Extraction of effective parameters of anisotropic optical materials using a decoupled analytical method

Thi-Thu-Hien Pham
Yu-Lung Lo
Extraction of effective parameters of anisotropic optical materials using a decoupled analytical method

Thi-Thu-Hien Pham and Yu-Lung Lo

Abstract. A decoupled analytical technique based on the Mueller matrix method and the Stokes parameters is proposed for extracting effective parameters of anisotropic optical materials in linear birefringence (LB), linear dichroism (LD), circular birefringence (CB), and circular dichroism (CD) properties. This technique is essential in determining the optical properties of opto-electric or biomedical materials for the development of advanced inspection and/or diagnostic applications. The error and resolution analysis of the proposed approach is demonstrated by extracting the effective parameters given an assumption of errors ranging ±0.005 in the values of the output Stokes parameters. The results confirm the ability of the proposed method to yield full-range measurements of all the optical parameters. The decoupled nature of the analytical model yields several important advantages, including an improved accuracy and the ability to extract the parameters of optical samples with only LB, CB, LD, or CD property without using compensation technique or pretreatment. Moreover, by decoupling the extraction process, the “multiple solutions” problem inherent in previous models presented by the current group is avoided.

Keywords: dichroism; birefringence; polarimetry; Mueller matrix; Stokes polarimeter.

1 Introduction

Methods for accurately determining the optical properties of opto-electric materials or biosamples are essential in facilitating the development of advanced inspection and/or diagnostic applications. For example, linear birefringence (LB) measurements provide a useful insight into the characteristics of liquid crystal display (LCD) compensator films or the photo-elasticity of human tissue, while circular birefringence (CB) measurements of human blood provide a reliable indication of diabetes. Similarly, linear dichroism (LD) measurements of human tissue can facilitate tumor diagnosis, while circular dichroism (CD) measurements are an effective means of characterizing and classifying protein structures.

CD analyses provide a reliable means of classifying different proteins. In addition, CD spectroscopy is also extensively used to probe a wide range of optically active (chiral) materials, ranging from small molecules to macromolecules. For example, Kuroda et al. designed and constructed a CD spectrophotometer (USC-1: J-800KCM) featuring a special sample holder designed to eliminate the parasitic artifacts caused by macroscopic anisotropies, such as LB and LD, that are unique to solid state samples. In a later study, the same group presented a universal chirooptical spectrophotometer (UCS-2: J-800KCMF) for the in-situ chirality measurement of solid samples. As in the spectrophotometer proposed in Kuroda et al., the signal output arising from macroscopic anisotropies unique to the solid state were removed using two lock-in amplifiers. The effectiveness of the proposed device was demonstrated by measuring the microcrystallites of both enantiomers of ammonium camphorsulfonate. However, while the spectrophotometer was capable of measuring both the CD and the CB properties of anisotropic optical samples, the Mueller matrix formulations used to extract the optical parameters were not decoupled. As a result, the device was restricted to samples with pure CD properties only or pure CB properties only.

Asahi and Kobayashi proposed an ellipsometry method using the general high accuracy universal polarimeter (HAUP) theory for determining the LB/CB and LD/CD of an anisotropic optically active material. The experimental results of the proposed approach showed that the LB and LD, CB and CD properties of crystals using a polarimetric imaging technique and an analytical model based on a Jones calculus formulation. However, in characterizing the samples, different tools were used for each optical property. For example, the LB and LD properties were extracted using a Metripol microscope, while the CB properties were analyzed using a HAUP or Scanning-HAUP, and the CD properties were examined using a circular dichroism imaging microscope. As a result, the optical parameters were not decoupled in the analytical model. That is, in extracting each optical property, the knowledge of one or more of the other properties was required. Consequently, the precision of the extraction results was highly sensitive to the effects of accumulated errors.

In addition, Chenault and Chipman proposed a technique for measuring the LD and LB spectra of infrared solid materials in transmission based on the Mueller matrix decomposition method. In the proposed approach, the intensity of the detected
signal was modulated by rotating the sample, and the LD and LB properties were calculated from the Fourier series coefficients of the detected signal at each wavelength. However, in extracting the sample parameters, an assumption was made that the principal birefringence axis and diattenuation axis were aligned. Ghosh et al.\textsuperscript{20-23} proposed a method using a Mueller matrix decomposition method to extract the polarization properties (linear retardance, optical rotation angle, diattenuation, and depolarization coefficient) of a complex turbid media. In the experimental studies, a photoelastic modulation polarimeter was used to record Mueller matrices from polycrylamide phantoms having strain-induced birefringence, sucrose-induced optical activity, and polystyrene microspheres-induced scattering. Wang and his group\textsuperscript{24} also presented comparisons of Mueller matrix elements of light backscattering from birefringent anisotropic turbid media containing glucose in single-scattering model and a double-scattering model with the Monte Carlo model. Huang and Knighton\textsuperscript{25,26} proposed a method to measure the diattenuation spectrum and the birefringence of the isolated rat retinal nerve fiber layer (RNFL) for glaucoma diagnosis using scanning laser polarimetry at wavelengths from 440 to 830 nm. The degree of polarization for reflection from the RNFL was also measured. It is noted that the above studies\textsuperscript{20-26} were based on the Mueller matrix method; studies by Ghosh et al. and Wood et al.\textsuperscript{20-23} were based on the Mueller matrix polar decomposition method in Lu and Chipman.\textsuperscript{27} Additionally, differential matrix formalism for an anisotropic medium in parallelism with Jones’ matrix formalism was proposed by Azzam.\textsuperscript{28} Ossikovski\textsuperscript{35} extended the differential matrix formalism including depolarizing media. Ortega-Quijano and Arce-Diego\textsuperscript{30,31} also proposed the differential Mueller matrices for general depolarizing media for measurements in transmission and backward direction.

Recently, Chen et al.\textsuperscript{32} proposed a technique for measuring the effective LB and LD properties of an optical sample using an analytical model based on the Mueller matrix and the Stokes parameter. More recently, the same group extended the proposed analytical model based on the Mueller matrix and the Stokes parameter to measure the effective LB, LD, and CB properties of an optical sample using an analytical model of the anisotropic material, it is assumed that the CD and LB components of the sample are positioned in front of the CB and LB components.\textsuperscript{34,35} For the nondepolarizing Mueller matrix, Ossikovski\textsuperscript{35} proposed that the decomposition allows for a straightforward interpretation and parameterization of an experimentally determined Mueller matrix in terms of an arrangement of polarization devices and their characteristic parameters: diattenuations, retardances, and axis azimuths.

According to Chen et al., Lo et al.,\textsuperscript{33} and Savenko et al.,\textsuperscript{36} the Mueller matrix for a LB material with an orientation angle $\alpha$ and phase retardance $\beta$ can be expressed as

$$M_{\theta \beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(4\alpha)\sin^2(\beta/2) + \cos^2(\beta/2) & \sin(4\alpha)\sin^2(\beta/2) & -\sin(2\alpha)\sin(\beta) \\ 0 & \sin(4\alpha)\sin^2(\beta/2) + \cos^2(\beta/2) & -\cos(4\alpha)\sin^2(\beta/2) + \cos^2(\beta/2) & \cos(2\alpha)\sin(\beta) \\ 0 & \sin(2\alpha) & -\cos(2\alpha) & \cos(\beta) \end{pmatrix}.$$  \hspace{1cm} (1)

The Mueller matrix for a CB material with an optical rotation angle $\gamma$ can be expressed as

$$M_{\theta \gamma} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\gamma) & \sin(2\gamma) & 0 \\ 0 & -\sin(2\gamma) & \cos(2\gamma) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$  \hspace{1cm} (2)

Meanwhile, the Mueller matrix for a LD material with an orientation angle $\theta_d$ and linear dichroism $D$ has the form
Finally, the Mueller matrix for a CD material with circular
dichroism $R$ has the form

$$M_{cd} = \begin{bmatrix}
1 + R^2 & 0 & 0 & 2R \\
0 & 1 - R^2 & 0 & 0 \\
2R & 0 & 1 - R^2 & 0 \\
0 & 0 & 0 & 1 + R^2
\end{bmatrix}. \quad (4)$$

Note that $R$ is equal to $(r_R - r_L)/(r_R + r_L)$, where $r_R$ and $r_L$ are the absorptions of right- and left-hand circular polarization light, respectively.\(^\text{13}\) Table 1 presents the symbols and definitions of six effective parameters used in the proposed study.\(^\text{12,37,38}\)

From Eqs. (1)–(4), it follows that to characterize an anisotropic optical material with hybrid properties, it is necessary to extract a total of six effective parameters. Figure 1 presents a schematic illustration of the setup used in the present study to extract these parameters for a typical anisotropic sample. Note that $P$ and $Q$ are a polarizer and quarter-wave plate, respectively, and are used to produce various linear and circular polarization lights, while $S_c$ and $S_l$ are the input and output Stokes vectors, respectively.

