Theoretical analysis of a method for extracting the phase of a phase-amplitude modulated signal generated by a direct-modulated optical injection-locked semiconductor laser

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Abstract. The phase modulation (PM) and amplitude modulation (AM) of optical signals can be achieved using a direct-modulated (DM) optical injection-locked (OIL) semiconductor laser. We propose and theoretically analyze a simple method to extract the phase component of a PM signal produced by a DM-OIL semiconductor laser. The pure AM component of the combined PM–AM signal can be isolated by square-law detection in a photodetector and can then be used to compensate for the PM–AM signal based on an optical homodyne method. Using the AM compensation technique, we successfully developed a simple and cost-effective phase extraction method applicable to the PM–AM optical signal of a DM-OIL semiconductor laser. © *The Authors. Published by SPIE under a Creative Commons Attribution 3.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI.* [DOI: 10.1117/1.OE.56.5.056112]

1 Introduction

The generation and processing of high-speed optical signals are critical issues in current and next-generation information technology applications that require ultrahigh data rates, such as high-capacity communications inside data centers, optical interconnects, and supercomputers.[1,2] Among various optical signal generation and processing techniques, the phase modulation (PM) of optical carriers has been widely investigated because of its promise to satisfy the ever-increasing requirements for high-speed data transmission.[3–6] The PM of an optical carrier is typically achieved using an external optical modulator, such as an acoustooptic or electrooptic modulator.[7,8] However, these external modulators exhibit several limitations, including large-form factors and high cost, and are typically difficult to integrate with other photonic devices.[9–11]
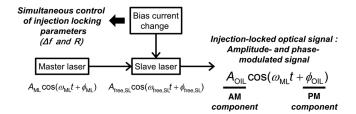
To overcome these limitations, optical injection-locked (OIL) lasers have been proposed and found effective for generating signals with PM simply by changing the injection-locking parameters, specifically, the detuning frequency between the master and free-running slave lasers and the injection ratio between the powers of the master and slave lasers.[12–14] We recently reported the theoretical analysis of the PM of an OIL laser based on the direct modulation (DM) of a slave laser, the results of which demonstrated the successful enhancement of the PM range up to 360 deg using a cascaded connection of injection-locked laser stages.[15] Although use of a DM-OIL laser is a simple and efficient means for producing a PM optical signal, the output signal contains a combination of both PM and amplitude modulation (AM) components, which limits the application of the resulting PM optical signal. To address this limitation, the PM component must be isolated and extracted.

Herein, we present a simple phase extraction method applicable to the PM–AM combined signal produced by a DM-OIL. We analyze the PM–AM combined effect of the DM-OIL using coupled rate equations and perform a theoretical investigation of pure PM extraction using an optical homodyne method. The pure AM component can be captured by square-law photodetection and can then be used to compensate for the combined PM–AM signal. Using the AM compensation method, we successfully isolated the pure PM component. The proposed pure PM extraction technique enables the use of DM-OILs in real-field PM applications, such as complex-format optical signal generation, optical signal processing, coherent optical communications, and light detection and ranging (LIDAR) systems.

2 Operating Principle

Figure 1 shows a schematic, the principles of a DM-OIL laser, and the mechanism of PM–AM combined optical signal generation. To generate a PM–AM optical signal using an OIL laser, two injection-locking parameters must be controlled, namely, the detuning frequency $\Delta f(= f_{\mathrm{ML}} - f_{\mathrm{free,SL}})$ and the injection ratio $R(= S_{\mathrm{ML}}/S_{\mathrm{free,SL}})$, where f_{ML} and $f_{\mathrm{free,SL}}$ are the frequencies of the optical signals from the master and free-running slave lasers, respectively, and S_{ML} and $S_{\mathrm{free,SL}}$ are the photon numbers of the master and the free-running slave lasers, respectively. We recently proposed a simple and efficient method for producing a PM optical signal by controlling the injection-locking parameters of a directly modulated slave laser.[15] The phase and amplitude of the injection-locked laser can be modulated by changing the bias current of the slave laser. The magnitude of the AM component in the PM–AM combined optical signal can be readily evaluated using affordable optical instruments, such

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Fig. 1 Schematic of PM and AM using the DM of an OIL semiconductor laser. In the equations, A_{ML} is the field amplitude of the master laser, ω_{ML} is the angular frequency of the master laser, ϕ_{ML} is the initial phase of the master laser, $A_{free,SL}$ is the field amplitude of the free-running slave laser, $\omega_{free,SL}$ is the angular frequency of the free-running slave laser, $\phi_{free,SL}$ is the initial phase of the free-running slave laser, A_{OIL} is the field amplitude of the OIL slave laser, and ϕ_{OIL} is the phase of the OIL slave laser.

as an optical spectrum analyzer or optical power meter; however, the magnitude of the PM component is not so easily measured.

The optical homodyne method can be used to determine the magnitude of the PM, as shown in Fig. 2.[16] The optical signal from the master laser is characterized by a field amplitude of A_{ML}, an angular frequency of ω_{ML}, and an optical phase of ϕ_{ML}. A portion of the master laser signal is used to achieve injection locking in the slave laser, while the remainder is sent to an external optical modulator. The OIL signal simultaneously modulates the field amplitude of A_{OIL} and phase of ϕ_{OIL} by changing the bias current of the slave laser, whereas the angular frequency of ω_{ML} is locked to the angular frequency of the master laser within a stable injection-locking range. The portion of the master laser signal sent to the external optical modulator is modulated by an electrical radio-frequency (RF) signal with an angular frequency of ω_{mod}. The modulated optical signal produced by the external modulator exhibits optical double sidebands, as shown in Fig. 2. The PM–AM optical signal from the DM-OIL is mixed with an RF-modulated optical signal in an optical combiner and detected using a photodetector. The amplitude A_{ML} and phase ϕ_{ML} of the master laser and the amplitude A_{mod} of the RF signal are all fixed at constant values, whereas the amplitude A_{OIL} and phase ϕ_{OIL} of the DM-OIL are modulated by changing the bias current of the slave laser. The signal amplitude detected by the typical homodyne

detection technique contains both the AM and PM optical components. However, to be useful in many real applications, the AM and PM components must be separated.

To accomplish this, we propose a pure-PM extraction method in a DM-OIL semiconductor laser, as shown in Fig. 3. The signal from a DM-OIL semiconductor laser is divided into two separate signals, which is different than the typical homodyne detection configuration shown in Fig. 2. A portion of the signal is detected by a photodetector PD_1, from which the pure AM component A_{OIL} is extracted. The remainder of the signal is combined with an RF-modulated optical signal to achieve homodyne detection using the same procedure as that shown in Fig. 2. The PM–AM combined optical signal is detected by a photodetector PD_2, and the AM component is compensated for by the signal A_{OIL} acquired from PD_1. Correspondingly, the amount of PM can be determined from the AM compensation because the compensated signal contains only the variable ϕ_{OIL}, which is the pure PM of the DM-OIL semiconductor laser.
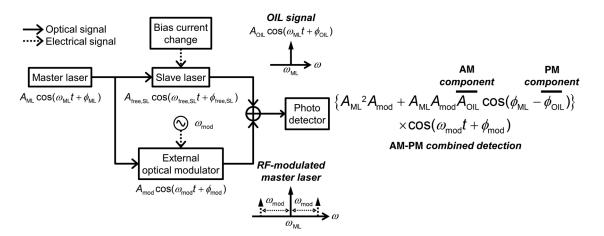
3 Theory

We performed a theoretical analysis of the method used to extract the PM from the output of a DM-OIL semiconductor laser by AM compensating for the typical homodyne detection method. Our analysis was based on

$$E_{ML}(t) = A_{ML}\cos(\omega_{ML}t + \phi_{ML}), \qquad (1)$$

$$E_{OIL}(t) = A_{OIL}\cos(\omega_{ML}t + \phi_{OIL}), \qquad (2)$$

$$\phi_{OIL} = \phi_{ML} + \Delta\phi, \qquad (3)$$

where $E_{ML}(t)$ and $E_{OIL}(t)$ are the time-varying optical field expressions of the master and injection-locked lasers, respectively; A_{ML}, ω_{ML}, and ϕ_{ML} are the field amplitude, angular frequency, and initial phase of the master laser, respectively; and A_{OIL}, ϕ_{OIL} are the amplitude and phase of the OIL laser, respectively. The ϕ_{OIL} term can be expressed as $(\phi_{ML} + \Delta\phi)$, where $\Delta\phi$ is the phase shift of the OIL laser. The A_{OIL} and ϕ_{OIL} terms can be calculated using the standard coupled-rate equation that describes the behavior of an OIL laser[17]

