Absolute distance measurements by laser interferometry

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ABSTRACT - A double-channel Michelson type interferometer is presented which allows absolute distance measurement up to 3 m with an uncertainty of 0.1 um. It uses the principle of synthetic wavelength and can be used under vacuum or in an aseptic medium, with the help of a new type of source, called an air-wavelength standard. The CNES (Centre National d'Etudes Spatiales) is interested in an absolute measurement both in space (vacuum) and on the ground, in air, to characterize optical instruments.

1 - INTRODUCTION

Laser interferometry allows one to realize dimensional measurement with a low uncertainty, at the 10^-7 level can be reached when the measurements are made under vacuum. In air, however, an additional uncertainty arises from the knowledge of the refractive index of the medium n.

To determine n, one needs to measure the temperature, the pressure, the carbon dioxide concentration and the relative humidity with the help of calibrated sensors and the Edlen formula. Nevertheless, the relative uncertainty is still about 1 part in 10^7, which limits the accuracy of dimensional measurement.

In this paper, we describe an interferometer which enables one to measure distances between 0.4 and 3 m in air and in vacuum, with a relative uncertainty close to 1 part in 10^8.

2 - PRINCIPLE [Jun97] [Ikes92] [Dänd88][Gill83]

Fig. 1 shows the principle of the apparatus. It is based on a double channel Michelson-type interferometer illuminated by the beam of a frequency tuneable laser diode around 1.55 µm of frequency ν. The beam splitter-compensator assembly is composed of 3 identical Fresnel parallelepipeds, optically adhered together and forming an optical block. One of the parallelepipeds has a semireflecting coating on the internal face. Only one of the arms of the interferometer has a totally reflecting glass-air or vacuum interface, the glass part of the other arm behaves as a compensating plate. The first arm of the interferometer is ended with a corner-cube reflector mounted on a piezoelectric transducer. The second one is ended with an identical reflector situated a distance D further from the optical block.
Any incident beam whose polarization is oriented at 45° to the plane of incidence (equivalent to the plane of Fig. 1), can be split into two orthogonal polarizations, p and s, respectively parallel and perpendicular to the plane of incidence. We can consider that these two orthogonally polarized output beams interfere independently inside the interferometer. After a simple total reflection for each direction of propagation, these beams acquire a phase difference $\Delta \phi$ derived from the Fresnel formulae

$$\tan \left( \frac{\varphi_p}{2} \right) = \frac{n_\perp \sqrt{n_\parallel \sin^2 \vartheta - 1}}{\cos \vartheta}$$

$$\tan \left( \frac{\varphi_s}{2} \right) = \frac{\sqrt{n_\perp^2 \sin^2 \vartheta - 1}}{n_\parallel \cos \vartheta}$$

from which we deduce

$$\tan \left( \frac{\varphi_p - \varphi_s}{2} \right) = \tan \left( \frac{\Delta \phi}{4} \right) = \frac{\cos \sqrt{\sin^2 \vartheta - 1/n_\parallel^2} \sin \vartheta}{\sin \vartheta}$$

where $\vartheta$ is the beam angle of incidence on the total reflecting surface and $n_g$ is the refractive index of the medium. An angle of incidence $\vartheta = 55^\circ$ is necessary to obtain a 90° phase shift at $\lambda = 633\text{nm}$ with an optical block made from Schott BK7 glass ($n_g = 1.515$).

At the output of the optical block, a polarizing beam splitting cube separates the p and s components of the polarization. The two output intensities are

$$I_p = \frac{I_0}{2} (1 + \cos \varphi)$$

$$I_s = \frac{I_0}{2} (1 + \sin \varphi)$$

where $I_0$ is proportional to the intensity. The quantity $\varphi = 2\pi \frac{V_0}{c} 2nD = (k + \varepsilon)2\pi$ is the output phase of the interferometer, $c$ is the speed of light in vacuum, $n$ and $V_0$ are defined above, $D$ is the distance to be measured, $k$ is the integer part of the interference order and $\varepsilon$ is the fractional part. From the output beam of the interferometer and using two perpendicular polarizers set in front of detectors, we obtain the two signals in phase quadrature given in (2 3).
Fig. 1 Interferometric set-up: OB optical block, CC corner cube, P polarizer, D, and D, cosine and sine detectors, M and DM multiplexer and demultiplexer, D distance to measure, v_a and v_b source A frequencies, v_b source B frequency.