The output Stokes vector in Fig. 1 can be calculated as

$$S_c = M_{lb}M_{cb}M_{ld}M_{cd}S_l$$

where $M_{lb}$, $M_{cb}$, and $M_{cd}$ are the Mueller matrices corresponding to the effective LD, LB, CB, and CD properties of the anisotropic sample, respectively. In Eq. (5), elements $m_{11} - m_{44}$ are all non-zero. As a result, solving the Mueller matrix product is highly complex. Accordingly, in the present study, a method is proposed for extracting the effective LD/CD of the sample using only elements $m_{11}$, $m_{12}$, $m_{13}$, and $m_{14}$. In the setup shown in Fig. 1, the sample is illuminated using six different input polarization lights, namely four linear polarization lights (i.e., $S_{00} = [1, 1, 0, 0]^T$, $S_{45} = [1, 0, 1, 0]^T$, $S_{90} = [1, -1, 0, 0]^T$, and $S_{135} = [1, 0, -1, 0]^T$)

\(^{13}\)n is refractive index, $\mu$ is absorption coefficient, $l$ is path length through the medium (thickness of material), $\lambda_0$ is vacuum wavelength. Subscripts $f$ and $s$ represent the fast and slow linearly polarized waves when neglecting the circular effects, and $+ \text{ and } -$ the right and left circular polarized waves when neglecting the linear effects.

### Table 1 Symbols and definitions of six effective parameters\(^\text{12,37,38}\)

| Name | Symbol | Range | Definition
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Orientation angle of fast axis of LB</td>
<td>$\alpha$</td>
<td>[0 deg, 180 deg]</td>
<td>$2\pi(n_i - n_s)/\lambda_0$</td>
</tr>
<tr>
<td>Linear birefringence of LB</td>
<td>$\beta$</td>
<td>[0 deg, 360 deg]</td>
<td>$2\pi(n_i - n_s)/\lambda_0$</td>
</tr>
<tr>
<td>Optical rotation of CB</td>
<td>$\gamma$</td>
<td>[0 deg, 180 deg]</td>
<td>$2\pi(n_i - n_s)/\lambda_0$</td>
</tr>
<tr>
<td>Orientation angle of LD</td>
<td>$\theta_d$</td>
<td>[0 deg, 180 deg]</td>
<td>$2\pi(\mu_i - \mu_s)/\lambda_0$</td>
</tr>
<tr>
<td>Linear dichroism of LD</td>
<td>$D$</td>
<td>[0, 1]</td>
<td>$2\pi(\mu_i - \mu_s)/\lambda_0$</td>
</tr>
<tr>
<td>Circular dichroism of CD</td>
<td>$R$</td>
<td>[-1, 1]</td>
<td>$2\pi(\mu_i - \mu_s)/\lambda_0$</td>
</tr>
</tbody>
</table>
and two circular polarization lights (i.e., right-handed \( \vec{S}_{RHC} = [1, 0, 0, 1]^T \) and left-handed \( \vec{S}_{LHC} = [1, 0, 0, -1]^T \)). The corresponding output Stokes vectors are obtained from Eq. (5) as

\[
S_0 = \begin{bmatrix} m_{11} + m_{12}, & m_{21} + m_{22}, & m_{31} + m_{32}, & m_{41} + m_{42} \end{bmatrix}^T \\
S_{45^\circ} = \begin{bmatrix} m_{11} + m_{13}, & m_{21} + m_{23}, & m_{31} + m_{33}, & m_{41} + m_{43} \end{bmatrix}^T \\
S_{90^\circ} = \begin{bmatrix} m_{11} - m_{12}, & m_{21} - m_{22}, & m_{31} - m_{32}, & m_{41} - m_{42} \end{bmatrix}^T \\
S_{135^\circ} = \begin{bmatrix} m_{11} - m_{13}, & m_{21} - m_{23}, & m_{31} - m_{33}, & m_{41} - m_{43} \end{bmatrix}^T \\
S_{RHC} = \begin{bmatrix} m_{11} + m_{14}, & m_{21} + m_{24}, & m_{31} + m_{34}, & m_{41} + m_{44} \end{bmatrix}^T \\
S_{LHC} = \begin{bmatrix} m_{11} - m_{14}, & m_{21} - m_{24}, & m_{31} - m_{34}, & m_{41} - m_{44} \end{bmatrix}^T
\]

where

\[
m_{11} = \frac{1}{2} \left( \frac{1 + 1 - D}{1 + D} \right) (1 + R^2) \tag{12}
\]

\[
m_{12} = \frac{1}{2} \left( \frac{1 - 1 - D}{1 + D} \right) \cos(2\theta_d)(1 - R^2) \tag{13}
\]

\[
m_{13} = \frac{1}{2} \left( \frac{1 - 1 - D}{1 + D} \right) \sin(2\theta_d)(1 - R^2) \tag{14}
\]

\[
m_{14} = \left( 1 + \frac{1 - D}{1 + D} \right) R. \tag{15}
\]

From the values of \( m_{11}, m_{12}, m_{13}, \) and \( m_{14} \), Eqs. (12)–(15) are then used to obtain the LD and CD properties of the sample. Specifically, the orientation angle \( \theta_d \) of LD is obtained as

\[
2\theta_d = \tan^{-1} \left( \frac{S_{45^\circ}(S_0) - S_{135^\circ}(S_0)}{S_{0^\circ}(S_0) - S_{90^\circ}(S_0)} \right). \tag{16}
\]

From Eqs. (12)–(15), the linear dichroism \( D \) is obtained as either

\[
D = \frac{\left[ S_{0^\circ}(S_0) - S_{90^\circ}(S_0) \right]^2 + \left[ S_{45^\circ}(S_0) - S_{135^\circ}(S_0) \right]^2}{\left[ S_{0^\circ}(S_0) + S_{90^\circ}(S_0) \right]^2 - \left[ S_{RHC}(S_0) - S_{LHC}(S_0) \right]^2}. \tag{17}
\]

or

\[
D = \frac{\left[ S_{45^\circ}(S_0) - S_{135^\circ}(S_0) \right]}{\sin(2\theta_d) \left[ S_{0^\circ}(S_0) + S_{90^\circ}(S_0) \right]^2 - \left[ S_{RHC}(S_0) - S_{LHC}(S_0) \right]^2}. \tag{19}
\]

Again, the circular dichroism \( R \) is obtained as

\[
R = \frac{S_{0^\circ}(S_0) + S_{90^\circ}(S_0) - \sqrt{\left[ S_{0^\circ}(S_0) + S_{90^\circ}(S_0) \right]^2 - \left[ S_{RHC}(S_0) - S_{LHC}(S_0) \right]^2}}{S_{RHC}(S_0) - S_{LHC}(S_0)}. \tag{20}
\]
Significantly, the values of θ_d, D, and R obtained from Eqs. (16), (17), and (20), respectively, are totally decoupled manner. Moreover, for values of θ_j other than 0-deg or 45-deg Eqs. (18) and (19) yield the same theoretical solution, and thus the equality (or otherwise) of the results obtained from the two equations provides the means to check the correctness of the experimental results.

