2\Delta R(x)}{c} - \frac{2(x - x_A) \sin \varphi}{c} \right) \cdot \exp \left\{ -j\frac{4\pi f_c \Delta R(x)}{c} \right\}. \]  

where \( \Delta R(x) = R(x) - r \) and \( f_c \) denotes the carrier frequency. Because the position of the range-compressed signal is changed with \( \Delta T(i) \), the wave number distribution varied accordingly. By applying the principle of stationary phase (POSP) to Eq. (7), \( h_r(\tau, x) \) is changed from the range time domain to the range frequency domain

\[ h_r(f_c, x) = \exp \left\{ -j4\pi \frac{f_c}{c} (\Delta R(x) + x \sin \varphi) \right\} \cdot \exp \left\{ j4\pi \frac{f_c x}{c} \sin \varphi \right\} \cdot \exp \left\{ j\Phi_1(f_c, x_A) \right\}, \]  

where \( \Phi_1(f_c, x_A) = 4\pi f_c x_A \sin \varphi / c \). As \( \Phi_1(f_c, x_A) \) is independent of \( x \), it can be compensated for by a matched filter in the range frequency and azimuth time domain. Therefore, \( h_r(f_c, x) \) can be rewritten after compensation as follows:

\[ h_r(f_c, x) = \exp \left\{ -j4\pi \frac{f_c}{c} (\Delta R(x) + x \sin \varphi) \right\} \cdot \exp \left\{ j4\pi \frac{f_c x}{c} \sin \varphi \right\}. \]  

where the azimuth wave number \( k_x = 2\pi f_d / v \), \( k_0 = 2\pi f_d / v \), the range wave number \( k_{\text{rc}} = 4\pi (f_c + f_d) / c \), and \( f_d \) is the Doppler frequency. Then, the signal \( h_r(f_c, x) \) is converted to the range frequency and azimuth wave number domain

\[ H_r(f_c, k_x) = \int \exp \left\{ -jk_{rc}(\Delta R(x) + x \sin \varphi) \right\} \cdot \exp \left\{ -j2\pi f_d \frac{x}{v} \right\} \exp \left\{ -jk_{ac}x \right\} dx. \]  

Substituting \( k_0 = 2\pi f_d / v \) into Eq. (10),

\[ H_r(f_c, k_x + k_0) = \int \exp \left\{ -jk_{rc}(\Delta R(x) + x \sin \varphi) \right\} \cdot \exp \left\{ -j(k_x + k_0)x \right\} dx. \]
In squint mode, the center of the azimuth wave number is \(-k_0x\), and \(k_x \in [-\pi(f_{PRF} - f_d)/v, \pi(f_{PRF} - f_d)/v]\) in Eq. (11), where \(f_{PRF}\) is equal to PRF. By using variable substitution, Eq. (11) can be written as follows:

\[
H_r(f, k_x) = \int \exp\{-jk_r c (\Delta R(x) + x \sin \varphi)\} \cdot \exp\{-jk_x x\} dx,
\]

(12)

where \(k_x \in (-\pi f_{PRF}/v, \pi f_{PRF}/v)\). Then, by using POSP, the stationary phase point \(x_0\) is given by

\[
x_0 = r \sin \varphi - \frac{(k_{rc} \sin \varphi + k_x)r \cos \varphi}{\sqrt{k_{rc}^2 - (k_{rc} \sin \varphi + k_x)^2}} - x_A.
\]

(13)

So, the expression of the 2-D matched filtering function in the range frequency domain and azimuth wave number domain is

\[
H_r(f, k_x) = \exp\{-jr [\cos \varphi \sqrt{k_{rc}^2 - (k_{rc} \sin \varphi + k_x)^2} + \sin \varphi (k_{rc} \sin \varphi + k_x) - k_{rc}]\}. \tag{14}
\]

Then, taking FFT of Eq. (14) with respect to \(r\), we can get the expression of the 2-D matched filtering function in the wave number domain as

\[
H_r(k_r, k_x) = \delta(k_r + \cos \varphi \sqrt{k_{rc}^2 - (k_{rc} \sin \varphi + k_x)^2} + \sin \varphi (k_{rc} \sin \varphi + k_x) - k_{rc}). \tag{15}
\]

### 3.2 New Stolt-Mapping Relationship

Several modified Stolt-mapping relationships have been proposed in Refs. 14 and 15, which rewrite the expression of RCM. But they do not essentially change the mapping relationship between the frequency domain and wave number domain. For the image-formation algorithm of the squint spotlight ASRW-SAR, a new mapping relationship should be found immediately due to the sliding receive window. Assuming the range compress signal is \(p(f_r, k_x)\), an IFFT was implemented with respect to \(f_r\) after the matched filtering operation

\[
P(r, k_x) = \int p(f_r, k_x) H^*_r(f_r, k_x) \exp\left(j \frac{4\pi f_r}{c} r\right) df_r,
\]