After an appropriate electronic treatment (subtraction of DC levels and intensity normalisation), these two signals are reduced to

$$\begin{align*}
I_x &= \cos \varphi \\
I_y &= \sin \varphi
\end{align*}$$

For control purposes, these two quadrature signals can be visualised by sending them to the inputs of an oscilloscope set to the XY configuration. The position of the spot gives directly a value of $\varphi$ (Fig. 2). Practical values of $\varphi$ are obtained by adequate computer treatment of these signals.

![Lissajou figure](https://www.spiedigitallibrary.org/conference-proceedings-of-spie)

After computing, $I_x$ and $I_y$ are sent to an A/D converter and to a sine and cosine inputs of a reversible counter. This counter determines the integer number of half fringes during the scan of the laser frequency.
To determine the fractional part, we have to
- measure \( \varphi \) over 255 steps uniformly distributed over \( 2 \times 2 \pi \) rad (the digital scan is ensured by a computer controlled piezoelectric transducer upon which the reference corner cube is mounted)
- normalise the sine and cosine experimental data by a procedure which determines the extreme values (suppression of error gain adjustment \( G_x \) and \( G_y \))
- determine the means-squares-circle [Raze 97] of the data recorded over \( 2 \pi \) rad (centre of the circle \( O(O_x, O_y) \) and the radius \( R_0 \))
Such that we have finally

\[
X = \frac{I_x}{G_x} - O_x \tag{25}
\]
\[
Y = \frac{I_y}{G_y} - O_y
\]

The value of \( \varphi \) is given by \( \varphi = \tan^{-1}(Y/X) \) From which we extract \( \varepsilon = \varphi / 2\pi \) (where \( 0 < \varepsilon < 1 \)) with an uncertainty \( \delta \varepsilon \) of order \( 10^{-4} \)

3 - MEASUREMENT UNDER VACUUM

3 - 1 - Description

The frequency \( v_\lambda \) of the tuneable laser diode is scanned from \( v_a \) to \( v_a^- \). reference frequencies corresponding to two optical transitions near 1.5\( \mu \)m of the acetylene molecule. The absolute distance \( D \) can be determined for each frequency

\[
D = (k + \varepsilon) \frac{c}{2v_a} \tag{31}
\]

where \( k \) is the integer part of the interference order and \( \varepsilon \) is the fractional part. \( D \) is also given by

\[
D = (\Delta k + \Delta \varepsilon) \frac{c}{2\Delta v} \tag{32}
\]

where \( \Delta k = k_a - k_a^- \), \( \Delta \varepsilon = \varepsilon_a - \varepsilon_a^- \) and \( \Delta v = v_a - v_a^- \). The ratio \( c/\Delta v \) is often called the synthetic wavelength. In our case \( v_a - v_a^- \) is of order 292 GHz.

In this step, \( \Delta k \) is known unambiguously and fractional parts at the beginning and end of the scan are determined by the method explained above. From equation (32) we calculate a first value of \( D \) with an uncertainty around 1.5\( \mu \)m derived from the uncertainty of the frequency difference \( \Delta v \) (97 kHz in our case) and from the uncertainty of \( \Delta \varepsilon \) (1.4 10^{-4}). With this uncertainty it is not possible however to obtain a better accuracy in the measurement of \( D \) by using equation (31) directly since the determination of \( k \) remains ambiguous.