Once the LD and CD properties are known, the product of the linear dichroism and circular dichroism Mueller matrix can be calculated as

\[
M_D = [M_{ld}][M_{cd}] = \begin{pmatrix}
B_{11} & B_{12} & B_{13} & B_{14} \\
B_{12} & B_{22} & B_{23} & B_{24} \\
B_{13} & B_{23} & B_{33} & B_{34} \\
B_{14} & B_{24} & B_{34} & B_{44}
\end{pmatrix}, \tag{21}
\]

where \(B_{ij} - B_{44}\) are all functions of \(\theta_d, D,\) and \(R,\) and can be extracted from Eqs. (16), (17), and (20), respectively. Note that all of the elements other than \(B_{42} \) and \(B_{43}\) are non-zero. The Mueller matrix of retardance has the form

\[
M_R = [M_{R0}][M_{Q}] = \begin{pmatrix}
1 & A_{12} & A_{13} & A_{14} \\
A_{21} & A_{22} & A_{23} & A_{24} \\
A_{31} & A_{32} & A_{33} & A_{34} \\
A_{41} & A_{42} & A_{43} & A_{44}
\end{pmatrix}, \tag{22}
\]

where \(A_{12} \) and \(A_{44}\) are functions of the orientation angle (\(\alpha\)) and phase retardance (\(\beta\)) of LB and optical rotation angle (\(\gamma\)) of CB. It is noted that elements \(A_{12}, A_{13}, A_{14}, A_{21}, A_{31}, A_{41}\) are all equal to zero. From Eqs. (21) and (22), it follows that the effective Mueller matrix for the LB, CB, LD, and CD properties of the anisotropic sample can be expressed as

\[
M_{RD} = M_R M_D = \begin{pmatrix}
B_{11} & B_{12} & B_{13} & B_{14} \\
A_{22}B_{12} + A_{23}B_{13} + A_{24}B_{41} & A_{22}B_{22} + A_{23}B_{23} & A_{22}B_{23} + A_{23}B_{33} & A_{22}B_{24} + A_{23}B_{34} + A_{24}B_{44} \\
A_{32}B_{12} + A_{33}B_{13} + A_{34}B_{41} & A_{32}B_{22} + A_{33}B_{23} + A_{34}B_{33} & A_{32}B_{23} + A_{33}B_{33} & A_{32}B_{24} + A_{33}B_{34} + A_{34}B_{44} \\
A_{42}B_{12} + A_{43}B_{13} + A_{44}B_{41} & A_{42}B_{22} + A_{43}B_{23} + A_{44}B_{33} & A_{42}B_{23} + A_{43}B_{33} & A_{42}B_{24} + A_{43}B_{34} + A_{44}B_{44}
\end{pmatrix}. \tag{23}
\]

It is noted that all of the elements in the effective Mueller matrix \([MRD]\) be obtained from Eqs. (6)–(11) (i.e. from the output Stokes vectors of experiment), while only \(m_{11}, m_{12}, m_{13}, m_{14}\) of \([MRD]\) are used to calculate the LD/CD values. In other words, the elements in \([MD]\) and \([MRD]\) can be obtained from Eqs. (6)–(11). Once all of the elements in \([MD]\) and \([MRD]\) are known, those in the Mueller matrix of retardance \([MR]\) can be inversely derived. Once all of the elements in \([M_D]\) and \([M_{RD}]\) are known, those in the Mueller matrix of retardance \([MR]\) can be inversely derived. It is noted that the elements of Mueller matrix in Eq. (21) can be easily extracted by Eqs. (6)–(11).

In this study, two methods are proposed for calculating the LB and CB properties of an anisotropic optical material. In the first method (the default method), the values of \(\alpha, \beta, \) and \(\gamma\) are derived using elements \(A_{22}, A_{23}, A_{24}, A_{32}, A_{33}, A_{42} \) and \(A_{44}\) in the Mueller matrix of retardance \([MR]\). Specifically, the orientation angle of LB is obtained as

\[
\alpha = \frac{1}{2} \tan^{-1}\left(-\frac{A_{34}}{A_{33}}\right). \tag{24}
\]

Meanwhile, the phase retardance is obtained as either

\[
\beta = \tan^{-1}\left(\frac{A_{14}}{\cos(2\alpha)A_{44}}\right). \tag{25}
\]

or

\[
\beta = \cos^{-1}(A_{44}). \tag{26}
\]

The optical rotation angle of CB can be calculated as follows:

\[
\gamma = \frac{1}{2} \tan^{-1}\left(\frac{A_{23} - A_{32}}{A_{22} + A_{33}}\right). \tag{27}
\]

The equality (or otherwise) of the results obtained for the phase retardance using Eqs. (25) and (26), respectively, provides the means to check the correctness of the experimental results. Once the orientation angle and retardance of LB are known, the other ways to obtain the optical rotation (\(\gamma\)) of CB are expressed as

\[
\gamma = \frac{1}{2} \tan^{-1}\left(\frac{-C_2A_{22} + C_1A_{23}}{C_1A_{22} + C_2A_{23}}\right), \tag{28}
\]

or

\[
\gamma = \frac{1}{2} \tan^{-1}\left(\frac{-C_3A_{32} + C_2A_{33}}{C_2A_{32} + C_3A_{33}}\right), \tag{29}
\]

where

\[
C_1 = \cos(2\alpha)^2 + \cos(\beta)\sin(2\alpha)^2, \tag{30}
\]

\[
C_2 = \cos(2\alpha)(1 - \cos(\beta))\sin(2\alpha), \tag{31}
\]

\[
C_3 = \cos(2\alpha)^2\cos(\beta) + \sin(2\alpha)^2. \tag{32}
\]

The equality of the results obtained for the optical rotation using Eqs. (27)–(29) provides the means to check the correctness of the experimental results.

For optical samples with a linear dichroism close to one \((D \approx 1),\) elements \(B_{41}\) and \(B_{44}\) in the product of the linear/ circular dichroism Mueller matrix [Eq. (21)] are close to zero. In other words, the Mueller matrix of \([MR]\) is a singular matrix. As a result, the Mueller matrix of \([MR]\) [Eq. (22)] cannot be found. Consequently, the values of the orientation angle, phase retardance and optical rotation of LB/CB in elements

---

Journal of Biomedical Optics 025006-5 February 2012 • Vol. 17(2)
of Mueller matrix [MR] in the first method are unreliable. Therefore, an alternative method is proposed for calculating the LB and CB properties of anisotropic optical samples with high linear dichroism. In the proposed approach, elements of Mueller matrix of retardance \([M_R]\) (except \(A_{32}, A_{34}\) and \(A_{44}\)) are obtained as:

\[
\begin{bmatrix}
B_{22} & B_{23} & 0 & 0 & 0 & 0 \\
B_{23} & B_{33} & 0 & 0 & 0 & 0 \\
0 & 0 & B_{32} & B_{33} & 0 & 0 \\
0 & 0 & B_{32} & B_{33} & 0 & 0 \\
0 & 0 & 0 & B_{22} & B_{23} & 0 \\
0 & 0 & 0 & 0 & B_{23} & B_{33}
\end{bmatrix}
\begin{bmatrix}
A_{22} \\
A_{33} \\
A_{32} \\
A_{31} \\
A_{42} \\
A_{43}
\end{bmatrix}
= \begin{bmatrix}
m_{22} \\
m_{33} \\
m_{32} \\
m_{31} \\
m_{42} \\
m_{43}
\end{bmatrix}.
\]

(33)

Therefore, the optical rotation angle of CB is calculated from elements \(A_{22}, A_{33}\) and \(A_{33}\) using Eq. (27). Meanwhile, the phase retardance is obtained as

\[
\beta = \sin^{-1} \left[ -\frac{A_{43}}{\cos(2\alpha + 2\gamma)} \right],
\]

(34)

where

\[
2\alpha + 2\gamma = \tan^{-1} \left( -\frac{A_{42}}{A_{43}} \right).
\]

(35)

Having extracted the optical rotation angle and phase retardance of the sample, the orientation angle (\(\alpha\)) of LB can be obtained as

\[
\alpha = \frac{1}{2} \tan^{-1} \left( \frac{C_{11}A_{42} + C_{22}A_{43}}{C_{22}A_{42} - C_{11}A_{43}} \right),
\]

(36)

where

\[
C_{11} = \cos(2\gamma) \sin(\beta),
\]

(37)

\[
C_{22} = \sin(2\gamma) \sin(\beta).
\]

(38)

In summary, in the decoupled analytical model proposed in this study, the orientation angle (\(\alpha\)) and phase retardance (\(\beta\)) of LB, optical rotation (\(\gamma\)) of CB, orientation angle (\(\theta_L\)) and linear dichroism (\(D\)) of LD, and circular dichroism (\(R\)) of CD are extracted using Eqs. (24), (26), (27), (16), (17), and (20), respectively. Also, it is noted that Eqs. (24), (26), (27), (16), (17), and (20) can be simplified as only the function of the measured Mueller elements. For samples with a linear dichroism of \(D \approx 1\), the optical rotation of CB, phase retardance and orientation angle of LB are extracted using Eqs. (27), (34), and (36), respectively. As a result, the robustness of the extracted results toward experimental measurement errors is reduced and the “coupling” and the “multiple solutions” problems in Chen et al.\(^{32}\) and Lo et al.\(^{31}\) are resolved. Importantly, the model provides the means to extract the properties of samples with only LB, CB, LD, or CD properties without the need for any form of compensation process. Furthermore, in similar to the models presented in Chen et al.\(^{32}\) and Lo et al.\(^{31}\) the proposed methodology does not require the principal birefringence axes and dichroism axes to be aligned.