(16)

where \(H^*_r(f_r, k_x)\) is the complex conjugate of \(H_r(f_r, k_x)\). Then, by applying FFT with respect to \(r\)

\[
P(k_r, k_x) = \int \int p(f_r, k_x) H^*_r(f_r, k_x) \cdot \exp\left(j \frac{4\pi f_r}{c} r\right) \cdot \exp(-j k_r r) df_r df_r.
\]

(17)

Substituting Eq. (15) into Eq. (17),

\[
P(k_r, k_x) = \int \delta \left(k_r - \left[\cos \varphi \sqrt{k_{rc}^2 - (k_{rc} \sin \varphi + k_x)^2} + \sin \varphi (k_{rc} \sin \varphi + k_x) - \frac{4\pi f_r}{c}\right]\right)
\]

\[
\cdot p(f_r, k_x) df_r.
\]

(18)

So, a new mapping relationship is illustrated as follows

\[
k_r = \cos \varphi \sqrt{k_{rc}^2 - (k_{rc} \sin \varphi + k_x)^2} + \sin \varphi (k_{rc} \sin \varphi + k_x) - \frac{4\pi f_r}{c}.
\]

(19)

According to the new mapping relationship, it is easy to convert \(p(f_r, k_x)\) from the range frequency domain to the 2-D wave number domain \(p(k_r, k_x)\) by interpolation. Finally,
3.3 Processing Scheme of Fast Image-Formation Algorithm

As Fig. 4 shows, the processing steps of the fast image-formation algorithm are divided into six stages as follows: time-shift compensation, azimuth deramping, data transform, bulk compression, modified Stolt mapping, and geometric correction.

As analyzed in Sec. 2, the time-shift $\Delta T_{\text{shift}}(t)$ causes a phase error. So, the first step of the proposed algorithm is to compensate for the phase error as in Eq. (5).

The overlapped azimuth spectrum limits the direct application of the available stripmap imaging algorithm in the spotlight SAR mode. To resolve the problem, azimuth deramping is adopted to eliminate the azimuth spectrum aliasing. Azimuth deramping shares the same processing flow as in Ref. 16.

The data transform stage includes four processing steps, namely, range FFT, phase compression, azimuth FFT, and residual phase compensation. For ASRW-SAR, the receive-window starting time is changed pulse-by-pulse in the azimuth direction. So, the Doppler history is variable in azimuth, which means that the target located only in the scene center is precisely focused. As analyzed in Sec. 3.1, in order to guarantee the focusing accuracy of all the targets located in different azimuth positions, the third exponential phase term in Eq. (8) should be compensated for, which is given by

$$\Omega_2(f_r, x) = \exp \left\{-j \frac{4\pi f_r x_A \sin \varphi}{c}\right\}.$$  (20)

Only the targets apart from the scene center at the distance $x_A$ in azimuth direction will be precisely focused. To ensure the imaging quality over the whole scene, a method based on block...
processing was proposed in Fig. 5. Echo data can be divided into \( n \) blocks along the azimuth direction, and each block can be compensated for by using different \( x_A \) values.

Moreover, after performing the azimuth FFT, the residual phase caused by azimuth deramping processing\(^{16} \) should be compensated for by

\[
\Omega_3(f_a) = \exp\left(\frac{j\pi f_a^2}{4f_r}\right). \tag{21}
\]

In the bulk compensation stage, the reference function is different from the traditional function in Ref. 1. Therefore, in a practical procedure, the reference function in bulk compensation is given by

\[
\Omega_4(f_r, k_x) = \exp\left(jr \cdot \cos \varphi \sqrt{k_r^2 - (k_c \sin \varphi + k_x)^2} + \sin \varphi (k_c \sin \varphi + k_x)\right) \cdot \exp\left(-j \frac{4\pi f_r r_{\min}}{c}\right), \tag{22}
\]

where \( r_{\min} \) is minimum slant range, \( r_{\min} = r - c \cdot N_r / (2f_s) \), and \( N_r \) denotes the range resample point. In Eq. (22), the first exponential term compensates for the phase at the reference slant range as the 2-D matched filtering function \( H_r(f_r, k_x) \) shows in Eq. (14). The second exponential term finishes the range compression and the third exponential term allows for redundant phase compensation, which are all caused by digital processing. After the bulk compression, the target at the reference range is properly focused, but a residual phase still exists for the targets at other ranges.

Different from the conventional Stolt mapping, the proposed Stolt mapping is modified as in Eq. (19) due to the change in the data-mapping relationship caused by the ASRW technique. The modified Stolt mapping performs the differential RCMC, differential SRC, and differential azimuth compression. After the modified Stolt mapping processing, the whole scene is accurately focused.