This ambiguity can be solved by using a third frequency standard \( v_h \) such that \( v_h - v_a \) (900 GHz in our case) is three times larger than \( v_a - v_a^- \). This allows one to reduce the uncertainty of \( D \) to 0.5\( \mu \)m. For this latter case, since it is not possible to scan continuously the laser frequency from \( v_a \) to \( v_h \), the value \( k_a - k_a^- \) cannot be measured directly. It is however calculated without ambiguity from the values determined using the first step of the measurement.
Equation (3.1) constitutes the last step of the measurement since now the uncertainty of \( D \) obtained by step 2 permits a determination of \( k \). The final uncertainty in \( D \) is obtained using equation (3.1) and becomes 2.5nm (10\(^{-6}\) relative uncertainty for \( D=3m \)).

3 - 2 - Experimental results

For the moment, we realised the first step of the measure, which consists in:
- determining \( \varepsilon_a \)
- counting the number of fringes during the scan of the laser diode frequency from \( \nu_a \) to \( \nu_a ' \)
- determining \( \varepsilon_a ' \)

Because the measure of \( \varepsilon_a \) and \( \varepsilon_a ' \) is not simultaneous, we have to take into account any vibration or thermal expansion of our interferometer during the measuring time of around 30s. So that we use a second laser source which is a He-Ne laser at \( \lambda=633nm \). It works as a classical interferometer.

At last, the absolute distance \( D \) is given by

\[
D = \left( \Delta k + \Delta \varepsilon - d_1 \right) \frac{c}{2 \Delta \nu} \\
d_1 = \frac{V_a}{V_a} \left( \varepsilon_a - \varepsilon_a ' \right)
\]

The quantity \( d_1 \) is the correction determined from the calculation of \( \varepsilon_a \) and \( \varepsilon_a ' \) at the beginning and the end of the scan.

Distance measurements are made under vacuum (10\(^{-4}\)mbar and 0.1mbar) at about 0.41 and 3m. The reference hollow corner cube in then moved from about 77um by mean of the piezoelectric transducer. The panel below summarizes the results obtained by 100 measurements each time. The mean value, the experimental standard deviation (esd), the repeatability, the reproducibility and the global uncertainty are calculated.

<table>
<thead>
<tr>
<th>Distance (um)</th>
<th>10(^{-4})mbar</th>
<th>0.41mbar</th>
<th>0.1mbar</th>
</tr>
</thead>
<tbody>
<tr>
<td>D (um)</td>
<td>2.956901</td>
<td>0.411941</td>
<td>2.956900</td>
</tr>
<tr>
<td>esd (um)</td>
<td>1.5</td>
<td>2.1</td>
<td>1.1</td>
</tr>
<tr>
<td>repeatability (um)</td>
<td>1.2</td>
<td>1.7</td>
<td>2.5</td>
</tr>
<tr>
<td>reproducibility (um)</td>
<td>2.8</td>
<td>1.7</td>
<td>2.5</td>
</tr>
<tr>
<td>uncertainty (um)</td>
<td>1.4</td>
<td>1.1</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Corner cube displacement (um) is around

<table>
<thead>
<tr>
<th>Distance (um)</th>
<th>10(^{-4})mbar</th>
<th>0.41mbar</th>
<th>0.1mbar</th>
</tr>
</thead>
<tbody>
<tr>
<td>D (um)</td>
<td>2.956821</td>
<td></td>
<td>0.411857</td>
</tr>
<tr>
<td>esd (um)</td>
<td>-</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>repeatability (um)</td>
<td>-</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>reproducibility (um)</td>
<td>-</td>
<td>1</td>
<td>0.72</td>
</tr>
<tr>
<td>uncertainty (um)</td>
<td>-</td>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>
The uncertainty obtained here is of the same order than theoretical uncertainty calculated into 3. The uncertainty is greater at 10⁻³ mbar than 0.1 mbar because we worked with the vacuum pump on, we suppose that it is the cause of interference vibrations. The displacement of the corner cube measured by the interferometer is 79µm.

