Optical coherence tomography images simulated with an analytical solution of Maxwell’s equations for cylinder scattering

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Abstract. An algorithm for the simulation of image formation in Fourier domain optical coherence tomography (OCT) for an infinitely long cylinder is presented. The analytical solution of Maxwell’s equations for light scattering by a single cylinder is employed for the case of perpendicular incidence to calculate OCT images. The A-scans and the time-resolved scattered intensities are compared to geometrical optics results calculated with a ray tracing approach. The reflection peaks, including the whispering gallery modes, are identified. Additionally, the Debye series expansion is employed to identify single peaks in the OCT A-scans. Furthermore, a Gaussian beam is implemented in order to simulate lateral scanning over the cylinder for two-dimensional B-scans. The fields are integrated over a certain angular range to simulate a detection aperture. In addition, the solution for light scattering by layered cylinders is employed and the various layers are identified in the resulting OCT image. Overall, the simulations in this work show that OCT images do not always display the real surface of investigated samples. © 2016 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.JBO.21.4.045001]

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1 Introduction

Optical coherence tomography (OCT) is an emerging tool in clinical diagnostics, which enables high-resolution imaging of the internal structure of biological tissue.1–3 It is, for example, used to detect glaucoma in ophthalmology and coronary intervention can be facilitated in cardiology. In a clinical environment, the processed interferograms are interpreted based on experience because the link between the microscopic properties of the scatterers influencing light propagation and the generated images is not fully understood.4–7 Therefore, the formation of images in OCT is investigated with computational models. Depending on the accuracy needed for simulated images, different OCT models are employed in literature. Many OCT simulations use Monte Carlo approaches for the radiative transfer equation8–13 or they use a numerical solution of Maxwell’s equations without accounting for the interferometer setup.14,15 Several proceedings dealing with Maxwell’s equations and OCT exist as well.16–19 In general, the use of the radiative transfer equation in a Monte Carlo approach is an approximation that neglects interference phenomena. Additionally, omitting the interferometer setup changes the properties of the resulting images. To the knowledge of the authors, only recently a full-wave approach including the interferometer setup has been implemented for simulation of image formation in OCT.19,20

In this work, the analytical solutions of Maxwell’s equations for scattering of a plane wave and of a two-dimensional (2-D) Gaussian beam by an infinitely long dielectric homogeneous cylinder have been used to implement an interferometer setup with broadband illumination. The plane wave solution generates one-dimensional A-scans while the Gaussian beam can be scanned laterally over the cylinder to generate B-scans. To the knowledge of the authors, the analytical Maxwell solution for cylinder scattering has not yet been used for the simulation of OCT images. Additionally, the time-resolved fields scattered by a cylinder are calculated so that the intensities correspond to measurements with an ultrafast detector without an interferometer setup. For comparison of the time-resolved scattered intensity with geometrical optics, a 2-D ray tracing algorithm based on Snell’s law has been implemented for plane wave illumination. The simulated images based on Maxwell’s equations show the whispering gallery modes (WGMs) that appear in scattering by a cylinder. There is an approach for simulating OCT images with a solution of Maxwell’s equations for spherical scatterers,21 but the appearing WGMs have not been investigated further in their work and the single signals have not been associated with specific pathways. In this work, the single pathways are labeled and the behavior of the surface waves is explained based on the simulated OCT images. Beyond the geometrical optics simulation, we employ the Debye series to identify the OCT peaks in the Maxwell solution. To the knowledge of the authors, the Debye series has not been used for the simulation of OCT scans before. The basic understanding of the effects of a single scatterer with a smooth surface is the first step toward understanding more complicated scattering processes that influence the features of OCT images. Based on this knowledge, it will be possible to investigate more complicated scatterers in the future.

1.1 Theory

OCT uses a broadband interferometer setup to reconstruct the internal structure of semitransparent scatterers. The changes in refractive index and polarization affect the backscattered
light so that the correlation between the reference beam and the probing beam contains depth information about the scatterer. Either the length of the reference arm is shifted so that an axial profile is recorded (time domain OCT) or the whole information is recorded in the Fourier domain since the depth profile of the scatterer is also contained in the frequency spectrum according to the Wiener-Khintchine theorem. As shown in Fig. 1, a Fourier-domain OCT setup is considered in this work. It is assumed that the incident pulse in time domain has the form \( E(t) = E_0 e^{-\frac{t^2}{2\Delta t^2}} \cos(\omega_0 t) \) so that the beam splitter with a splitting ratio of \( \frac{1}{2} : \frac{1}{2} \) yields

\[
E_r(t) = E_i(t) = E_0 e^{-\frac{t^2}{2\Delta t^2}} \cos(\omega_0 t)
\]

in the reference arm and the sample arm, respectively. \( E_r \) is the field in the reference arm and \( E_i \) is the field in the sample arm incident on the scatterer. \( E_0 \) is the field amplitude, \( \omega_0 \) is the central frequency, and \( 2\sqrt{\log 2b} \) is the amplitude-related FWHM of the envelope of the pulse in time domain. The Fourier transform yields

\[
E_r(\omega) = E_i(\omega) = E_0 \frac{b}{2\sqrt{2}} \left( e^{-\frac{(\omega + \omega_0)^2}{\Delta^2}} + e^{-\frac{(\omega - \omega_0)^2}{\Delta^2}} \right).
\]