3 Analytical Simulations and Error Analysis

In this section, the ability of the proposed analytical method to extract the six effective optical parameters over the measurement ranges is verified using a simulation technique. A further series of simulations is then performed to evaluate the accuracy of the results obtained from the proposed method given errors of \(\pm 0.005\) in the values of the output Stokes parameters.\(^{32,33}\)

(39)

(Note that the error range defined here is consistent with the measurement precision of a typical commercial polarimeter.) Finally, simulations are performed to investigate the resolution of the proposed method in extracting the properties of anisotropic samples with very low retardance, linear dichroism, optical rotation, and circular dichroism, respectively.

3.1 Analytical Simulations

In performing the analytical simulations, the theoretical values of the output Stokes parameters for the six input lights, namely \(S_0, S_45, S_90, S_{135}, S_{LHC},\) and \(S_{RHC}\), were calculated for a hypothetical sample using the Jones matrix formulation based on given values of the sample parameters and a knowledge of the input Stokes vectors. The theoretical Stokes values were then inserted into the analytical model derived in Sec. 2 in order to derive the effective optical parameters. Finally, the extracted values of the effective optical parameters were compared with the input values used in the Jones matrix formulation.

The ability of the proposed method is evaluated by extracting \(\alpha, \beta, \theta_L, D, \gamma,\) and \(R\) of an anisotropic sample, respectively. When a parameter is extracted, the input of this extracted parameter is changed over its full range (i.e., \(\alpha, \theta_L, \gamma; 0\)-deg to 180-deg; \(\beta; 0\)-deg to 360-deg; \(D; 0\) to 1; and \(R; -1\) to 1). The other input parameters were specified as follows: \(\alpha = 50\)-deg, \(\beta = 60\)-deg, \(\theta_L = 35\)-deg, \(D = 0.4, \gamma = 15\)-deg, and \(R = 0.1\). For example, to extract the principal axis angle of LB, the orientation angle of LB extracted using Eq. (24) was specified as \(\alpha = 0\)-deg to 180-deg and the other input parameters were specified as \(\beta = 60\)-deg, \(\theta_L = 35\)-deg, \(D = 0.4, \gamma = 15\)-deg, and \(R = 0.1\), respectively.\(^{32,33}\) It is found that the proposed model enables the orientation angle of LD in Eq. (16), linear dichroism in Eq. (17), and circular dichroism in Eq. (20) to be measured over the full range. Again, the analytical model proposed in this study enables the full-range measurement of the orientation angle of LB in Eq. (24), phase retardation in Eq. (26), and optical rotation in Eq. (27), respectively. The results confirm the ability of the proposed method to yield full-range measurements of six effective parameters.

3.2 Error Analysis of Proposed Analytical Model

To examine the robustness of the proposed analytical model toward errors in the output Stokes parameter values, the Jones matrix formulation was used to derive the theoretical output Stokes parameters \(S_0, S_{45}, S_{90}, S_{135}, S_{RHC},\) and \(S_{LHC}\) for a composite sample with given LB/CL/CD properties and known input polarization states. The 500 sets of error-affected Stokes parameters were then produced by applying random perturbations around \(\pm 0.005\) to the theoretical Stokes parameters.\(^{32,33}\) The perturbed Stokes parameter values were then inserted into the analytical model in order to extract the effective parameters of the sample. Finally, the extracted values of the optical properties were compared with the given values used in the Jones matrix formulation.

Journal of Biomedical Optics

025006-6

February 2012 • Vol. 17(2)
In deriving the theoretical values of the output Stokes parameters, the effective properties of the optical sample were assigned as follows: \( \alpha = 50\)-deg, \( \beta = 60\)-deg, \( \theta_d = 35\)-deg, \( D = 0.4\), \( \gamma = 15\)-deg, and \( R = 0.1\). The values of \( \alpha\), \( \beta\), \( \theta_d\), \( D\), \( \gamma\), and \( R\) were then extracted from the perturbed values of the output Stokes parameters using Eqs. (24), (26), (27), (16), (17), and (20), respectively. From inspection, the error bars of parameters \( \alpha\), \( \beta\), \( \theta_d\), \( D\), \( \gamma\), and \( R\) have values of just \( \pm 0.022\)-deg, \( \pm 0.038\)-deg, \( \pm 0.174\)-deg, \( \pm 0.005\), \( \pm 0.066\)-deg, and \( \pm 0.003\), respectively. Thus, it is inferred that the analytical model is robust toward experimental errors in the output Stokes parameters.

### 3.3 Resolution of Extracted Parameter Values for Samples with Low LB, LD, CB, and CD Properties

For samples with close to zero retardance \( \beta \approx 0\), the Mueller matrix of linear birefringence (Eq. 1) is a unit matrix for any value of the orientation angle of LB \( \alpha \) (0-deg to 180-deg). In other words, for a sample with \( \beta \approx 0\), the values obtained for the orientation angle of LB from Eq. (24) are unreliable. Similarly, for samples with a linear dichroism close to zero \( D \approx 0\), the Mueller matrix of linear dichroism [Eq. (2)] is a unit matrix for any value of the linear dichroism axis angle \( \theta_d \) (0-deg to 180-deg). In other words, the results obtained for \( \theta_d \) from Eq. (16) are unreliable. Therefore, the performance of the proposed analytical model in extracting the optical parameters of samples with a low LB, low LD, low CB, and low CD are evaluated as shown in Table 2.

The extracted values of the sample parameters are compared with the input values given assumed errors of \( \pm 0.005\) in the values of the output Stokes parameters. Significantly, the results presented in Table 2 show that even though the orientation angle of LB is highly sensitive to errors in the output Stokes parameters, the extracted values of the CB, LD, and CD properties deviate only slightly from the input values. In other words, the decoupled nature of the analytical model prevents the error in the orientation angle of LB from contaminating the extracted values of the remaining parameters, and improves their precision as a result. Similarly, in low LD case, the extracted values of the

### Table 2

The results of the extracted parameter for samples with low LB, LD, CB, and CD properties.