Fig. 2 Limitations of the typical homodyne detection method used to evaluate the magnitude of the PM of a DM-OIL laser. The detected signal contains both PM and AM components, where A_{mod} and ω_{mod} are the field amplitude and angular frequency, respectively, of the electrical RF signal.
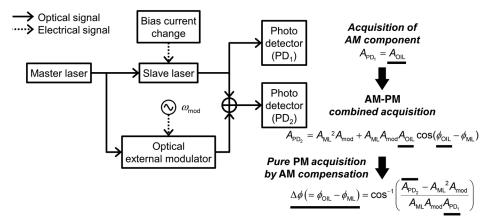
Fig. 3 Schematic showing the method of extracting the pure PM component by the AM compensation of the PM–AM combined optical signal generated by a DM-OIL semiconductor laser.

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = \{g[N(t) - N_{\mathrm{tr}}] - \gamma_p\}S(t)$$
$$+ 2\kappa\sqrt{S_{\mathrm{ML}}S(t)}\cos[\phi(t) - \phi_{\mathrm{ML}}], \quad (4)$$

$$\frac{\mathrm{d}\phi(t)}{\mathrm{d}t} = \frac{\alpha}{2}\{g[N(t) - N_{\mathrm{tr}}] - \gamma_p\} - \kappa\sqrt{\frac{S_{\mathrm{ML}}}{S(t)}}\sin[\phi(t) - \phi_{\mathrm{ML}}] - \Delta\omega, \quad (5)$$

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = J(t) - \gamma_n N(t) - g[N(t) - N_{\mathrm{tr}}]S(t), \quad (6)$$

where $S(t)$, $\phi(t)$, and $N(t)$ are the photon number, phase, and carrier number of an injection-locked slave laser, respectively, and N_{tr} is the transparency carrier number of a free-running slave laser, which is defined as $N_{\mathrm{tr}} = N_{\mathrm{th}} - \gamma_p/g$, where N_{th}, γ_p, and g are the threshold carrier number, photon decay rate, and linear gain, respectively. The term $\Delta\omega(= 2\pi\Delta f)$ is the angular frequency difference between the master and free-running slave lasers, α is the linewidth enhancement factor of the laser, $J(t)$ is the number of electrons in the DC bias of the slave laser, κ is the field coupling ratio between the master and free-running slave lasers, and γ_n is the carrier decay rate. The phase shift $\Delta\phi$ of the injection-locked laser can be derived as

$$\Delta\phi = \sin^{-1}\left\{-\frac{\Delta\omega}{\kappa\sqrt{1 + \alpha^2}}\sqrt{\frac{S_{\mathrm{OIL}}}{S_{\mathrm{ML}}}}\right\} - \tan^{-1}\alpha, \quad (7)$$

where S_{OIL} is the steady-state photon number given by

$$S_{\mathrm{OIL}} = \frac{S_{\mathrm{free,SL}} - (\gamma_n/\gamma_p)\Delta N_0}{1 + (g\Delta N_0/\gamma_p)}, \quad (8)$$

where ΔN_0 is the steady-state carrier number. From Eq. (7), $\Delta\phi$ can be modulated by either varying $S_{\mathrm{OIL}}/S_{\mathrm{ML}}$ or $\Delta\omega$, which are closely related to, or identical to, the definitions of the injection-locking parameters Δf and R. The modulated phase shift is in the range

$$-\frac{\pi}{2} \leq \Delta\phi \leq \cot^{-1}\alpha. \quad (9)$$

Although the optical PM obtained using an injection-locked semiconductor laser can be controlled by tuning the injection-locking parameters, the PM index is typically limited to π. The index can be enhanced to 2π if a cascaded OIL configuration is employed.[15] The steady-state photon number S_{OIL} can be modulated by varying either $S_{\mathrm{free,SL}}$ or ΔN_0, which changes the DC current of the slave laser. Using Eqs. (7) and (8), a time-varying optical field expression of the injection-locked laser can be obtained.[13,17–20] The master laser signal $E_{\mathrm{ML}}(t)$ is modulated by an external optical modulator with an electrical signal

$$E_{\mathrm{mod}}(t) = A_{\mathrm{mod}}\cos(\omega_{\mathrm{mod}}t + \phi_{\mathrm{mod}}), \quad (10)$$

where A_{mod}, ω_{mod}, and ϕ_{mod} are the field amplitude, angular frequency, and phase of the modulating RF signal, respectively. The corresponding modulated optical signal $E_{\mathrm{ML,RF-mod}}(t)$ is then

$$E_{\mathrm{ML,RF-mod}}(t) = [1 + A_{\mathrm{mod}}\cos(\omega_{\mathrm{mod}}t + \phi_{\mathrm{mod}})]$$
$$\times [A_{\mathrm{ML}}\cos(\omega_{\mathrm{ML}}t + \phi_{\mathrm{ML}})]. \quad (11)$$