(2)

For the spectrum needed, we only consider positive frequencies and in order to keep the original peak height, the half-sided spectrum is multiplied by a factor of 2 and by the phase corresponding to the distance the pulse travels in each case (compare with the caption in Fig. 1):

\[
E_r(\omega) = 2E_0 \frac{b}{2\sqrt{2}} \left( e^{-\frac{(\omega + \omega_0)^2}{\Delta^2}} + e^{-\frac{(\omega - \omega_0)^2}{\Delta^2}} \right) e^{i\omega_1 r_1}.
\]

(3)

where \( r_1 \) is the distance from the light source to the cylinder surface and \( r_3 \) is the total distance the reference pulse travels. Often, the central wavelength \( \lambda \) and the full-width half-maximum (FWHM) \( \Delta \) of the source pulse are the given parameters for OCT devices. The central frequency is related to the central wavelength over \( o_0 = (2\pic/\lambda) \) with \( c \) being the speed of light in vacuum. The FWHM in frequency is related to the FWHM in \( \lambda \) as \( \Delta \omega \approx (2\pic\Delta\lambda/\lambda^2) \). From the spectral width follows the temporal FWHM as \( b = (4\sqrt{\ln 2}/\Delta\omega) \) for the field amplitude. The scattered field is calculated for \( N \) spectral values of the pulse ranging from \( o_{\text{min}} \) to \( o_{\text{max}} \). The first frequency is set to \( o_{\text{min}} = 0 \). The last frequency value is the frequency belonging to the cutoff amplitude, where \( E_r(o_{\text{max}}) = e(b/2\sqrt{2})E_0 \). This yields

\[
o_{\text{max}} = o_0 - \frac{2 \ln e}{\sqrt{-b^2 \ln e}}.
\]

(5)

The spectral values used for further calculation are, therefore, equally spaced \( o_0 = (k/c)\alpha_n \). \( c \) is chosen to be \( 10^{-30} \). Given the spectrum, the next step is to calculate the scattered field and the reference field. The scattered field is the solution of the Helmholz equation for a homogeneous, dielectric cylinder. The detected light in the far field for a plane wave with perpendicular incidence \( E_i(\omega) \) scattered by the cylinder in the sample arm is given by Bohren and Huffman as

\[
\begin{bmatrix}
E_{s,0}(r_3, \theta, k) \\
E_{s,1}(r_3, \theta, k)
\end{bmatrix} = e^{i\omega r_3} \sqrt{\frac{2}{\pi(r_2 + a)}} e^{ik(r_2 + a)} \begin{pmatrix} T_1(\theta) & T_2(\theta) \\ T_3(\theta) & T_2(\theta) \end{pmatrix} \begin{pmatrix} E_{s,0}(k) \\
E_{s,1}(k)
\end{pmatrix},
\]

(6)

\( \parallel \) is the TM mode with polarization parallel to the cylinder axis and \( \perp \) is the TE mode with polarization perpendicular to the cylinder axis. The distance between the point detector and the origin at the cylinder center is \( r_2 + a \), where \( a \) is the cylinder radius, \( r_2 \) denotes the distance the scattered wave travels from the cylinder surface to the detector. The \( T \)-matrix elements can be calculated according to

\[
T_1(\theta) = \sum_{n=-\infty}^{\infty} b_{n,0} e^{-i\alpha n} = b_{0,0} + 2 \sum_{n=1}^{\infty} b_{n,0} \cos(n\theta),
\]

(7)

\[
T_2(\theta) = \sum_{n=-\infty}^{\infty} a_{n,1} e^{-i\alpha n} = a_{0,1} + 2 \sum_{n=1}^{\infty} a_{n,1} \cos(n\theta),
\]

(8)

\[
T_3(\theta) = \sum_{n=-\infty}^{\infty} a_{n,1} e^{-i\alpha n} = -2i \sum_{n=1}^{\infty} a_{n,1} \sin(n\theta),
\]

(9)

\[
T_4(\theta) = \sum_{n=-\infty}^{\infty} b_{n,1} e^{-i\alpha n} = -2i \sum_{n=1}^{\infty} b_{n,1} \sin(n\theta) = -T_3(\theta).
\]

(10)
The truncation criterion for the infinite sum and the shape of the expansion coefficients $a_n$ and $b_n$ as used in this work can be found in Ref. 22. The orientation of the coordinate system for the scattering problem is shown in Fig. 2. $z$ is the axial coordinate and $y$ is the lateral coordinate. We refer to the cylinder surface facing the illumination source (e.g., a plane wave coming from negative $z$, as shown in the figure) as the cylinder front side and the cylinder surface on the opposite side at positive $z$-values as the cylinder back side. The scatterer is symmetric with respect to the $z$-axis in Fig. 2. This symmetry axis is referred to as the optical axis.

In order to decompose the scattered field into different interactions, the Debye series based on Refs. 23-26 is employed. Again, only perpendicular incidence is considered. The general Debye series for all incident angles can be found in Ref. 26. The Debye series introduces a geometrical series with transmission coefficients $T_{n1}^{21}$ and $T_{n1}^{22}$ and reflection coefficients $R_{n1}^{11}$ and $R_{n1}^{22}$ in order to identify single interactions. The superscript 2 stands for the surrounding medium while the superscript 1 stands for the region filled with the scatterer.

For the scattering problem is shown in Fig. 2. This symmetry axis is referred to as the optical axis.

The factor $\frac{1}{4}$ stands for diffraction.26-28 Thus, a geometrical picture is used to understand the structure of the scattered fields, but the equations contain the exact Maxwell solution.

For simulations with a Gaussian beam in two dimensions in the paraxial approximation, the expansion coefficients given in Ref. 22 are modified, as described in Ref. 29. For simplicity, only the case of TM polarization and perpendicular incidence is considered here. The expansion coefficient can then be modified as

$$b_n^\parallel = C_{n1} b_n |,$$  

(13)

where

$$C_{n1} = \left(1 + \frac{n}{6} \left(s \sqrt{2iQ_0} \right)^3 (Z^3 - 3Z) \right) C_{n0},$$  

(14)

and

$$C_{n0} = e^{i k z_0} \sqrt{i Q_0} e^{-i Q_0 z_0 \omega^2 / \nu_0^2}.$$  

(15)

As shown in Fig. 3, $z_0$ is the focus position where the beam waist has the minimum value $w_0$, $y_0$ is the lateral position of the beam, used for scanning over the sample. Furthermore, the abbreviations $Q_0 = [i - (2z_0 / kw_0^2)]^{-1}$, $Z = s \sqrt{2iQ_0(n + k y_0)}$, and $s = (1 / kw_0)$ have been used (compare with Ref. 29). The validity of the paraxial approximation has been successfully tested for the used values for the beam waist $w_0$ by comparing the beam to the beam constructed by the angular spectrum of plane waves method and by evaluating the integral given in Ref. 29 numerically.

The last modification of the expansion coefficients used in this work is for the case of layered cylinders. Light scattering by a layered cylinder can be calculated with a matrix formulation, as described in Refs. 30-32. It is mentioned in passing that a part of the scattering algorithm for layered cylinders has been adapted from Ref. 31. For a layered cylinder with $N_L$ layers, radii $r_j$ (j = 1, . . . , $N_L$ counted from the inner layers to the outer layers), refractive indices $n_j$ and perpendicular incidence of the illumination beam, the expansion coefficients $a_{n1}$ and $b_{n1}$ are calculated as

$$a_{n1} = \frac{\det A}{\det B},$$  

(16)

$$b_{n1} = \frac{\det C}{\det D},$$  

(17)

where $A_{ij}$, $B_{ij}$, $C_{ij}$, and $D_{ij}$ are $2N_L \times 2N_L$ matrices.30

Fig. 2 Orientation of the coordinate system for the scattering problem.

The origin of the coordinate system is at the center of the cylinder cross section. $n_1$ is the refractive index of the surrounding medium and $n_2$ is the refractive index of the cylinder. The optical axis is the line between the cylinder front side and the cylinder back side at $y = 0 \mu m$.

Fig. 3 Gaussian beam in the paraxial approximation. The analytical solution allows scanning in y-direction and shift of the focus in z-direction (where the beam waist radius has the minimum value $w_0$ when the amplitude has dropped by $1/e$). The focus is at $z = -z_0$ and $y = y_0$. 

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2 Calculation of an Optical Coherence Tomography Signal

The described equations are used to calculate the scattered field \( E_s(\omega) \) (compare Fig. 1). In Fourier-domain OCT, the detector measures the interferogram of the scattered field and the reference field and the measured data are transformed with the inverse Fourier transform to obtain a depth profile. For the scattering angle \( \theta = 180 \) deg, the OCT intensity is given by

\[
I_{OCT}(t, \theta = 180 \deg) = \frac{1}{2} c \varepsilon_0 \mu_0 n_1 F^{-1}(n_1^2 (E_s(\omega, 180 \deg) + E_r(\omega))^2). \tag{18}
\]

Since the detector collects the spectral data coherently over a certain angular range, the integral describing a 2-D detection aperture can be written as

\[
I_{OCT}(t) = \frac{1}{2} c \varepsilon_0 \mu_0 n_1 F^{-1}\left( \int_0^{\theta_i} (E_s(\omega, \theta) + E_r(\omega)) d\theta \right)^2. \tag{19}
\]

This is valid in the paraxial approximation under the assumption that the light coming from the cylinder is imaged onto the detector through a perfect imaging system without effects from finite lenses or aberrations. The integral can be evaluated analytically as

\[
I_{OCT}(t) = \frac{1}{2} c \varepsilon_0 \mu_0 n_1 F^{-1}\left( \int_0^{\theta_i} (E_s(\omega, \theta) + E_r(\omega)) d\theta \right)^2.
\]

The single terms of the OCT signal can also be decomposed as

\[
F^{-1}\left\{ |E_s(\omega) + E_r(\omega)|^2 \right\} = F^{-1}\left\{ |E_s(\omega)|^2 + 2 |E_s(\omega)| |E_r(\omega)| \cos(\omega \tau_1) \right\}.
\]

These terms represent the autocorrelation term, the offset term at the origin \( z = 0 \), and the cross-correlation signals, respectively. Instead of increasing the distance between the autocorrelation and the cross-correlation in the image by increasing \( r_z \) relative to \( r_1 + r_2 \) as shown in Fig. 1, (which requires higher \( N \) because of the numerical Fourier transform), the term

\[
I_{cc}(t) = \frac{1}{2} c \varepsilon_0 \mu_0 n_1 F^{-1}(E_s(\omega)^* E_r(\omega)) \tag{22}
\]

can be directly evaluated for the cross-correlation intensity. For clarification, a detailed calculation is given for a simplified model for a Fourier-domain OCT A-scan: assuming for simplicity, a reference pulse of the form \( E_1 e^{j \omega t_1} \) with field amplitude \( E_1 \), temporal width \( b_1 \) and delay \( t_1 \) because of the length of the reference arm, the field in the Fourier domain can be expressed as

\[
E_1 \frac{b_1}{\sqrt{2}} e^{-j \omega t_1} e^{j \omega t_1}. \tag{23}
\]

Equation (23) can be simplified by using \( |\alpha + \beta| = |\beta + \gamma| = |\alpha + \gamma| \) and by executing the inverse Fourier transform. The constant \( (1/2)c \varepsilon_0 \mu_0 n_1 \) is omitted in the following explanation. The calculation gives nine terms: Three are of the form \( E_i^2 \sqrt{2} e^{j \omega t_1} \), \( i \in \{1, 2, 3\} \). These terms form the offset at the origin because they do not depend on \( r \). Furthermore, there are six correlation terms of the form:

\[
E_i E_j \frac{b_i b_j}{\sqrt{2} \sqrt{b_i^2 + b_j^2}} e^{-j \omega \frac{t_i - t_j}{2}}, \quad i \neq j, \quad i, j \in \{1, 2, 3\}.
\]

With \( t_1 \) being the time delay of the reference arm, the terms containing \( t_1 - t_1 \), \( t_1 - t_2 \), \( t_1 - t_3 \), \( t_2 - t_2 \) or much shorter \( t_1 < t_2, t_3 \) than the sample arm so that the cross-correlations and autocorrelations do not overlap. In the first case, the pulse with \( t_1 - t_1 \) is on the negative side of the \( z \)-axis; in the second case, it is on the positive side. It can be seen from Eq. (24) that the OCT signal is wider than the width of the illumination pulse in the sample arm when comparing the denominators in the exponential function of the OCT cross-correlation signal to the original pulse from Eq. (1).

The height of the OCT cross-correlation signal depends not only on \( E_1 E_2 \) but also on \( b_1 \) and \( b_j \). Based on the definitions in Refs. 1, 33, and 34 for the detected OCT signal, the maxima of the OCT cross-correlation signal \( I_{cc} \) are compared to the square root of the time-resolved backscattered intensity \( \sqrt{I_z} = |E_z|^2 \) (compare Fig. 1). We scale \( \sqrt{I} \) to the maxima of \( I_{cc} \) by a factor of \( C_{scan} \), which is defined by the relation [compare Eq. (24)]:

\[
E_i E_j b_i b_j \frac{1}{\sqrt{2} \sqrt{b_i^2 + b_j^2}} = C_{scan} E_j. \tag{25}
\]
\[ C_{\text{sc}} = E_i \frac{b_1}{2}. \]  

(26)

Therefore, the maxima of the OCT cross-correlation signal intensity are predicted to equal the square root of the scattered intensity \( \sqrt{I_i} \) as long as the width of the pulses is not changed in the scattering process.

It is mentioned in passing that the simulations for geometrical optics have been conducted with a ray tracing algorithm based on Snell’s law and Fresnel’s equations and that a random number is drawn for the starting position of the light package between \( z = -a, y = -a \) and \( z = -a, y = a \) to simulate a plane wave incident on the cylinder cross section. The detected counts are converted to intensities as

\[ I_{\text{mc}} = \frac{1}{2} c c_0 |E_0|^2 \frac{n_{\text{counts}}}{\sum n_{\text{counts}} A_{\text{II}}} A_{\text{II}}, \]

(27)

where \( n_{\text{counts}} \) is the number of counts, \( E_0 \) is the amplitude from Eq. (1), \( A_{\text{II}} = 2a \) is the geometrical illumination line from \( -a \) to \( a \), and \( A_{\text{bin}} = \left[ 2 \sigma (r_2 + a) \right] / n_0 \) is the angular bin size when \( n_0 \) angles are calculated. Further literature concerning ray tracing and its modifications can be found in Govaerts et al.\(^{35}\) and in Lugovtsov et al.\(^{36}\)

### 3 Results

#### 3.1 Time-Resolved Intensity for Plane Wave Scattering

Before the OCT results are shown, the scattered intensities for plane wave illumination are discussed from the point of view of geometrical optics. This allows the attribution of pathways to every signal. Unless otherwise noted, only scattering has been considered in the simulations so that the imaginary part of the refractive index is zero throughout this work. Figure 4 shows the graphical output of the implemented ray tracing algorithm for a cylinder cross section based on Fresnel’s equations and Snell’s law for three selected pathways for plane wave illumination of a cylinder with radius \( a = 20 \mu m \) and \( n_2 = 1.4 \) in air \( n_1 \). The pathways are chosen so that the rays exit at \( \theta \approx 180 \) deg. Figure 4 shows that the light packages either travel along the optical axis (\( \gamma \approx 0 \mu m \)) so that the path lengths \( d \) counted from \( z = -a \) are multiples of the radius \( a \) times the refractive index \( n_2 \) so that \( d = 4mna_2 \) with \( m \in \mathbb{N}_0 \) or that the light packages follow pathways that form triangles or starlike shapes. For further reference, the rays in the geometrical optics picture are numbered according to how often the light package interacts with the boundary of the cross section. If the interaction is linked to a starlike pathway that can exit with \( \theta = 180 \) deg for the given refractive indices \( n_1 \) and \( n_2 \), the interaction number is denoted with a prime ('). Thus, the labels allow to distinguish between multiple reflections that occur on the optical axis and signals from more complex pathways. This number system is used throughout the rest of this work. The interactions in Fig. 4 are 1, 6' and 7', accordingly. In Fig. 5, the calculation is extended to all scattering angles \( \theta \) and the Maxwell solution is compared to the geometrical optics picture. \( E_0 \) is set to \( 1 \mu \text{m} \) throughout this work [compare Eq. (1)]. Figure 5 shows the scattered 2-D intensity that an ultrafast detector would register for each scattering angle \( \theta \) computed with (a) the implemented Maxwell solution from Eq. (6) and (b) the ray tracing algorithm for geometrical optics from Eq. (27) for a cylinder with radius \( a = 20 \mu m \) and a refractive index of \( n_2 = 1.4 \) in air \( n_1 \). The light is polarized parallel to the cylinder axis.

![Graphical illustration of a 2-D ray tracing simulation for three selected light packages for plane wave illumination of a cylinder with a radius \( a = 20 \mu m \) and \( n_2 = 1.4 \) in air. The incident geometrical rays are marked in red, the internal and the outgoing ones have different colors. The number of interactions with the boundary is 1 for the middle ray and 6' and 7' for the others, respectively. The light is polarized parallel to the cylinder axis.](https://www.spiedigitallibrary.org/journals/Journal-of-Biomedical-Optics)
along the circumference of the cylinder are the WGMs, which are well-described in literature\textsuperscript{37–40} with useful applications.\textsuperscript{41,42} The patterns in the Maxwell solution are in general broader and reach positions that are not reached in geometrical optics, which shows the wave behavior of light. The Maxwell solution gives an image that is fairly similar to the geometrical optics picture when the periodically appearing structures that are the WGMs radiating into the far field are ignored. The ray tracing solution and the Maxwell solution agree better when the cylinder diameter is very large compared to the wavelengths of the incident pulse.

3.2 Optical Coherence Tomography A-Scan for Plane Wave Scattering

With the basic effects of cylinder scattering explained for a plane wave at perpendicular incidence, the computational results for a plane wave OCT are discussed. With Eqs. (6) and (18), the A-scan shown in Fig. 6 is computed. The offset, the autocorrelations, and the cross-correlation terms are visible. The graph is symmetric as expected from Eq. (21). The $z$-axis is shifted so that the origin is at the cylinder front side just like in Fig. 5. The autocorrelation signals overlap slightly with the cross-correlation signals. This is due to the fact that $r_1$ is not different enough from $r_2$, but increasing $r_1$, $r_2$, or $r_3$ increases the calculation time because of the required $N$ (compare with Sec. 1.1). The cross-correlations are investigated in Fig. 7 with Eq. (22).

Figure 7 shows the OCT cross-correlation signal from Fig. 6 calculated with Eq. (22), the square root of the backscattered intensity, and the results of the ray tracing algorithm for $\theta = 180$ deg. The counts of the ray tracing algorithm are converted with Eq. (27). The ray tracing plot is drawn with a broader line for clarity. The square root of the backscattered field $\sqrt{I_r}$ (that an ultrafast detector for time-resolved measurements would detect) is scaled to the maxima of the OCT peaks with $\frac{1}{2} c e_q n_1 C_{\text{inc}}$ with Eq. (26) just like $I_{\text{mc}}$. The different insets show the signals 1 and 3 in detail as well as the signals 5, 6', 7', and 11' belonging to longer pathways and the WGMs. The first six WGMs are marked with “WGM.” It can be seen that the ray tracing algorithm for geometrical optics predicts the cylinder front side (interaction 1), the back side (interaction 3) and signals at $z = 112$ $\mu$m (interaction 5, compare Fig. 4), $z = 130.4$ $\mu$m (the ray that touches the cylinder boundary 6' times, compare Fig. 4), $z = 138.6$ $\mu$m (7' interactions), and $z = 217.8$ $\mu$m (11'). The intensity of the OCT signals that can be

![Fig. 5 The Maxwell solution is compared to the geometrical optics picture: (a) Maxwell and (b) ray tracing simulation for a cylinder with 20 $\mu$m radius. (c) The simulated setup. The noise from the inverse Fourier transform has been removed so that only intensities greater than or equal to $e^{-50}$ have been plotted in the Maxwell case. The 2-D intensities coded in the color values are on a logarithmic scale (to base e) in both plots. The distances are calculated from $z = tc$.](https://www.spiedigitallibrary.org/journals/Journal-of-Biomedical-Optics/045001-6-April-2016-Vol-21-4-Brenner-Reitzle-and-Kienle-Optical-coherence-tomography-images-simulated-with-an-analytical/045001-6-April-2016-Vol-21-4-Brenner-Reitzle-and-Kienle-Optical-coherence-tomography-images-simulated-with-an-analytical)

![Fig. 6 A-scan for an OCT system with $z = 845 \pm 300$ nm with $8 \times 10^4$ calculated spectral data points in backscattering direction ($\theta = 180$ deg). The offset in the middle, the autocorrelations and the cross-correlations are visible. The distances used are $r_1 = 6a$, $r_2 = 4a$, and $r_3 = 180a$. The cylinder has a refractive index of $n_2 = 1.4$ in air and its radius is $a = 20 \mu$m.](https://www.spiedigitallibrary.org/journals/Journal-of-Biomedical-Optics/045001-6-April-2016-Vol-21-4-Brenner-Reitzle-and-Kienle-Optical-coherence-tomography-images-simulated-with-an-analytical/045001-6-April-2016-Vol-21-4-Brenner-Reitzle-and-Kienle-Optical-coherence-tomography-images-simulated-with-an-analytical)
associated with a pathway from geometrical optics decreases faster than the intensity of the gallery modes. Furthermore, the intensity of the OCT cross-correlation signal \( I_{cc} \) is proportional to the square root of the backscattered intensity \( \sqrt{I_s} \), as calculated in Eqs. (24) and (26) as long as the width of the scattered pulses does not change. As expected from Eq. (24), the width of the OCT peaks is in general broader than the width of \( \sqrt{I_s} \).

### 3.3 Near Fields and Whispering Gallery Modes

Figure 8 shows snapshots of the cylinder near fields. The incident plane wave in (a) hits the cylinder boundary from the left and part of it is reflected and a part of it is transmitted into the cylinder. The wave outside where \( n_1 = 1 \) travels faster than the part of the wave that has to travel through the cylinder with \( n_2 = 1.4 \). At the cylinder back side, a part of the wave is transmitted, a part of it is reflected in 180-deg direction again and the WGMs appear. The part that is reflected backward is either reflected again or transmitted, but in both cases, the intensity has decreased a lot compared to the intensity of the gallery modes, which form at the back side of the cylinder in Fig. 8(b) and which start to travel along the circumference (c). Because of dispersion, their width changes the longer they travel (d). While the light travels a distance \( d = n_4 \pi n_2, n \in \mathbb{N}_0 \) along the optical axis when detected in backscattering direction \( \theta = 180 \text{ deg} \) (compare Fig. 7), the first gallery modes WGM 1 travel a distance \( d = 2 \pi n_2 + \pi a n_2 \). \( 2a \) appears because they first form at the boundary after the light has passed through the cylinder. \( n_2 \) indicates that the refractive index for the surface waves corresponds neither exactly to the refractive index \( n_2 \) of the cylinder nor \( n_1 \) of the surrounding medium. This is due to the fact that the WGMs do not exactly travel a circular pathway around the cylinder.\(^{44} \)

### 3.4 Plane Wave Optical Coherence Tomography with the Debye Series

Another method to understand the origin of the cross-correlation signals is to decompose the scattered field into single interaction terms. This is done in the Debye series for plane wave scattering (compare Refs. 23–26). To the knowledge of the authors, the Debye series has not been implemented for an OCT algorithm before. Figure 9 shows the OCT cross-correlation signal from Eq. (22) for different \( p \) (compare Eqs. (11) and (12)) for \( \theta = 180 \text{ deg} \). A chosen \( p \) includes all \( p + 1 \) interactions. It can be seen in Figs. 9(a)–9(f) that again the contributions from the cylinder front side, the back side, and the WGMs are recovered. When \( p = 2 \) is calculated in (b), not only the signal at \( z = 2an_1 \) appears from the optical axis but also the peak belonging to the WGM 1. This is in agreement with the observation in the near field FDTD simulation (Fig. 8) that the gallery modes WGM 1 are formed at the cylinder back side and that they travel along the cylinder circumference. Lock et al. show for calculations with the Debye series that scattered light reaches regions that are not accessible in geometrical optics, but their calculations are not within the context of OCT imaging.\(^{25,27,45} \)

### 3.5 Gaussian Beam Optical Coherence Tomography

In reality, most OCT devices employ a focused beam that scans over the sample to improve the lateral resolution of the images. To include the lateral scanning, a Gaussian beam has been implemented as shown in Fig. 3. The 2-D Gaussian beam has been implemented according to Eqs. (13) and (14) with an axial focus point at \( z_0 = 0 \mu \text{m} \) at the cylinder center. The used positions for the Gaussian beam in Fig. 10 in lateral

![Fig. 7](image-url)

\( \sqrt{I_{mc}}, \sqrt{I_s} \) and the OCT cross-correlation signal \( I_{cc} \) (compare Eqs. (6) and (22)) versus \( z \). The ray tracing simulation has 361 angular bins, 5001 temporal bins and \( 10^6 \) light packages. The OCT parameters are the same as in Fig. 6 except for \( N = 8 \cdot 10^9, r_1 = a, r_2 = a, \) and \( r_3 = 3a \). For the OCT signal, only the cross-correlation part according to Eq. (22) has been used.
direction range from \( y_0 = -26 \mu m \) to \( y_0 = 26 \mu m \) with a distance of \( \Delta y = 2 \mu m \) between the channels. Only the cross-correlation part of the 2-D intensity in the units \( \text{Wm}^{-1} \) has been considered according to Eq. (22). The zero-position \( y_0 = 0 \mu m \) is scanned as well. Thus, in total, 27 \( y_0 \)-values have been used. The cylinder is outlined in black. The distances in the OCT picture can be seen. Since only the scattering angle \( \theta \) is considered, no curvature of the cylinder surfaces is visible. Signals from other angles are not detected. The distance between the cylinder front and back side (interactions 1 and 3) at \( y_0 = 0 \mu m \) is the expected value \( d = 2an_2 \). At the upper and lower lateral sides of Fig. 3 (when \( y_0 \neq 0 \mu m \)), patterns that are not labeled can be seen in the background. It has been shown in previous work\(^4\) that they stem from the limited length of the numerical inverse Fourier transform and that they decrease for higher \( N \). At large axial depths \( z \), the gallery modes show again the expected change in width.

Figure 11 shows the pulse that is reflected from the first surface (interaction 1) during the lateral scan over \( y_0 \). The values are the same as those used for Fig. 10 except for \( z_0 = a \). Thus, we can compare the lateral width of the reflected signal to the minimum width \( w_0 \) of the incident Gaussian beam. The width that is found in Fig. 11 has the width \( w_0 \) of the incident Gaussian pulse in \( y_0 \) direction. This is due to the fact that only the part of the Gaussian that is backscattered in \( \theta = 180\text{-deg} \) direction is detected. Since the focus with the minimum beam waist \( w_0 \) is located at the cylinder surface \( z_0 = a \), the beam waist \( w_0 \) is the width that is found by the fitting algorithm with the fitted function \( f(x) = \alpha_1 e^{\left(\frac{x}{w_0}\right)^2} \).

### 3.5.1 Optical coherence tomography with an aperture

In Fig. 12, the OCT cross-correlation signal simulated with an aperture is shown for a Gaussian beam with a beam waist of \( w_0 = 3.9 \mu m \). The angular range included in Eq. (19) is \( 180 \pm 80 \text{-deg} \). The integral from Eq. (19) is solved numerically for a discrete number of angles in the first two images (from left to right) and the third image shows the analytical solution from Eq. (20). It can be seen that the numerical solution with 1001 angles on the left and the analytical solution on the right agree well with each other. The numerical solution with only 11 angles has not converged to the correct result yet. For 1001 angles and for the analytical solution, the signals at the lateral cylinder sides do not lie exactly on the surface and additionally, fewer signals are observed compared to 11 angles. The interference in the
coherent detection mode prevents imaging of the curvature of the smooth cylinder surface. The curvature of the cylinder surface is visible for the range of $\theta = 180 \pm 80$ deg in Fig. 5. This is due to the fact that in Fig. 5, no interferometer setup and no aperture integral for coherent detection is calculated. Similar simulated and experimental OCT images are presented in Yi et al. for calculations with spheres, but the WGMs and the individual pathways according to geometrical optics were not discussed in depth. For rough surfaces, the curvatures of the scatterers are visible both in simulated and experimental OCT images. We expect to see the WGMs and the missing curvature in the tomograms when the scatterer is similar to a cylinder or a sphere with a smooth surface (when absorption is negligible). For example, we predict that measuring a glass fiber with a smooth surface will produce OCT images showing the gallery modes.

3.5.2 Optical coherence tomography simulation for a layered cylinder

Figure 13 shows the simulated cross-correlation OCT image for a layered cylinder with three layers for TM polarization.
according to Eq. (17). Equation (20) has been used to simulate
the collection of the signal over a certain angular range ... 2016 • Vol. 21(4)
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Fig. 12 OCT cross-correlation signal for calculation with an aperture according to Eq. (19). The two images (a) and (b) are numerical solutions and image (c) is the analytical solution with Eq. (20). The used values are \( N = 8000 \), \( \lambda = 1300 \pm 300 \text{ nm} \), \( y_0 \) from \(-76 \mu \text{m}\) to \(76 \mu \text{m}\) with \( \Delta y_0 = 2 \mu \text{m} \), \( z_0 = 0 \mu \text{m}, r_1 = a, r_2 = a \) and \( r_3 = 3a \). The cylinder radius is \( a = 70 \mu \text{m} \). The logarithm of the 2-D intensity is plotted to base \( e \).

Fig. 13 Cross-correlation signal of an OCT B-scan for a layered cylinder. \( z \) has again been shifted and divided by a factor of 2. The incident Gaussian beam has a beam waist of \( w_0 = 5 \mu \text{m} \) and \( \lambda = 845 \pm 300 \text{ nm} \) has been used with \( N = 8000 \) values. The used positions for the Gaussian beam in lateral direction range from \(-9 \mu \text{m}\) to \(9 \mu \text{m}\) with a distance of \( 1 \mu \text{m} \) between the channels. In total, 19 \( y_0 \)-values have been used. The focus is at the cylinder center. The logarithm to base \( e \) of the 2-D intensity is shown. The distances in the OCT system are \( r_1 = a, r_2 = a \), and \( r_3 = 3a \). The cylinder layers are drawn in black.

4 Conclusions
A 2-D algorithm for the simulation of image formation in OCT for scattering of a plane wave and of a Gaussian beam by an infinitely long homogeneous dielectric cylinder has been implemented. The basic effects of cylinder scattering have been investigated and the pathways from the geometrical optics picture have been compared to the full-wave Maxwell solution. The relation between the OCT signals and the backscattered light has been calculated and the scaling factor is used to compare
the square root of the backscattered light and the OCT signal directly. The single contributions from the scattering interactions in geometrical optics have been labeled and linked to photon pathways. With the Debye series and with the ray pathways calculated from geometrical optics, the single signals have been explained and the differences between geometrical optics and the wave nature of light have been linked to the observed OCT signals. Both for a plane wave and for a scanning Gaussian beam, the WGMs, which travel along the circumference of the cylinder, have been identified in the Maxwell solution. The simulated OCT image of a layered cylinder has shown the different layers scaled with their respective refractive indices. The integral over a certain angular range for the 2-D aperture has shown that the curvature of the smooth cylinder in lateral direction is not visible in the computed simulations. It is shown that OCT images do not always display the real surface of the investigated sample.

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References


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