<table>
<thead>
<tr>
<th>Case</th>
<th>The input values</th>
<th>Error bars of ( \alpha), ( \beta), ( \theta_d), ( D), ( \gamma), and ( R), respectively</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low LB</td>
<td>( \alpha = 3)-deg, ( \beta = 3)-deg, ( \theta_d = 35)-deg, ( D = 0.4), ( \gamma = 15)-deg, and ( R = 0.1)</td>
<td>( \pm 0.366)-deg, ( \pm 0.04)-deg, ( \pm 0.176)-deg, ( \pm 0.003), ( \pm 0.012)-deg, and ( \pm 0.001)</td>
</tr>
<tr>
<td>Low LD</td>
<td>( \alpha = 50)-deg, ( \beta = 60)-deg, ( \theta_d = 35)-deg, ( D = 0.05), ( \gamma = 15)-deg, and ( R = 0.1)</td>
<td>( \pm 0.018)-deg, ( \pm 0.033)-deg, ( \pm 1.472)-deg, ( \pm 0.003), ( \pm 0.01)-deg, and ( \pm 0.002)</td>
</tr>
<tr>
<td>Low CB</td>
<td>( \alpha = 50)-deg, ( \beta = 60)-deg, ( \theta_d = 35)-deg, ( D = 0.4), ( \gamma = 0.1)-deg, and ( R = 0.1)</td>
<td>( \pm 0.028)-deg, ( \pm 0.032)-deg, ( \pm 0.185)-deg, ( \pm 0.003), ( \pm 0.027)-deg, and ( \pm 0.001)</td>
</tr>
<tr>
<td>Low CD</td>
<td>( \alpha = 50)-deg, ( \beta = 60)-deg, ( \theta_d = 35)-deg, ( D = 0.4), ( \gamma = 15)-deg, and ( R = 0.01)</td>
<td>( \pm 0.028)-deg, ( \pm 0.032)-deg, ( \pm 0.185)-deg, ( \pm 0.003), ( \pm 0.027)-deg, and ( \pm 0.001)</td>
</tr>
</tbody>
</table>

In deriving the theoretical values of the output Stokes parameters, the effective properties of the optical sample were assigned as follows: \( \alpha = 50\)-deg, \( \beta = 60\)-deg, \( \theta_d = 35\)-deg, \( D = 0.4\), \( \gamma = 15\)-deg, and \( R = 0.1\). The values of \( \alpha\), \( \beta\), \( \theta_d\), \( D\), \( \gamma\), and \( R\) were then extracted from the perturbed values of the output Stokes parameters using Eqs. (24), (26), (27), (16), (17), and (20), respectively. From inspection, the error bars of parameters \( \alpha\), \( \beta\), \( \theta_d\), \( D\), \( \gamma\), and \( R\) have values of just \( \pm 0.022\)-deg, \( \pm 0.038\)-deg, \( \pm 0.174\)-deg, \( \pm 0.005\), \( \pm 0.066\)-deg, and \( \pm 0.003\), respectively. Thus, it is inferred that the analytical model is robust toward experimental errors in the output Stokes parameters.

### Table 3

The comparison between nondepolarizing method and depolarizing method with low depolarization case.

<table>
<thead>
<tr>
<th>Input value (Case #1)</th>
<th>( M = \begin{pmatrix} 1.0000 &amp; 0.0185 &amp; 0.0029 &amp; 0.0042 \ 0.0172 &amp; 0.7569 &amp; -0.0405 &amp; 0.0462 \ 0.0034 &amp; 0.5024 &amp; 0.5450 &amp; -0.5466 \ 0.0024 &amp; -0.0070 &amp; 0.6244 &amp; 0.5967 \end{pmatrix} ) [22]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td>Input control values</td>
</tr>
<tr>
<td>Orientation angle of LB ( \alpha )</td>
<td>( X )</td>
</tr>
<tr>
<td>Linear birefringence ( \beta )</td>
<td>0.83 (rad)</td>
</tr>
<tr>
<td>Optical rotation ( \gamma )</td>
<td>2.14-deg</td>
</tr>
<tr>
<td>Orientation angle of LD ( \theta_d )</td>
<td>( X )</td>
</tr>
<tr>
<td>Linear dichroism ( D )</td>
<td>0</td>
</tr>
<tr>
<td>Circular dichroism ( R )</td>
<td>0</td>
</tr>
</tbody>
</table>

\textsuperscript{4}Note: the notation \( X \) in the tables indicates none value.
LB, CB, and CD properties deviate only slightly from the input values despite the error in the extracted value of $\theta_d$. Overall, the ability of the proposed method to extract the orientation angle of LB of samples with a low degree of birefringence can be reliable when retardance is larger than 3-deg. Moreover, the values of orientation angle of LD can be reliable when linear dichroism is larger than 0.05. The ability of the proposed method to extract the optical parameters of samples with CB larger than 0.1-deg or CD larger than 0.01 are reliable with the input Stokes parameters given assumed errors of $\pm 0.005$.

4 Comparison Between the Proposed Nondepolarizing Method and Depolarizing Method

To compare the results by the proposed nondepolarizing Mueller matrix method and depolarizing Mueller matrix method, one demonstration was established by using the results of Mueller matrix in Ghosh et al.$^{23}$ In Table 3, the Mueller matrix was experimentally recorded in the forward scattering geometry from the polycrylamide phantom having degree of strain-induced birefringence (extension = 2 mm applied along the vertical direction), chiral (concentration of sucrose = 1 M corresponding to magnitude of optical activity = 1.96 deg cm$^{-1}$), turbid (scattering coefficient = 3 mm$^{-1}$), value for anisotropy parameter of particles in polycrylamide calculated from Mie theory = 0.95.$^{23}$ All elements of the Mueller matrix was inserted into Eqs. (24), (26), (27), (16), (17), and (20) in order to extract the values of $\alpha, \beta, \gamma, \theta_d, D$, and $R$, respectively. In Table 3, the results show that with depolarization coefficient equal to 0.19 (low depolarization), the extracted parameters by the proposed method are equivalent to those by the method in Ghosh et al.$^{23}$

Another comparison was established by using the results of Mueller matrix in Ghosh et al.$^{23}$ with high depolarization (i.e. depolarization coefficient equal to 0.798). The Monte Carlo-generated Mueller matrix and the decomposed matrices for a birefringent, chiral, turbid medium (anisotropy in refractive index = $1.36 \times 10^{-5}$, which corresponds to a value of linear retardance = 1.35 rad for a path length of 10 mm, optical activity = 1.96 deg cm$^{-1}$, scattering coefficient = 6 mm$^{-1}$, and average cosine of scattering angle = 0.935) are introduced in Ghosh et al.$^{23}$ The axis of linear birefringence was kept along the vertical ($Y$) direction (orientation angle of LB = 90-deg) in the simulation. Table 4 compares the values of $\alpha, \beta, \gamma, \theta_d, D$, and $R$ obtained from Ghosh et al.$^{23}$ and the proposed study. It is observed that a good agreement exists between the extracted values of LD/CD (i.e., $\theta_d, D$, and $R$) by the proposed method and those by the method in Ghosh et al.$^{23}$ with high depolarization. It is explained that the first element of the depolarization Mueller matrix equals one and other elements (except diagonal) are closed to zero.$^{20-23,27,34,35}$ Thus, the first row of effective Mueller matrix $[M_{\text{MD}}]$ (i.e. $m_{11}, m_{12}, m_{13}$, and $m_{14}$) in Eq. (23) does not change when the depolarization Mueller matrix is multiplied in front of the effective Mueller matrix. In other words, Eqs. (16), (17), and (20) also can be used to extract the values of $\theta_d, D$, and $R$ in the case of the depolarizing Mueller matrix.

Interestingly, the extracted values of LB/CB (i.e., $\alpha, \beta, \gamma$) are also closed to the corresponding input values (i.e., orientation angle of LB [$\alpha$] 90-deg and value of linear retardance [$\beta$] 1.35 rad in the simulation). With a separate series of comparisons, it is found that when the difference between the two linear depolarization values and circular depolarization value are small (see the depolarization Mueller matrix in Ghosh et al.$^{23}$), the extracted values of LB/CB are still reliable in high depolarization case.

5 Influence of the Order of $[M_R]$ and $[M_D]$

Researchers have proved that for the depolarizing Mueller matrix, the decomposition in $[M_\Delta][M_R][M_D]$ is a natural generalization of the polar decomposition.$^{20,23,34,38,39,40}$ Thus, for the nondepolarizing Mueller matrix, the decomposition in $[M_R][M_D]$ is generally used for the decomposition. The decomposition in $[M_\Delta][M_R][M_D]$ clearly separates the depolarizing component ($[M_\Delta]$) from the nondepolarizing component ($[M_R][M_D]$).$^{20}$

### Table 4: The comparison between nondepolarizing method and depolarizing method with high depolarization case.

<table>
<thead>
<tr>
<th>Parameters Estimated values of (Ghosh et al.$^{23}$)</th>
<th>Extracted values (the proposed study)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orientation angle of LB ($\alpha$)</td>
<td>88.9-deg</td>
</tr>
<tr>
<td>Linear birefringence ($\beta$)</td>
<td>0.905 [rad]</td>
</tr>
<tr>
<td>Optical rotation ($\gamma$)</td>
<td>2.09-deg</td>
</tr>
<tr>
<td>Orientation angle of LD ($\theta_d$)</td>
<td>X</td>
</tr>
<tr>
<td>Linear dichroism ($D$)</td>
<td>0.019 (diattenuation)</td>
</tr>
<tr>
<td>Circular dichroism ($R$)</td>
<td>0.004</td>
</tr>
<tr>
<td>Depolarization coefficient ($D$)</td>
<td>0.798</td>
</tr>
</tbody>
</table>

$^*$Note: the notation X in the table indicates none value.
For completely polarized incident light, the decrease of the polarization degree is solely caused by the depolarizing component. Thus, $[M_R][M_I][M_D]$ is useful to have the depolarizing component following the nondepolarizing component for the interpretation of experimental data. Also, Morio and Goudail showed that the decomposition proposed by Lu and Chipman always leads to physical Mueller matrices.