The last processing part is the geometric correction, which compensates for the change in the receive-window starting time caused by the ASRW technique. After the modified Stolt mapping, the azimuth IFFT is performed to transform the data into the azimuth time and range frequency domain. After that, geometric distortion is corrected by

\[
\Omega_5(f_r, x) = \exp\left\{j 4\pi f_r \frac{\sin \varphi}{c} x\right\}. \tag{23}
\]

Finally, a range IFFT is applied to get the fine-focusing and a nondistorted SAR image.

4 Experimental Results

In this section, a Ka-band linear frequency modulation transmission signal is adopted for simulation, and the azimuth resolution \( \rho_a \) is 0.1 m. First, to prove the effectiveness of the ASRW technique, the experiments comparing the raw echo between the ASRW-SAR and conventional-SAR are carried out. Then, to verify the validity of the proposed image-formation algorithm,
the experiments with point targets imaging experiments are carried out. The specific simulation parameters are listed in Table 1, and the simulation scene (500 × 500 m², resulting in 5000 × 5000 pixels in the SAR image) is illustrated by Fig. 6.

In ASRW-SAR, the ASRW technique changes the starting time of the receive window in each azimuth direction by removing the range-walk. Also, the length of the range dimension is dependent on the RCM, whose main part is range-walk. Figure 7 shows the raw echo data of ASRW-SAR and conventional-SAR systems at different squint angles ($\phi = 40 \times \text{deg}$ / $70 \times \text{deg}$). Also, the range sample number in a conventional-SAR and ASRW-SAR are $N_r$ and $N'_r$, respectively. As shown in Fig. 7, the RCM is reduced by ASRW, and the range sampling number decreases significantly. Further, the higher the squinted angle, the bigger the range sampling number scaling and the more immediate the demand of ASRW.

In the imaging experiments part, the imaging results of a conventional WDA and the proposed imaging algorithm are shown in Fig. 8. The comparison results of the proposed imaging algorithm with a conventional WDA shows that the proposed method is effective. Also, Figs. 9 and 10 show the interpolated contour plots of the corresponding focusing targets A, C, and E in different squint angles. The spatial resolution (azimuth resolution $\rho_a$, range resolution $\rho_r$), peak side lobe ratio, and integrated side lobe ratio of the three targets are compared.

**Fig. 6** Simulation scene.

**Fig. 7** Comparing the echo data of ASRW-SAR and conventional SAR systems at different squint angles. (a) conventional SAR, $\phi = 40 \times \text{deg}$, $N_r = 32768$, (b) ASRW SAR, $\phi = 40 \times \text{deg}$, $N'_r = 8192$, (c) conventional SAR, $\phi = 70 \times \text{deg}$, $N_r = 262144$, and (d) ASRW SAR, $\phi = 70 \times \text{deg}$, $N'_r = 16384$.
point targets processing results are listed in Table 2. Both the contour plots and image evaluation results demonstrate that the fine-focusing performance of the proposed improved image-formation algorithm and all the evaluation results indicate that the image results are slightly affected by the spatial variant slant range. Besides, the comparison of the computation load of conventional-SAR and ASRW-SAR is represented in the Appendix, and the computation efficiency is $\gamma = 4.1/17.0$ in $\varphi = 40\text{ deg}/70\text{ deg}$. The advantage of less-computation load indicates its potential suitability in a real-time imaging system.

$\rho_r$ denotes the slant range resolution.

Fig. 8 Comparing imaging results of conventional WDA and improved algorithm. (a) Conventional WDA. (b) Proposed imaging algorithm.

Fig. 9 Imaging results of the improved algorithm in $\varphi = 40\text{ deg}$. (a) Target A. (b) Target C. (c) Target E.

Fig. 10 Imaging results of the improved algorithm in $\varphi = 70\text{ deg}$. (a) Target A. (b) Target C. (c) Target E.
5 Conclusion

In this article, an ASRW technique was introduced for efficiently acquiring the airborne highly squint spotlight SAR echo. The airborne squint spotlight ASRW-SAR varies its receive-window starting time pulse-by-pulse as a function of range-walk, namely, the linear term of RCM. So, the RCM of ASRW-SAR is reduced dramatically and the range sampling number is decreased, which means the data volume and the computation loads are reduced. Therefore, the ASRW technique decreases the SAR system complexity, and has great potential value in real-time imaging systems.

In order to deal with the varying receive-window starting times in airborne squint spotlight ASRW-SAR, a fast image-formation algorithm is presented for highly precise focusing of the ultrahigh-resolution (e.g., 0.1 m) airborne squint spotlight ASRW-SAR data. First, azimuth deramping preprocessing is performed to eliminate the azimuth spectrum aliasing. Then, bulk compression and modified Stolt mapping are utilized for precise focusing. At last, geometric correction is implemented to remove the effect of the varying receive-window starting time by the ASRW technique. The validity and accuracy of the proposed algorithm have been demonstrated by point targets simulation data. Also, the processing results of point targets at different squint angles indicate that the proposed algorithm is suitable for high-resolution and high squint angle airborne spotlight SAR image formation. Further, the computation load is reduced, which would be more markedly decreased in high squint angle, e.g., 17 times improvement of computation efficiency at a 70-deg squint angle.

### Appendix: Comparison of Computation Load of Conventional-SAR and ASRW-SAR

The number of multiplications is adopted as a computation load indicator. Let the interpolation kernel length be $M_{\text{ken}}$. Also, the azimuth sample numbers in conventional SAR and ASRW-SAR are $N_a$ and $N_a'$, respectively. The particular computational load in each imaging stage is listed in Table 3.

$C_{\text{con}}$ denotes the entire computational load for conventional-SAR.

$$
C_{\text{con}} = (12 + 2M_{\text{ken}})N_aN_r + 6N_aN_r\log_2 N_a + 4N_aN_r\log_2 N_r. \quad (24)
$$

### Table 2 Targets A, B, and C evaluation results in different squint angles.

<table>
<thead>
<tr>
<th></th>
<th>Azimuth</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_a$ (m)</td>
<td>Peak side lobe ratio (PSLR) (dB)</td>
</tr>
<tr>
<td>A</td>
<td>0.100</td>
<td>−13.28</td>
</tr>
<tr>
<td>B</td>
<td>0.100</td>
<td>−13.26</td>
</tr>
<tr>
<td>C</td>
<td>0.100</td>
<td>−13.26</td>
</tr>
<tr>
<td>D</td>
<td>0.100</td>
<td>−13.26</td>
</tr>
<tr>
<td>E</td>
<td>0.100</td>
<td>−13.27</td>
</tr>
<tr>
<td>A</td>
<td>0.102</td>
<td>−12.88</td>
</tr>
<tr>
<td>B</td>
<td>0.101</td>
<td>−12.86</td>
</tr>
<tr>
<td>C</td>
<td>0.101</td>
<td>−12.85</td>
</tr>
<tr>
<td>D</td>
<td>0.101</td>
<td>−12.87</td>
</tr>
<tr>
<td>E</td>
<td>0.103</td>
<td>−12.89</td>
</tr>
</tbody>
</table>

$\varphi = 40$ deg

$\varphi = 70$ deg
Also, the entire computational load $C_{ASRW}$ for ASRW-SAR is expressed as

\[
C_{ASRW} = (16 + 2M_{ken})N_aN_r + 6N_aN_r\log_2 N_a + 4N_aN_r\log_2 N_r.
\]  

(25)

Assume that the computational efficiency $\gamma$ is defined as

\[
\gamma = \frac{C_{con}}{C_{ASRW}}.
\]  

(26)

With the condition of the same parameters as listed in Table 1, $N_a$ and $N'_a$ are equivalent, $N_a = N'_a = \left[ (v \cdot \text{PRF})/(\rho_a \cdot f_r) \right]$ assuming an 8-point interpolation is adopted ($M_{ken} = 8$). And in different squint angle $\varphi = 40 \text{ deg} / 70 \text{ deg}$, $N_a$ (or $N'_a$) is equal to 17,500/87,792. Also, the $N_r$ and $N_r'$ are listed in Fig. 6 in different squint angles. Therefore, the computation efficiency $\gamma$ is calculated by Eq. (26) in different squint angles, and the final value $\gamma$ is listed in Table 2.

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### References


*Table 3* Computation load comparison.

<table>
<thead>
<tr>
<th></th>
<th>Conventional SAR</th>
<th>ASRW-SAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth deramping</td>
<td>$8N_aN_r + 2N_aN_r\log_2 N_a$</td>
<td>$8N_aN_r + 2N_aN_r\log_2 N'_a$</td>
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<tr>
<td>Range FFT</td>
<td>$2N_aN_r\log_2 N_r$</td>
<td>$2N_aN_r\log_2 N'_a$</td>
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<td>Azimuth FFT</td>
<td>$2N_aN_r\log_2 N_a$</td>
<td>$2N_aN_r\log_2 N'_a$</td>
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<tr>
<td>Bulk compression</td>
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<td>$4N_aN'_r$</td>
</tr>
<tr>
<td>Stolt mapping</td>
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<td>$2M_{ken}N_aN'_r$</td>
</tr>
<tr>
<td>Azimuth IFFT</td>
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<td>$2N_aN_r\log_2 N'_a$</td>
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<tr>
<td>Geometric correction</td>
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<td>$4N_aN'_r$</td>
</tr>
<tr>
<td>Range IFFT</td>
<td>$2N_aN_r\log_2 N_r$</td>
<td>$2N_aN_r\log_2 N'_a$</td>
</tr>
</tbody>
</table>


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