The signal can be combined with $E_{\mathrm{OIL}}(t)$ in an optical combiner to achieve homodyne detection, and the combined signal can be detected using a square-law photodetector. The photodetector signal $E_{\mathrm{PD}}(t)$ is

$$E_{\mathrm{PD}}(t) = \{[1 + A_{\mathrm{mod}}\cos(\omega_{\mathrm{mod}}t + \phi_{\mathrm{mod}})]$$
$$\times [A_{\mathrm{ML}}\cos(\omega_{\mathrm{ML}}t + \phi_{\mathrm{ML}})] + A_{\mathrm{OIL}}\cos(\omega_{\mathrm{ML}}t + \phi_{\mathrm{OIL}})\}^2. \quad (12)$$

The corresponding amplitude for an angular frequency ω_{mod} term detected by the photodetector is

$$A_{\mathrm{PD}} = A_{\mathrm{ML}}^2 A_{\mathrm{mod}} + A_{\mathrm{ML}}A_{\mathrm{mod}}A_{\mathrm{OIL}}\cos\Delta\phi. \quad (13)$$

The signal at ω_{mod} detected by the square-law photodetector contains two variables A_{OIL} and ϕ_{OIL} as in Eq. (13). This is because both the amplitude and phase of the OIL laser can be simultaneously modulated by changing the injection-locking parameters. The output of the DM-OIL laser clearly

exhibits a PM–AM combined optical signal, which can be readily measured by a square-law photodetector, as in Eq. (13). Finally, the pure phase amount $\Delta\phi(= \phi_{OIL} - \phi_{ML})$ is

$$\Delta\phi = \cos^{-1}\left(\frac{A_{PD} - A_{ML}^2 A_{mod}}{A_{ML}A_{mod}A_{OIL}}\right). \qquad (14)$$
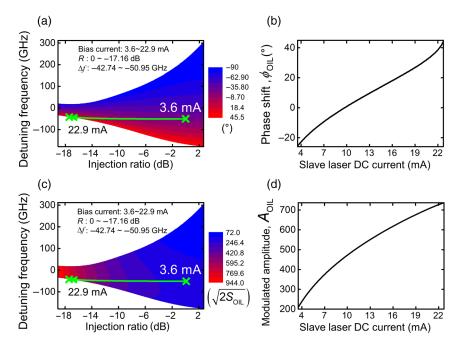
4 Simulation and Results

Figure 4(a) shows the injection-locking map as a function of the detuning frequency and injection ratio. The shaded region is the stable locking regime. We used the standard coupled-rate equations [Eqs. (4)–(6)] to characterize the behavior of an OIL laser, as we did in our previous work.[15] The simulation parameters are shown in Table 1.[17] The slave laser DC bias current was used as a tuning parameter to modify the injection-locking conditions. When the slave laser DC bias current was varied, the two injection-locking parameters, namely, the detuning frequency Δf and injection ratio R, changed simultaneously. The arrow lines in Fig. 4(a) show the evolution of the locking conditions when the slave laser DC current changed from 3.6 to 22.9 mA. The two values of the bias currents, 3.6 and 22.9 mA, were chosen because the maximum phase shift of 70.1 deg can be achieved between these bias values. A linewidth enhancement factor α of 1 was used for all analyses in this manuscript. The color map in Fig. 4(a) corresponds to a phase shift $\Delta\phi$ of the injection-locked laser as a function of the injection-locking parameters. The PM of the OIL laser caused by the DC current change of the slave laser is shown in Fig. 4(b) and ranges from 25.6 deg to 44.5 deg. Similarly, the color map in Fig. 4(c) shows the square root of the photon number of the OIL laser. This corresponds to the AM of the OIL laser caused by the change in the slave laser DC bias current. Figure 4(d) shows the amount of AM caused by

the DC bias current change in the slave laser. Figures 4(b) and 4(d) clearly show the PM–AM combined effect in the DM-OIL semiconductor laser.