Since Mueller matrices do not commute, the influence of the order in nondepolarizing Mueller matrices is investigated in this section. The influence of the order of nondepolarizing Mueller matrices for extracting the six effective optical parameters is verified using genetic algorithms (GAs). Thereafter, simulations are performed to compare the two sets of results obtained from product of $[M_R][M_I]$ and $[M_D][M_R]$ using GAs with the extracted results using the proposed method.

The GAs provide a powerful technique for computing the exact or approximate solutions to a wide variety of optimization and classification type problems. In the present study, the candidate solution strings contain six elements corresponding to $\alpha_S$, $\beta_S$, $\gamma_S$, $\theta_S$, $D_S$, and $R_S$. In generating the candidate solutions, the search spaces for $\alpha_S$, $\beta_S$, $\gamma_S$, $\theta_S$, $D_S$, and $R_S$ were specified as $0^\circ \leq \alpha_S \leq 180^\circ$, $0^\circ \leq \beta_S \leq 360^\circ$, $0^\circ \leq \gamma_S \leq 180^\circ$, $0^\circ \leq \theta_S \leq 180^\circ$, $0 \leq D_S \leq 1$, and $-1 \leq R_S \leq 1$, respectively. The quality of each candidate solution is evaluated using a fitness function based on the distance between the elements of the input Mueller matrix and the elements of the product of $[M_R][M_D]$ or $[M_D][M_R]$, respectively. In other words, the error function has the form:

$$E_\psi = \sum_{i=1}^{16} (\varphi_i[M_{\text{true}}] - \varphi_i[M_{\text{est}}])^2$$

### Table 5: Influence of order in Mueller matrix using GA and the proposed study.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimated values ([$M_I$][$M_R$] using GA)</th>
<th>Estimated values ([$M_I$][$M_D$] using GA)</th>
<th>Estimated values (the proposed study)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orientation angle of LB ($\alpha_S$)</td>
<td>$\alpha_S = 59.999$-$\text{deg}$</td>
<td>$\alpha_S = 60.954$-$\text{deg}$</td>
<td>$\alpha_S = 59.997$-$\text{deg}$</td>
</tr>
<tr>
<td>Linear birefringence ($\beta_S$)</td>
<td>$\beta_S = 44.999$-$\text{deg}$</td>
<td>$\beta_S = 45.683$-$\text{deg}$</td>
<td>$\beta_S = 45.001$-$\text{deg}$</td>
</tr>
<tr>
<td>Optical rotation ($\gamma_S$)</td>
<td>$\gamma_S = 15.001$-$\text{deg}$</td>
<td>$\gamma_S = 14.637$-$\text{deg}$</td>
<td>$\gamma_S = 15.000$-$\text{deg}$</td>
</tr>
<tr>
<td>Orientation angle of LD ($\theta_{ds}$)</td>
<td>$\theta_{ds} = 59.998$-$\text{deg}$</td>
<td>$\theta_{ds} = 38.657$-$\text{deg}$</td>
<td>$\theta_{ds} = 59.994$-$\text{deg}$</td>
</tr>
<tr>
<td>Linear dichroism ($D_S$)</td>
<td>$D_S = 0.3$</td>
<td>$D_S = 0.353$</td>
<td>$D_S = 0.3$</td>
</tr>
<tr>
<td>Circular dichroism ($R_S$)</td>
<td>$R_S = 0.1$</td>
<td>$R_S = 0.015$</td>
<td>$R_S = 0.1$</td>
</tr>
<tr>
<td>Note: Subscript “s” indicates “sample”</td>
<td>($E_\psi = 2.04 \times 10^{-8}$)</td>
<td>($E_\psi = 2.06 \times 10^{-8}$)</td>
<td>Note: Input Mueller matrix of $[M]$ into Eqs. (24), (26), (27), (16), (17) and (20)</td>
</tr>
</tbody>
</table>

For Case #1:

| Orientation angle of LB ($\alpha_S$) | $\alpha_S = 29.999$-$\text{deg}$ | $\alpha_S = 31.114$-$\text{deg}$ | $\alpha_S = 30.001$-$\text{deg}$ |
| Linear birefringence ($\beta_S$)   | $\beta_S = 60.002$-$\text{deg}$ | $\beta_S = 62.237$-$\text{deg}$ | $\beta_S = 60.008$-$\text{deg}$ |
| Optical rotation ($\gamma_S$)      | $\gamma_S = 24.999$-$\text{deg}$ | $\gamma_S = 24.415$-$\text{deg}$ | $\gamma_S = 24.998$-$\text{deg}$ |
| Orientation angle of LD ($\theta_{ds}$) | $\theta_{ds} = 30.001$-$\text{deg}$ | $\theta_{ds} = 0.236$-$\text{deg}$ | $\theta_{ds} = 30.001$-$\text{deg}$ |
| Linear dichroism ($D_S$)            | $D_S = 0.4$                             | $D_S = 0.522$                            | $D_S = 0.4$                            |
| Circular dichroism ($R_S$)          | $R_S = 0.2$                             | $R_S = 0.001$                            | $R_S = 0.2$                            |
| ($E_\psi = 3.8 \times 10^{-8}$)    | ($E_\psi = 0.016$)                      | |
where $\phi_i;\frac{1}{2}M_{\text{Input/C138}}$ represents the elements of the input Mueller matrix and $\phi_i;\frac{1}{2}M_{\text{MD/C138}}$ and $\phi_i;\frac{1}{2}M_{\text{MR/C138}}$ represent the corresponding elements of $[M_R][M_D]$ and $[M_D][M_R]$. In other words, the objective of the GA is to determine the values of $\alpha_S, \beta_S, \gamma_S, \theta_S, D_S$, and $R_S$, which minimize the error function.

Table 5 compares the extracted values of $\alpha_S, \beta_S, \gamma_S, \theta_S, D_S$, and $R_S$ obtained from the GA ($[M_R][M_D]$ and $[M_D][M_R]$) and from the proposed study method. With a separate series of simulations, it is observed that a good agreement exists between the extracted values in $[M_R][M_D]$ and the proposed study method. In the $[M_D][M_R]$ case, only the extracted values of $\alpha_S, \beta_S, \gamma_S$ are equivalent to the extracted values of two cases. Thus, the order of $[M_R]$ and $[M_D]$ affected the effective parameters of anisotropic optical materials in LD/CD, but not in LB/CB properties.

6 Experimental Verification of Proposed Analytical Model

6.1 Experimental Setup

Figure 2 presents a schematic illustration of the experimental setup used to verify the performance of the proposed analytical model. In performing the experiments, the input light was provided by a frequency-stable He-Ne laser (SL 02/2, SIOS Co.) with a central wavelength of 632.8 nm. In addition, a polarizer (GTH5M, Thorlabs Co.) and quarter-wave plate (QWP0-633-04-4-R10, CVI Co.) were used to produce four linear polarization lights (0-deg, 45-deg, 90-deg and 135-deg) and two circular polarization lights (right-handed and left-handed). A neutral density filter (NDC-100-2, ONSET Co.) and power meter detector (8842A, OPHIT Co.) were used to ensure that each of the input polarization lights had an identical intensity. Note that for samples with no linear dichroism, the output Stokes parameters can be normalized as $S_C/S_0$ since the terms $m_{12}, m_{13}$ and $m_{14}$ in Eq. (5) are non-zero. Thus, there is no need to ensure that the six input lights have an identical optical intensity before entering the sample. However, for samples with linear dichroism, the output Stokes parameters cannot be normalized in this way, and thus the neutral density filter and power meter detector are required.

