Finding new local minima by switching merit functions in optical system optimization

Alexander Serebriakov
Florian Bociort, MEMBER SPIE
Joseph Braat, MEMBER SPIE
Delft University of Technology
Optics Research Group
Lorentzweg 1
NL-2628 CJ Delft, The Netherlands

Abstract. A strategy to escape from poor local minima by switching merit functions during local optimization is discussed. As a switching partner, we define a new auxiliary merit function, which also tends to zero for ideal systems, but differs significantly from traditional merit functions. The examples include high-dimensional optimization problems. © 2005 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/1.2074827]

Subject terms: optical design; geometrical optics; optimization; merit function; aberrations.

Paper 050610LR received Jul. 26, 2005; revised manuscript received Aug. 26, 2005; accepted for publication Aug. 26, 2005; appeared online Aug. 26, 2005; published online Oct. 24, 2005.

1 Introduction

The presence of multiple local minima during optimization is one of the major challenges of optical system design. If the number of optimization variables is not too large, recent achievements in global optimization give the optical designer very powerful tools to find good solutions.1–6 These methods tend however to be very time-consuming if the dimensionality of the optimization problem is large. In this work we describe a computationally effective strategy which, despite the fact that it cannot guarantee success, can often improve the result of local optimizations that would otherwise converge to poor local minima.

2 Modifying the Merit Function

One of the empirical strategies to find a new solution or to escape from stagnation is to modify the conditions under which local optimization algorithms operate. This can be achieved by changing, for instance, some system parameters, some parameters of the local optimization algorithm, or the merit function. Many computer programs allow an easy switch between merit functions (MFs) based on transverse and wavefront aberrations. Although occasionally successful, this strategy is limited by the fact that when system parameters change, the behavior of these two merit functions is usually very similar. Experience shows that the chances of success for finding a new solution increase when the two MFs both tend to zero for ideal systems but differ sufficiently from another.

If a standard merit function leads to an unsatisfactory solution, an auxiliary merit function that could be tried out in the first stage of optimization can be defined as follows: Consider an arbitrary ray (A′I′ in Fig. 1) that has in the image space the direction cosines L and M with respect to the x and y axes, respectively. The two components of the transverse aberration of the ray (defined with respect to the chief ray) are denoted by \( \delta \hat{x} \) and \( \delta \hat{y} \). The ray intersects in \( I' \) the image plane and in \( A' \) a sphere centered in the intersection point \( I \) of the chief ray with the image plane. The radius \( R \) of the sphere should be chosen larger, but still of the same order of magnitude as the length \( II' \) of the transverse aberration vector. The length \( R' \) of the segment \( A'I' \) is then given by

\[
R/R = a + \sqrt{1-b} = a + 1 - b/2 - b^2/8 - \ldots,
\]

where we have used the abbreviations

\[
a = (L\delta x + M\delta y)/R
\]

and

\[
b = [(\delta \hat{x}^2 + \delta \hat{y}^2) - (L\delta x + M\delta y)^2]/R^2.
\]

Since for ideal imaging \( R' \) tends to \( R \), the quantity \( R'/R - 1 \), averaged over all rays, can be used as an auxiliary merit function. (Since the chief ray is the reference, distortion has no effect on the auxiliary MF.) The power series expression should be used in Eq. (1) instead of the exact expression in order to avoid abnormal termination when for certain rays that have large aberrations \( b \) becomes larger than 1.

Starting the optimization with the auxiliary MF defined above and ending it with the standard MF required by the application often leads to a different, sometimes better, solution than when only the standard MF is used. In addition, switching back and forth between the standard and the auxiliary MF can be useful to escape from stagnation. Switching to the auxiliary MF in an optimization stage where the standard MF does not change any more may be sufficient for escaping from stagnation but not for escaping from an

Fig. 1 Definition of the auxiliary merit function.
unsatisfactory solution. To maximize the chances of success for obtaining a different solution, optimization with the auxiliary MF should be started at a poor configuration for which the value of the standard MF is high enough. Often, the initial configuration used for standard optimization is adequate. The optimization can then take a path in the variable space that differs sufficiently from the one in the case of the standard MF.

3 Results

We have implemented the auxiliary MF as a user-defined merit function in the optical design program CODE V. Figure 2 shows the evolution of the auxiliary MF as well as that of the default MFs of CODE V, root mean square (RMS) spot size and RMS wavefront, during an optimization driven by the auxiliary MF. It can be observed that, while the auxiliary MF always decreases, the other two MFs actually increase beyond point A. Moreover, the RMS spot size and RMS wavefront appear to be strongly correlated, i.e., they increase or decrease at the same time. In several other tests we have found that the behavior of the transverse aberration along trajectories in the variable space is much stronger correlated with that of the wavefront than with that of the auxiliary MF. Therefore, the auxiliary MF is a more successful switch partner for RMS spot size than RMS wavefront.

Several examples where the use of the auxiliary MF has moved the local optimization into the basin of attraction of a new minimum of a standard MF are shown in Figs. 3–5.
In the examples shown in Figs. 3 and 4 the default RMS spot size of CODE V after the two-step optimization (auxiliary+default) is lower than after optimization with the default MF only. In the intermediate stage after optimization with the auxiliary MF (not shown in Figs. 3 and 4) the solutions already have almost the same shape as that shown after the final reoptimization with the default MF.

For complex systems with a large number of variables, computational efficiency becomes extremely important in the search for new solutions. Since the auxiliary MF uses only standard data for the rays to be traced \( L, M, \delta x, \delta y \) it is computationally efficient. A new local minimum obtained with our method in an optimization problem with 83 variables is shown in Fig. 5. For this lithographic lens, distortion control has also been included. The new two-stage solution may be interesting even when, as in the example shown in Fig. 5, it has a higher MF than the one obtained directly with the standard MF. Sometimes, the new solution differs sufficiently from the known ones, and, taken as a starting system for other design techniques, it can lead to unexpected design forms.

4 Conclusion

In this paper we have defined a new type of merit function that differs significantly from the known ones. If the use of a standard merit function leads to an unsatisfactory result (poor local minimum or stagnation), switching between the new and the standard merit function can lead to escape. Even if success cannot be guaranteed, the switching can be rapidly tried out in design problems such as complex systems with many optimization variables where global optimization methods are too slow or inapplicable for other reasons. The new merit function can be easily implemented in existing optical software.

Acknowledgment

The first author gratefully acknowledges the support of this research by ASML Netherlands BV. We also acknowledge the use for this work of an educational license of CODE V.

References