Figure 5(a) shows the amplitude of the PM–AM combined signal detected by a square-law photodetector using the typical homodyne detection method shown in Fig. 2. The signal amplitude contains both PM and AM components. The pure AM component in Fig. 4(d) can be measured either by an optical power meter or photodetector, whereas the pure PM component in Fig. 4(b) cannot be easily measured by commercially available optical instruments. Therefore, the PM–AM combined signal in Fig. 5(a) requires a compensation process based on the pure AM component to fully extract the PM component.

Table 1 Simulation parameters.
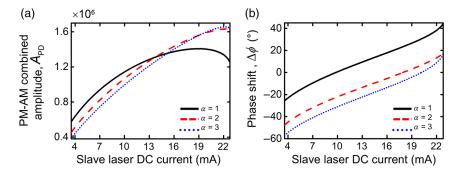
Parameter name	Symbol	Quantity	Unit
Center wavelength	λ	1550	nm
Linear gain	g	4.7×10^4	1/s
Threshold carrier number	N_{th}	2×10^7	—
Transparency carrier number	N_{tr}	9.36×10^6	—
Electron decay rate	γ_n	1	1/ns
Photon decay rate	γ_p	500	1/ns
Cavity length	L	250	μm
Reflectivity of facet	r	0.25	—
Field coupling ratio	κ	225	1/ns

Fig. 4 (a) Locking map showing the phase shift of the OIL laser as a function of the injection locking parameters. (b) Phase shift of the OIL laser as a function of the slave laser DC bias current. (c) Locking map showing the photon number change. (d) Amplitude of the OIL laser as a function of the slave laser DC bias current.

Fig. 5 (a) Simulation results of the amplitude of the PM–AM combined signal as a function of the slave laser DC current. (b) Phase shift $\Delta\phi(=\phi_{ML}-\phi_{OIL})$ as a function of the slave laser DC current using the AM compensation method. Both the red solid and blue dashed curves are potential solutions.

Fig. 6 (a) Amplitude of the PM–AM combined signal as a function of the slave laser DC bias for various α values. (b) Phase shift as a function of the slave laser DC current for various α values.

Figure 5(b) shows the adjusted amount of PM $\Delta\phi(=\phi_{ML}-\phi_{OIL})$ after application of the AM compensation method using Eq. (14). The absolute phase shift range from 25.6 deg and 44.5 deg can be extracted, as shown by the red solid and blue dashed curves in Fig. 5(b). The phase of the OIL laser moves from a negative to positive value when the slave laser DC bias current increases. It can be seen that a lower bias than 9.9 mA produces a negative phase shift, whereas a higher bias causes a positive phase shift. The phase change from −25.6 deg to 44.5 deg can be detected when the slave laser DC current changes from 3.6 to 22.9 mA, as shown by the red solid curve in Fig. 5(b). The results in Fig. 5(b) where the AM compensation method was employed are well matched with those in Fig. 4(b) and, thus, clearly demonstrate that the AM compensation technique can be used to extract the pure PM amount from the PM–AM combined signal.

The degree of PM achieved by the DM-OIL semiconductor laser is significantly affected by the linewidth enhancement factor α of the slave laser. For this reason, we performed phase extraction of the DM-OIL semiconductor laser for various α values of slave lasers. Figure 6(a) shows the compensated homodyne-detected amplitude of the PM–AM combined signal as a function of the slave laser DC bias for various α values. Figure 6(b) shows the extracted phase shift using the AM compensation method. The phase shift exhibited significantly different values depending on both the slave laser DC bias current and the α parameter, although the shift range was maintained at 70 deg. In the future, it is possible that the relationship between the phase shift and DC bias current can be used to experimentally extract the laser α parameter.

5 Conclusion

We performed a theoretical analysis of a method for extracting the PM of a DM-OIL semiconductor laser. First, we found that a DM-OIL semiconductor laser exhibits a PM–AM combined effect based on the coupled-rate laser equations. However, the typical homodyne detection method provides a PM–AM combined product that limits the application of DM-OIL semiconductor lasers. Second, we proposed a simple AM compensation technique based on the square-law detection of a photodetector to extract the PM amount, and the resulting pure AM component was used to compensate for the PM–AM combined component in an RF-modulated homodyne detection signal. The compensation technique enabled us to successfully extract the exact PM amplitude of the DM-OIL semiconductor laser. The application of this simple PM extraction technique to the PM optical signal of a DM-OIL semiconductor laser enables the use of these lasers in coherent optical communications, optical processing, and complex-format signal generation applications.

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