The output Stokes parameters were computed from the intensity measurements obtained using a commercial Stokes polarimeter (PAX5710, Thorlabs Co.) at a sampling rate of 30 samples per second. A minimum of 1024 data points were obtained for the effective parameters ($\alpha, \beta, \theta, D, \gamma$, and $R$) of each sample. Of these data points, 100 points were then chosen in order to calculate the mean value of each parameter.

The validity of the proposed measurement method was evaluated using different optical samples, namely a quarter-wave plate (QWP0-633-04-4-R10, CVI Co.), a half-wave plate (QWP0-633-04-2-R10, CVI Co.), the de-ionized water with containing D-glucose, a polarizer (GTH5M, Thorlabs Co.), a polarization controller, and a polymer polarizer (LLC2-82-18s, OPTIMAX Co.) baked in an oven at 150 °C for 100 min. Also, a composite sample comprising a quarter-wave plate, a half-wave plate and a polarizer in aligned or non-aligned principal axis were tested. Note that the polarization controller was used to evaluate the performance of the proposed method in extracting the parameters of samples with circular dichroism only. Meanwhile, the baked polarizer was chosen in order to evaluate the performance of the proposed method in measuring the optical parameters of samples with both linear birefringence and linear dichroism.

6.2 Experimental Results

6.2.1 Quarter-wave plate (LB property only)

Figure 3 illustrates the experimental results obtained for the effective properties of the quarter-wave plate (QWP). From inspection, the standard deviations of the orientation angle and phase retardance measurements are found to be just 0.04-deg and 0.013-deg, respectively. In other words, the ability of the proposed method to extract the properties of samples with linear birefringence only is confirmed. As expected, the linear dichroism, optical rotation angle and circular dichroism parameters have a value close to zero at all values of the orientation angle. As discussed in Sec. 3.3, the proposed analytical model yields reliable results for the orientation angle of LD only for samples with a linear dichroism greater than or equal to 0.05. In the present sample, the linear dichroism is close to zero, and thus the orientation angle of LD varies randomly in the range of 0-deg to 180-deg as the orientation angle of LB is increased [see Fig. 3(b)]. As expected, Figs. 3(c) and 3(d) show that the optical rotation angles and values of circular dichroism of the quarter-wave plate are also close to zero.

6.2.2 Half-wave plate and de-ionized water containing D-glucose (CB property only)

Half-wave plate (CB property only). In general, the elements in the Mueller matrix for an optically active material are different from those in the Mueller matrix for a half-wave plate. The Mueller matrix of a half-wave plate with an optical rotation $\gamma_H$ has the form as

$$M_{\text{HP}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(4\gamma_H) & \sin(4\gamma_H) & 0 \\ 0 & \sin(4\gamma_H) & -\cos(4\gamma_H) & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{41}$$

Thus, in computing the optical rotation angle of the half-wave plate, the formula of linear retardance ($\beta$) is revised as:

$$\beta = \tan^{-1}\left(\frac{A_{34}}{\cos(2\varphi)A_{44}}\right). \tag{42}$$
or

\[ \beta = \cos^{-1}(-A_{44}). \]  

(43)

Then, the formula of optical rotation (\( \gamma \)) is revised as:

\[ \gamma = \frac{1}{2} \tan^{-1} \left( \frac{A_{23} + A_{32}}{A_{22} - A_{33}} \right). \]  

(44)

or

\[ \gamma = \frac{1}{2} \tan^{-1} \left( \frac{C_2A_{22} + C_1A_{23}}{C_1A_{22} - C_2A_{23}} \right). \]  

(45)

Figure 4 presents the experimental results for the effective properties of the half-wave plate. Figure 4(c) shows that a good agreement is obtained between the measured value of the optical rotation angle and the actual value of the optical rotation angle over the considered range of 0-deg to 90-deg. From inspection, the standard deviation of the optical rotation angle is found to be just 0.008-deg. As discussed in Sec. 3.3, the analytical model yields reliable results for the orientation angle of LB only for samples with a linear dichroism greater than or equal to 0.05. Figures 5(a) and 5(b) show that both the retardance and the linear dichroism of the de-ionized water with containing D-glucose are close to zero. Thus, the extracted values of the orientation angle of LB and orientation angle of LD vary randomly as the concentration of glucose is increased. As expected, Fig. 5(d) shows that the circular dichroism of the de-ionized water with containing D-glucose is also close to zero.

De-ionized water containing D-glucose (CB property only). Figure 5 illustrates the experimental results obtained for the effective parameters of the de-ionized water with containing D-glucose (Merck Ltd.). The D-glucose was poured into a container of de-ionized water. The average measured values of the optical parameters of the sample with different concentration of glucose from 0 to 1 M in increments of 0.1 M are summarized. The container is glass and its width is 12.5 mm outside and 10 mm inside. Distance from center of sample to surface of detector is 23 mm. Figure 5(c) shows the measured value of the optical rotation angle regarding to the concentration of glucose over the considered range of 0 to 1 M. The sensitivity of the D-glucose measurement is estimated 1.9 M/l, and it is a good agreement with Lo and Yu.43 From inspection, the standard deviation of the optical rotation angle is found to be just 0.01-deg. As discussed in Sec. 3.3, the analytical model yields reliable results for the orientation angle of LB only for samples with a phase retardance greater than or equal to 3-deg. Similarly, reliable results for the orientation angle of LD are obtained only for samples with a linear dichroism greater than or equal to 0.05. Figures 5(a) and 5(b) show that both the retardance and the linear dichroism of the de-ionized water with containing D-glucose are close to zero. Thus, the extracted values of the orientation angle of LB and orientation angle of LD vary randomly as the concentration of glucose is increased. As expected, Fig. 5(d) shows that the circular dichroism of the de-ionized water with containing D-glucose is also close to zero.

6.2.3 Polarizer (LD property only)

Figure 6 presents the experimental results for the effective parameters of the polarizer. As expected, the linear dichroism
Fig. 4 Experimental results for effective parameters of half-wave plate (HP).

Fig. 5 Experimental results for effective parameters of de-ionized water with containing D-glucose.
has a value close to one [see Fig. 6(b)]. Moreover, a good agreement is observed between the measured values of the orientation angle of LD and the known values. From inspection, the standard deviations of $\theta_{L}$ and $D$ are found to be just 0.007-deg and 1.42 $\times$ 10$^{-5}$, respectively. In other words, the ability of the proposed analytical model to extract the parameters of samples with pure LD properties is confirmed. Figures 6(c) and 6(d) confirm that the CB and CD properties of the polarizer are close to zero. As discussed in Sec. 3.3, reliable results are obtained for the orientation angle of LB provided that the phase retardance has a value greater than or equal to 3-deg. As shown in Fig. 6(a), the polarizer has a retardance of less than 2-deg. Thus, the extracted value of the orientation angle of LB varies randomly in the range of 0 to 180-deg as the orientation angle of LD is increased. Note that in previous studies by the current group,32,33 both the retardance and the orientation angle of LB of the polarizer were found to vary randomly with the orientation angle of LD. However, in the present study, the extracted value of the retardance is approximately constant. In other words, the decoupled nature of the analytical model proposed in this study successfully resolves the “multiple solutions” problem found in Chen et al.32 and Lo et al.33 for samples with a linear dichroism of $D = 1$.

6.2.4 Polarization controller (CD property only)

In the present study, a sample with pure CD properties was simulated using a polarization controller comprising a half-wave plate sandwiched between two quarter-wave plates and a neutral density filter (NDF). In performing the measurement process, the experimental settings of the polarization controller and NDF required to replicate a pure CD sample were determined using the genetic algorithm (GA) method described in Refs. 41, 42. That is, having specified the desired value of the circular dichroism (e.g., $R = 0.2$), the orientation angle of the two quarter-wave plates ($\alpha_1$ and $\alpha_2$), the optical rotation angle of the half-wave plate ($\gamma_1$), and the output intensity of the NDF were tuned in accordance with the results obtained from the GA such that the following condition was satisfied for each of the six input polarization lights.

$$S_c = [M_{QW1}] [M_{HP}] [M_{QW2}] [M_{NDF}] S_c \approx [M_{cd}] S_c$$

where $S_c$ are the output Stokes parameters obtained when using the simulated CD sample, $[M_{QW1}]$ and $[M_{QW2}]$ are the Mueller matrices of the two quarter-wave plates, $[M_{HP}]$ is the Mueller matrix of the half-wave plate, $[M_{NDF}]$ is the Mueller matrix of the neutral density filter, and $[M_{cd}]$ is the theoretical Mueller matrix of a sample with CD properties only. Six different values of $\alpha_1$, $\alpha_2$, $\gamma_1$, and the NDF output intensity were obtained for input lights of $S_{0-deg}$, $S_{45-deg}$, $S_{90-deg}$, $S_{135-deg}$, $S_{RHC}$, and $S_{LHC}$, respectively. In performing the experiments, the polarization controller and NDF were set accordingly, and the resulting output Stokes parameters were measured using a commercial polarimeter (PAX5710, Thorlabs Co.). The circular dichroism of the simulated sample was then calculated using Eq. (20) in Sec. 2.

Figure 7(d) shows that a good agreement is obtained between the measured values of the circular dichroism and the simulated values. From inspection, the standard deviation of the measured values is just 2.94 $\times$ 10$^{-4}$. Thus, the ability of the proposed
method to extract the parameters of an optical sample with CD properties only is confirmed. As expected, Fig. 7(b) shows that the linear dichroism of simulated sample is close to zero. Thus, the extracted values of the orientation angle of LD vary randomly as the simulated circular dichroism value is increased. Note that for a sample with pure CD properties, the phase retardance and optical rotation angle are very small (zero, ideally). However, the polarization controller and NDF do not provide sufficient parameters for the actual LB and CB properties of the simulated sample to be explored. As shown in Figs. 7(a) and 7(c), the extracted value of the orientation angle and the linear retardance of LB and the optical rotation of CB varies randomly in the range of 0 to 180-deg as the simulated circular dichroism value is increased.

6.2.5 Baked polarizer (LB and LD properties)

Figure 8 illustrates the experimental results obtained for the LB and LD properties of the baked polarizer (BP). As expected, the measured values of the optical rotation angle and circular dichroism are close to zero [see Figs. 8(c) and 8(d)]. Due to the prolonged exposure of the polarizer to a high-temperature environment, the input light leaks through one of the LD axes. Thus, as shown in Fig. 8(b), the linear dichroism has a value close to 1. Moreover, it can be seen that a good agreement exists between the measured values of the orientation angle of LD and the given values. The average value of the phase retardance is found to be 16.92-deg [see Fig. 8(a)]. In addition, a good correlation is observed between the measured values of the orientation angle of LB and the given values. From inspection, the standard deviations of the extracted values of $\alpha$, $\beta$, $\theta_d$, and $D$ are found to be just 0.03-deg, 0.03-deg, 0.01-deg and $4.16 \times 10^{-4}$, respectively. In other words, the proposed analytical model enables the parameters of hybrid samples with both LB and LD properties to be accurately determined.

6.2.6 Composite sample comprising quarter-wave plate, half-wave plate, and polarizer (LB, CB, and LD properties)

**Aligned principal axis of quarter-wave plate, half-wave plate and polarizer.** Figure 9 shows the experimental results obtained for a composite sample comprising a polarizer (LD), a half-wave plate (CB) and a quarter-wave plate (LB). As shown in Figs. 9(a)–9(c), a good agreement is obtained between the measured values and the known values of the orientation angle of LB, orientation angle of LD, and optical rotation, respectively. Moreover, as expected, the quarter-wave plate has a phase retardance of approximately $\beta = 90$-deg, the polarizer has a linear dichroism of approximately $D \approx 1$. As expected, the measured values of the circular dichroism are close to zero [see Fig. 9(d)]. Thus, the ability of the proposed measurement method to extract the effective parameters of samples with both linear/circular birefringence and linear dichroism is confirmed.

**Non-aligned principal axis of quarter-wave plate, half-wave plate, and polarizer.** Figure 10 illustrates the experimental results obtained for the effective properties of a composite sample comprising a polarizer (LD), a half-wave plate (CB) and a quarter-wave plate (LB) in non-aligned principal axis angle. It is noted that the average measured values of the effective parameters of the composite sample with different principal axis angle of quarter-wave plate from 0 to 90-deg in increments of 15-deg whereas the principal axis angle of polarizer fixed to
Fig. 8 Experimental results obtained for effective parameters of baked polarizer (BP).

Fig. 9 Experimental results for effective parameters of composite sample with aligned principal axis.
25-deg and the principal axis angle of half-wave plate fixed to 15-deg are summarized.

As shown in Figs. 10(a)–10(c), a good agreement is obtained between the measured values and the known values of the orientation angle of LB, orientation angle of LD, and optical rotation, respectively. Moreover, as expected, the quarter-wave plate has a phase retardance of approximately $\beta = 90$-deg, the polarizer has a linear dichroism of approximately $D \approx 1$. As expected, the measured values of the circular dichroism are close to zero [Fig. 10(d)]. Once again, the ability of the proposed measurement method to extract the effective parameters of samples with LB, CB, and LD in non-aligned principal axis angle is confirmed.

In summary, the experimental results confirm that the decoupled nature of the analytical model improves accuracy and the ability to extract the parameters of optical samples with only linear birefringence, circular birefringence, linear dichroism, or circular dichroism property without using compensation technique or pretreatment. Moreover, the unreliable result of one effective parameter problem inherent in others is avoided [see Eqs. (24), (26), (27), (16), (17), and (20)]. Furthermore, the “multiple solutions” problem when extracting the LB properties of a sample with a linear dichroism of $D \approx 1$ is resolved. Thus, the ability of the proposed measurement method to extract the effective parameters of samples with LB/CB/LD/CD is confirmed.

7 Conclusions and Discussions

This study has proposed a decoupled analytical technique based on the Mueller matrix method and the Stokes parameters for extracting LB, CB, LD, and CD properties of anisotropic optical materials. The effective LB, CB, LD, and CD parameters of the sample are fully decoupled in the extraction process. Thus, the “multiple solutions” problem observed in Chen et al. and Lo et al. when extracting the LB properties of a sample with a linear dichroism of $D \approx 1$ is resolved. In addition, the experimental results have shown that the decoupled nature of the extraction process localizes the effects of measurement errors and enables the properties of pure LB, LD, CB, or CD samples to be extracted without the need for any form of compensation process. As a result, the proposed approach has significant potential for applications such as collagen and muscle structure characterization (based on LB measurement only), protein structure characterization (based on CB/CD measurements), or diabetes detection (based on CB measurement only) without using a compensation technique or pretreatment. In contrast to existing analytical models based upon the Mueller matrix decomposition method, the six effective parameters are uniquely extracted in a totally decoupled manner. Also, the principal birefringence axis and diattenuation axis are not required to be aligned. Additionally, it is found that Eqs. (6)–(11) are all function of Mueller elements, thus those effective optical parameters also can be extracted by a Mueller polarimetry. Moreover, as similar to Sec. 3.2, the error bars of Mueller elements ($m_{ij}$, $i, j = 1 \sim 4$) obtained by Eqs. (6)–(11) have values less than $\pm 0.006$. As compared to $\pm 0.005$ random perturbations in the output Stokes parameters, the extracted Mueller elements are still quite robust. Although only four different input polarization lights, namely three linear polarization lights (i.e., $S_4$, $S_{45}$, and $S_{90}$) and one circular polarization lights (i.e., $S_{RHC}$) are enough in obtaining all Mueller elements. However, the over-determined system in obtaining all Mueller elements by using six input polarized
lights can provide an important means to verify the correctness of values extracted from the experimental results. In future work, the proposed method will be extended in extracting nine effective parameters, not only on the orientation angle and phase retardance of LB, the orientation angle and linear dichroism of LD, the optical rotation of CB, and the circular dichroism of CD properties, but also on two linear depolarizations and the circular depolarization properties of turbid media.

Acknowledgments
The authors gratefully acknowledge the financial support provided to this study by the National Science Council of Taiwan under Grant No. NSC99-2221-E-006-034-MY3. Also, the related patent was issued in 2011.

References