We also examined the effect of the external temperature T (the variation is about 2°C) on the absolute distance D and we observe a linear relation between D and T. The factor is 12.5µm/°C at 3m (Fig 3).

\[
D = 1.250E-06T + 2.957E-00
\]

\[
R^2 = 9.436E-01
\]

We submitted the laser source at temperatures of 10, 20 and 30°C and we made absolute distance measurements at 3m as described above. The panel below shows the results.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>10°C</th>
<th>20°C</th>
<th>30°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>D (µm)</td>
<td>2.956904</td>
<td>2.956906</td>
<td>2.956905</td>
</tr>
<tr>
<td>Uncertainty (µm)</td>
<td>2.0</td>
<td>1</td>
<td>1.5</td>
</tr>
</tbody>
</table>

We also examined the correlation between the absolute distance measured as described above and the displacement determined by the visible source working as a classical interferometer (Fig 4).
We can see that the two curves have the same behaviour. the visible measurement is 1.44µm and the infrared one is 1.61µm.

![Curve Plot]

**Fig. 4 Absolute distance and visible measurement vs time**

We prove that we can measure an absolute distance with an uncertainty of 2 µm. We show that a second source is necessary to make a correct measurement of D. We show too that our system can detect any variation of D around a face value, 0.41 or 3 m.

If the method is applied to distance measurement in air the relative uncertainty will be increased to $10^{-7}$ because of the unknowledge of the refractive index n.

**4- MEASUREMENT IN AIR OR ANY GASEOUS MEDIUM**

In order to make distance measurement in air (or another gaseous medium), a new type of source is used with the interferometer described above. The source in question is an air wavelength standard developed at the BNM-INM (Bureau National de Metrologie-Institut National de Metrologie). Its relative uncertainty $\frac{d\lambda}{\lambda}$ is about 1 part in $10^9$ and it is insensitive to the refractive index of the medium, generally air. Fig. 5 shows the principle of this wavelength standard.

![Diagram]

**Fig 5 Air wavelength standard**
The source is based on a plane-plane Fabry-Perot cavity with a zerodur spacer to which the silica mirrors are optically adhered. The gold coated mirrors have a reflectivity of 97% at 633nm and even higher in the infrared (1550nm).

The design of this system allows one to determine unambiguously the interference order \( k \) of the transmission peak to which the frequency \( v_1 \) of a laser diode is locked. After locking, the frequency of the laser source tracks in real time the refractive index fluctuations such that \( n v_1 = \text{constant} \). The wavelength of the source given by \( \frac{2e}{k} \) remains unaltered. This wavelength is determined by measuring \( e \) under vacuum, with the help of an optical standard and the method of exact fractions.

Using this system, the three preceding reference frequencies \( v_1, v'_1, \) and \( v_2 \) are replaced by the reference wavelength given by a laser diode locked on different transmission peaks of known interference orders. The distance \( D \) is determined as described above.

The measurement uncertainty is limited by the wavelength standard (about \( 10^{-8} \)) and is free of refractive index corrections.

5. CONCLUSION AND PERSPECTIVES

In this paper we have outlined the principles of distance measurements using a device based on a fringe counting sigmameter and the concept of synthetic wavelengths. A prototype interferometer has been developed at CSO Mesure in order to prove the feasibility of the first step of the measurement. We prove that we can measure an absolute distance up to 3m with an uncertainty of \( 2 \mu m \) in using one synthetic wavelength and the continuous scan between two frequencies. It will allow us to link two discrete frequencies (synthetic wavelength about 310\( \mu m \)) thus reach an uncertainty of 0.1 \( \mu m \). Finally, we will use the basic wavelength (1.5\( \mu m \)) to measure the absolute distance with an absolute uncertainty about 3nm.

The next step consists in associating our interferometer and the air-wavelength standard and to show that we can make distance measurement in air without any problem.

REFERENCES: