Nonstationary interference suppression in spread spectrum communication systems

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ABSTRACT

This paper discusses nonstationary interference excision techniques in spread spectrum communications systems. Excision techniques to remove, or at least suppress, sinusoidal, auto-regressive, and digital communication signals have matured relative to those dealing with nonstationary interferers. Time-frequency domain methods for analysis and estimation of the frequency modulated (FM) interference are summarized. Domains other than time and frequency, such as the Gabor-domain, the Wavelet-domain, and quadratic time-frequency signal representations, are appropriate for non-traditional smart jamming in which the interference parameters are highly dependent on time. Excision methods can be linear, bilinear, or nonlinear with performance dependent on the interference power relative to the desired signal and noise. The receiver SINR expressions and curves are presented in this paper for some of the key interference excision techniques.

Keywords: interference suppression, spread spectrum, direct sequence, anti-jamming, notch filtering, transform-domain, time-frequency distribution.

1. INTRODUCTION

Suppression of correlated interference is an important aspect of modern broadband communication platforms. For wireless communications, in addition to the presence of benign interferers, relatively narrowband cellular systems, employing time division multiple access (TDMA) or advanced mobile phone system (AMPS) may coexist within the same frequency band of the broadband code division multiple access (CDMA) systems. Hostile jamming is certainly a significant issue in military communication systems. Global positioning system (GPS) receivers potentially experience a mixture of both narrowband and wideband interference, both intentionally and unintentionally.

One of the fundamental applications of spread spectrum (SS) communication systems is its inherent capability of interference suppression. SS systems are implicitly able to provide a certain degree of protection against intentional or unintentional interferers. However, in some cases, the interference might be much stronger than the SS signal, and the limitations on the spectrum bandwidth render the processing gain insufficient to decode the useful signal reliably. For this reason, signal processing techniques are frequently used in conjunction with the SS receiver to augment the processing gain, permitting greater interference protection without an increase in the bandwidth. Although much of the work in this area has been motivated by the applications of SS as an anti-jamming method in military communications, it is equally applicable in commercial communication systems where SS systems and narrowband communication systems may share the same frequency bands.

The early work on narrowband interference rejection techniques in spread spectrum communications has been reviewed comprehensively by Milstein in 1. Milstein discusses in depth two classes of rejection schemes: 1) those based on least-mean square (LMS) estimation techniques, and 2) those based on transform domain processing structures. The improvement achieved by these techniques is subject to the constraint that the interference be relatively narrowband with respect to the SS signal waveform. Poor and Rusch 2 have given an overview of NBI suppression in SS with the focus on CDMA communications. They categorize CDMA interference suppression by linear techniques, nonlinear estimation techniques, and multiuser detection techniques. Laster and Reed 3 have provided a comprehensive review of interference rejection techniques in digital wireless communications, with the focus on advances not covered by the previous review articles.

This article discusses recent contributions to nonstationary interference suppression in direct-sequence spread spectrum (DS/SS). We focus on broadband, but instantaneously narrowband interference (INBI).

Interference suppression techniques for nonstationary signals, such as INBI, have been summarized by Amin and Akansu 4. Many ideas behind NBI suppression techniques 1, 2, 3 can be extended to account for the nonsta-

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tionary nature of the interference. For time-domain processing, time-varying notch filters and subspace projection techniques can be used to mitigate interferers characterized by their instantaneous frequencies and instantaneous bandwidths. Nonstationary interference suppression is generally achieved using linear and bilinear transforms, where the time-frequency domain, the wavelet domain, and the Gabor domain are typically considered. Several methods are available to synthesize the nonstationary interference waveform from the time-frequency domain and subtract it from the received signal.

For multi-antenna receivers, combined spatial and time-frequency signatures of signal arrivals can be used for interference suppression in DS/SS communications. Interference suppression techniques based on subspace projections can be extended to sensor array processing. With random PN spreading code and deterministic nonstationary interferers, the use of antenna arrays offers increased DS/SS signal dimensionality relative to the interferers. As such, interference mitigation through spatio-temporal subspace projections leads to reduced DS/SS signal distortion and improved performance over the case of a single antenna receiver. The angular separation between the interference and desired signals is shown to play a fundamental role in trading off the contribution of the spatial and time-frequency signatures to the interference mitigation process. The expressions of the receiver SINR implementing subspace projections were derived in. Interference synthesis techniques can also benefit from the spatial dimension. The distinction in the spatial signatures of multiple interference is used to simplify the recovery of the individual interference waveforms and as such enhances the cancellation effectiveness of the receiver.

The intent of this paper is to summarize recent important contributions in the area of nonstationary interference suppression. Section 2 provides the signal model assuming an antenna array at the receiver. A single-antenna receiver is simply a special case of the presented model. Section 3 gives an overall coverage of the different techniques over the past decade dealing with the nonstationary nature of the interference. We exclude in this coverage adaptive filtering, techniques due to their maturity and wide use. Section 4 shows how an antenna array can be used to enhance the suppression of interferers with time-varying frequency contents. Each of Section 3 and Section 4 includes examples for illustrations.

2. SIGNAL MODEL

In DS/SS communications, each symbol is spread into \( L = T/T_c \) chips, where \( T \) and \( T_c \) are, respectively, the symbol duration and chip duration. We use discrete-time form, where all signal arrivals are sampled at the chip-rate of the DS/SS signal. The symbol-rate source signal is denoted as \( s(n) \), and the aperiodic binary spreading sequence of the \( n \)th symbol period is represented by \( c(n, l) \in \{ \pm 1 \}, l = 0, 1, \ldots, L - 1 \). The chip-rate sequence of the DS/SS signal can be expressed as

\[
d(k) = s(n)c(n, l) \quad \text{with} \quad k = nL + l. \tag{1}
\]

For notation simplicity, we use \( c(l) \) instead of \( c(n, l) \) for the spreading sequence.

We consider an antenna array of \( N \) sensors. The propagation delay between antenna elements is assumed to be small relative to the inverse of the transmission bandwidth, so that the received signal at the \( N \) sensors are identical to within complex constants. The received signal vector of the DS/SS signal at the array is expressed by the product of the chip-rate sequence \( d(k) \) and its spatial signature \( h \),

\[
x_a(k) = d(k)h. \tag{2}
\]

The channel is restricted to flat-fading, and is assumed fixed over the symbol length, and as such \( h \) in the above equation is not a function of \( k \).

The array vector associated with a total of \( U \) interference signals is given by

\[
x_u(k) = \sum_{i=1}^{U} a_i u_i(k) \tag{3}
\]

where \( a_i \) is the array response to the \( i \)th interferer, \( u_i(k) \). We assume that and all \( U \) interferers have high power relative to the desired signal, i.e., high jammer-to-signal ratios (JSRs).
Without loss of generality, we set $||b||^2_F = N$ and $||a_i||^2_F = N$, $i = 1, 2, \cdots, U$, where $|| \cdot ||^2_F$ is the Frobenius norm of a vector. The input data vector is the sum of three components,

$$x(k) = x_a(k) + x_u(k) + b(k) = d(k)h + \sum_{i=1}^{U} a_i u_i(k) + b(k)$$  \hspace{1cm} (4)$$

where $b(k)$ is the additive noise vector. In regards to the above equation, we make the following assumptions.

A1) The information symbols $s(n)$ is a wide-sense stationary process with $E[s(n)s^*(n)] = 1$, where the superscript $^*$ denotes complex conjugation. The spreading sequence $c(k)$ is a binary random sequence with $E[c(k)c(k+l)] = \delta(l)$, where $\delta(l)$ is the delta function (This assumption is most suitable for military application and P-code GPS).

A2) The noise vector $b(k)$ is zero-mean, temporally and spatially white with

$$E[bb^T(k+l)] = 0 \text{ for all } l,$$

and

$$E[bb^H(k+l)] = \sigma^2 \delta(l)I_N,$$

where $\sigma$ is the noise power, the superscripts $^T$ and $^H$ denote transpose and conjugate transpose, respectively, and $I_N$ is the $N \times N$ identity matrix.

A3) The signal and noise are statistically uncorrelated.

3. NONSTATIONARY INTERFERENCE SUPPRESSION

Most interference excision techniques \cite{1,2,3} deal with stationary or quasi-stationary environment. The interference frequency signature, or characteristics, is assumed fixed or slowly time-varying. Such techniques are incapable of effectively incorporating the suddenly changing or evolutionary rapidly time-varying nature of the frequency characteristics of the interference. They all suffer from their lack of intelligence about interference behavior in the joint time-frequency (t-f) domain and therefore are limited in their results and their applicability. For the time-varying interference depicted in Fig. 1, frequency-domain methods remove the frequency band $\Delta f$ and ignore the fact that only few frequency bins are contaminated by the interference at a given time. Dually, time domain excision techniques, through gating or clipping the interference over $\Delta t$, do not account for the cases where only few time samples are contaminated by the interference for a given frequency. Applying either method will indeed eliminate the interference but at the cost of unnecessarily reducing the desired signal energy. Adaptive excision methods might be able to track and remove the nonstationary interference, but would fail if the interference is highly nonlinear FM or linear FM, as in Fig. 1, with high sweep rates. Further, the adaptive filtering length or block transform length trades off the temporal and the spectral resolutions of the interference. Increasing the step size parameter increases the filter output errors at convergence, and causes an unstable estimate of the interference waveform.

The above example clearly demonstrates that nonstationary interferers, which have model parameters that rapidly change with time, are particularly troublesome due to the inability of single-domain mitigation algorithms to adequately ameliorate their effects. In this challenging situation, and others like it, joint t-f techniques can provide significant performance gains, since the instantaneous frequency (IF), the instantaneous bandwidth, and the energy measurement, in addition to myriad other parameters, are available. The objective is then to estimate the t-f signature of the received data using t-f analysis, attenuating the received signal in those t-f regions that contain strong interference. This is depicted by the region in between the dashed lines in Fig. 1.

An FM interference in the form $u(n) = e^{j\phi(n)}$ is solely characterized by its IF, which can be estimated using a variety of IF estimators, including the time-frequency distributions (TFDs) \cite{7,8}.

The TFD of the data, $x(n)$, at time $t$ and radian frequency $\omega$, is given by

$$C_f(t, \omega, \phi) = \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \phi(m, l)x(n + m + l)x^*(x + m - l)e^{-j2\omega t}$$  \hspace{1cm} (5)$$

where $\phi(m, l)$ is the time-frequency kernel which is a function of the lag $l$ and time-lag $m$. Several requirements have been imposed on $\phi(m, l)$ to satisfy desirable distribution properties, including power localization at the IF. As shown in eqn. (5), the TFD is the Fourier transform of a time-average estimate of the autocorrelation function.
A time-frequency notch filter can be designed, in which the position of the filter notch is synchronous with the interference IF estimate. Based on the IF, two constraints should exist to construct an interference excision filter with desirable characteristics. First, an FIR filter with short impulse response must be used. Long extent filters are likely to span segments of changing frequency contents and, as such, allow some of the interference components to escape to the filter output. Second, at any given time, the filter frequency response must be close to an ideal notch filter to be able to null the interference with minimum possible distortion of the signal. This property, however, requires filters with infinite or relatively long impulse responses.

Amin \(^9\) has shown that a linear-phase five-coefficient filter is effective in FM interference excision. Assuming exact IF values, the corresponding receiver SINR is given by

$$\text{SINR} = \frac{L}{11/8 + 9\sigma/4}. \quad (6)$$

The above expression shows that full interference excision comes at the expense of a change in the noise variance in addition of a self-noise form, as compared with the non-interference case. The main objective of any excision process is to reduce both effects. The SINR in (6) assumes a random IF with uniform distribution over \([0, 2\pi]\). For an interference with fixed frequency \(\omega_0\), the receiver SINR becomes dependent on \(\omega_0\). The receiver performance sensitivity to the interference frequency is discussed in detail in \(^9\).

Wang and Amin \(^10\) considered the performance analysis of the IF-based excision system using a general class of multiple-zero FIR excision filters showing the dependence of the BER on the filter order and its group delay. The effect of inaccuracies in the interference IF on receiver performance was also considered as a function of the filter notch bandwidth. Closed form approximations for SINR at the receiver are given for the various cases.

One of the drawbacks to the notch filter approach in \(^9\) is the infinite notch depth due to the placement of the filter zeros. The effect is a “self-noise” inflicted on the received signal by the action of the filter on the PN sequence underlying the spread information signal. This problem led to the design of an open-loop filter with adjustable notch depth based on the interference energy. The notch depth is determined by a variable embedded in the filter coefficients chosen as the solution to an optimization problem which maximizes receiver SINR. The TFD is necessary for this work, even for single component signals, because simple IF estimators do not provide energy information. Amin, Wang, and Lindsey accomplished this work in \(^11\), incorporating a “depth factor” into the analysis and redeveloping all the SINR calculations. The result was a significant improvement in SINR, especially at mid-range interference-to-signal ratios (ISR’s), typically around 0 to 20 dB.

Instead of using time-varying excision filters, Barbarossa and Scaglione \(^12\) proposed a two-step procedure based on dechirping techniques commonly applied in radar algorithms. In the first step the time varying interference is converted to a fixed frequency sinusoid eliminated by time invariant filters. The process is reversed. In the
second step and the interference-free signal is multiplied by the interference t-f signature to restore the DS/SS signal and noise characteristics which have been strongly impacted in the first phase.

Similar to predictor/subtractor method for stationary and quasi-stationary interferers, Lach, Amin, and Lindsey proposed synthesis/subtractor technique for FM interference using TFD\(^{13}\). A replica of the interference can be synthesized from the t-f domain and subtracted from the incoming signal to produce an essentially interference-free channel.

Another synthesis/subtractor method is introduced in\(^{14}\) where the discrete evolutionary and the Hough transforms are used to estimate the IF. The interference amplitude is found by conventional methods such as linear filtering or singular value decomposition. This excision technique applies equally well to one or multi-component chirp-interferers with constant or time-varying amplitudes and with instantaneous frequencies not necessarily parametrically modeled.

To overcome the drawbacks of the potential amplitude and phase errors produced by the synthesis methods, Amin, Ramineni and Lindsey\(^{15}\) proposed a projection filter approach in which the FM interference subspace is constructed from its t-f signature. Since the signal space at the receiver is not specifically mandated, it can be rotated such that a single interferer becomes one of the basis functions. In this way, the interference subspace is one dimensional and its orthogonal subspace is interference-free. A projection of the received signal onto the orthogonal subspace accomplishes interference excision with a minimal message degradation. The projection filtering methods compare favorably over the previous notch filtering systems.

In\(^{16}\), Zhang, Amin, and Lindsey proposed a method to suppress more general INBI signals. The interference subspace is constructed using t-f synthesis methods. In different to the work in\(^{13}\), the interferer is removed by projection rather than subtraction. To estimate the interference waveform, a mask is constructed and applied such that the masked t-f region captures the interference energy, but leaves out most of the DS/SS signals.

Seong and Loughlin have also extended the projection method developed by Amin et. al.\(^{15}\) for exciting constant amplitude FM interferers from DS/SS signals to the case of AM-FM interferers\(^{17}\). Theoretical performance results (correlator SNR and BER) for the AM-FM projector filter show that FM estimation errors generally cause greater performance degradation than the same level of error in estimating the AM. The lower-bound for the correlator SINR for the AM-FM projection filter for the case of both AM and FM errors is given by

\[
\text{SINR} = \frac{L - 1}{L + \sigma + A^2 \left[ \frac{1}{1 + \sigma_{\Delta a}} (1 - e^{-\sigma_{\Delta a}}) + \sigma_{\Delta a} \right]},
\]

where \(L\) is the PN sequence length, \(A^2\) is the interference power, \(\sigma\) is the variance of AWGN, and \(\sigma_{\Delta a}\) and \(\sigma_{\Delta \phi}\) are the variances of the estimation errors in the AM and FM, respectively.

Linear t-f signal analysis has also been shown effective to characterize a large class of nonstationary interferers. Roberts and Amin\(^{18}\) proposed the use of the discrete Gabor transform (DGT) as a linear joint time-frequency representation. The DGT can attenuate a large class of nonstationary wideband interferers whose spectra are localized in the t-f domain. Compared to bilinear TFs, the DGT does not suffer from the crossterm interference problems, and enjoys a low computational complexity. In\(^{19}\), Wei, Harding, and Bovik devised a DGT-based, iterative time-varying excision filtering, in which a hypothesis testing approach was used to design a binary mask in the DGT domain. The time-frequency geometric shape of the mask is adapted to the time-varying spectrum of the interference. They show that such a statistical framework for the transform-domain mask design can be extended to any linear transform. Both the maximum likelihood test and the local optimal test are presented to demonstrate performance versus complexity.

The application of the short-time Fourier transform (STFT) to nonstationary interference excision in DS/SS communications is considered in\(^{20, 21}\). In both papers, due to the inherent property of STFT to trade off temporal and spectral resolutions, several STFTs corresponding to different analysis windows were generated. In\(^{20}\), Ouyang and Amin used a multiple-pole data window to obtain a large class of recursive STFTs. Subsequently, they employed concentration measures to select the STFT that localizes the interference in the t-f domain. This procedure is followed by applying a binary excision mask to remove the high-power t-f region. The remainder is synthesized to yield a DS/SS signal with improved signal-to-interference ratio (SIR).

In\(^{21}\), Krongold et. al proposed multiple overdetermined tiling techniques and utilized a collection of STFTs for the purpose of interference excision. Unlike the procedure in\(^{20}\), the authors in\(^{21}\) removed the high-value
coefficients in all generated STFTs, and used the combined results, via efficient least-square synthesis, to reconstruct an interference-reduced signal. Bultan and Akansu \textsuperscript{22} proposed a chirplet-transform-based exciser to handle chirp-like interference types in SS communications.

The block diagram in Fig. 2 depicts the various interference rejection techniques using the time-frequency methods cited above.

**Example**

At this point, in order to further illustrate these excision methods, the work in \textsuperscript{15} will be detailed since it includes comparisons between the two most prominent techniques based on TFDs currently being studied — notch filtering and projection filtering. The major theme of the work is to annihilate interference via projection of the received signal onto an “interference-free” subspace generated from the estimated interference characteristics. This paper includes a figure, reprinted here as Fig. 3, which clearly illustrates the trade-offs between projection and notch filtering based on the ISR. In the legend, the variable \( a \) represents the adaptation parameter for the notch filtering scheme and \( N \) represents the block size, in samples, for a 128 sample bit duration in the projection method. Thus, \( N = 128 \) means no block processing and \( N = 2 \) corresponds to 64 blocks per bit being processed for projection. Since the projection and non-adaptive notch filter techniques are assumed to completely annihilate the interference, their performance is decoupled from the interference power, and therefore correctly indicate constant SINR across the graph. The dashed line representing the notch filter with \( a = 0 \) is really indicating no filtering at all, since the adaptation parameter controls the depth of the notch.

It is evident from Fig. 3 that without adaptation a crossover point occurs around 2 dB where filtering with an infinitely deep notch is advantageous. Thus when the interference power exceeds this point, presumably a user would flip a switch to turn on the excision subsystem. However, with adaptation, this process happens automatically, while giving superior performance in the midrange. For the projection technique, the block size determines receiver performance conspicuously (ceteris paribus). Most important to note, however, is the superior performance of projection over all methods when the block size is equal to the bit duration, i.e. no block processing. It is feasible that computational complexity may warrant a trade-off between SINR and block size, in which case a hybrid implementation may be of benefit — one that automatically switches between adaptive notch filtering and projection depending on the desired SINR. In any case, this example illustrates the parameters involved in the design of modern excision filters for nonstationary interferers.
4. SUBSPACE PROJECTION ARRAY PROCESSING FOR NONSTATIONARY INTERFERENCE SUPPRESSION

Most receiver are equipped with antenna arrays. The extension of projection methods to cancel FM interferers in DS/SS communications using a multi-antenna receiver is discussed in 6. It is shown that, for a single FM interferer, the output SINR becomes

$$\text{SINR} = \frac{(L - |\beta_1|^2)^2}{\left(1 - \frac{2}{L}\right) |\beta_1|^4 + \frac{\sigma}{N} (L - |\beta_1|^2)}.$$  \hspace{1cm} (8)

where $\beta_i$ is the spatial correlation coefficient between the spatial signatures $h$ and $a_i$, $i = 1, 2, \cdots, U$, defined in Section 2, and is given by

$$\beta_i = \frac{1}{N} h^H a_i.$$  \hspace{1cm} (9)

It is easy to show that SINR in (8) monotonously decreases as $|\beta_1|$ increases, and the lower bound of the SINR is reached for $\beta_1 = 1$, which is the case of the DS/SS signal and the interference signal arriving from the same direction. With a unit value of $\beta_1$,

$$\text{SINR} = \frac{(L - 1)^2}{\left(1 - \frac{2}{L}\right) + \frac{\sigma}{N}(L - 1)} \approx \frac{N(L - 1)}{N} \frac{\sigma}{L + \sigma}.$$  \hspace{1cm} (10)

This result is the same as that of the single-sensor case developed in 15, except for the appearance of the array gain, $N$, for the desired DS/SS signal over the noise. That is, the independent multi-sensor subspace projection results in the same output SINR with the proposed multi-sensor subspace projection method when $|\beta_1| = 0$.

On the other hand, the maximum value in (8) corresponds to $\beta = 0$, and is equal to SINR = $LN/\sigma$, as discussed above. For the illustration of the SINR behaviour, we plot in Fig. 4 the SINR in (8) versus $|\beta_1|$ for a two-sensor array, where $L = 64$, and one FM jammer is considered with $M = 7$. The input SNR is 0dB.

A two-element array, $N = 2$, is considered with half-wavelength spacing. The DS/SS signal uses random spreading sequence with $L = 64$. The AOA of the DS/SS signal is 0 degree from broadside ($\theta_D = 0^\circ$).

We consider two interference signals. Each interference signal is assumed to be made up of uncorrelated FM component with $M_i = 7, i = 1, 2$. $M_i$ is the dimension of the subspace spanned by the $i$th interference. The
overall interference subspace is $M = 14$. The AOAs of the two interferers are $\theta_J = [40^\circ, 60^\circ]$. The respective spatial correlations in this example are $|\beta_1| = 0.53$ and $|\beta_2| = 0.21$. Note that, in the subspace projection method, the output SINR is independent of the input jammer-to-signal ratio (JSR), since the interferers are entirely suppressed, regardless of their power. Fig. 5 shows the receiver SINR versus the input SNR. The upper bounds correspond to interference-free data. For high input SNR, the receiver SINR is decided by the induced signal distortion. It is evident from Fig. 5 that the two-antenna receiver outperforms the single-antenna receiver case by a factor much larger than the array gain. Since the output SINR in the two-antenna receiver highly depends on the spatial correlation coefficients, the curves corresponding to a two-sensor array in Fig. 5 will assume different values upon changing $\beta_1$, or/and $\beta_2$. The best performance is achieved at $\beta_1 = \beta_2 = 0$. 

![Fig. 4. Output SINR versus $|\beta_1|$.](image)

![Fig. 5. Output SINR versus input SNR.](image)
Signal Synthesis

The Wigner-Ville distribution (WVD) is a spatial case of the TFD of equation (5). Signal synthesis using WVD is an effective technique to recover signals that are localizable in the t-f domain\textsuperscript{23}. However, the WVD synthesis technique suffers from the difficulty of identifying the time-frequency signatures of the different sources in the presence of noise and cross-terms. The latter are the by-product of the bilinear nature of the distribution. It is shown in\textsuperscript{6} that averaging the WVDs across the array reduces both the noise and cross-term levels and allows the difference in the spatial signatures of the sources to enhance the source respective t-f signatures. This enhancement is shown by the following example.

We consider several signals incident on an eight-sensor uniform linear array (ULA) with interelement spacing of half-wavelength. The additive noise is zero mean, Gaussian distributed, spatially and temporally white. The number of data samples is 128. In this example, three chirp signals, $s_1(t)$, $s_2(t)$ and $s_3(t)$, arrive at the array with AOAs of $-20^\circ$, $0^\circ$ and $20^\circ$, with the respective start and end frequencies given by $(0.9\pi, 0.5\pi)$, $(0.66\pi, 0.26\pi)$, and $(0.5\pi, 0.1\pi)$. In the t-f plane, the source signals have parallel signatures, emulating a multipath environment. The cross-term of $s_1(t)$ and $s_3(t)$ also forms a chirp-like cross-term structure whose frequency starts from $0.7\pi$ and ends with $0.3\pi$, and therefore lies closely to the t-f signature of $s_2(t)$. Fig. 6 depicts the WVD of the signals at the reference sensor (sensor #1) for the case of noise-free environment. It is clear that the t-f signature of all signal autoterms and crossterms are parallel in the t-f domain. The crossterms produced from the three source signals are even more dominant than the source autoterms. In the single sensor receiver, it becomes difficult to distinguish the source autoterms from the crossterms without any a priori knowledge of the sources.

Next, a 5dB noise is added to the data at each array sensor so that the input SNR is $-5$dB. Fig. 7 and Fig. 8 depict both the reference-sensor WVD and the array-averaged WVD. It is evident that the noise obscures both the signal autoterms and crossterms of the WVD at a single sensor. It is difficult, therefore, to retrieve the desired signal if we only synthesize from a single sensor.

Upon averaging, both noise and crossterms are sufficiently reduced to clearly manifest the individual source t-f signature, and the signals could be individually recovered if we place the appropriate masks in the t-f region. Fig. 9 shows the WVD of the synthesized signal $s_2(t)$ using the array averaging. Fig. 10 shows the synthesized signal using STFD\textsuperscript{24}, which is a technique that does not apply array averaging for signal recovery. Fig. 11 displays the real parts of the original signal $s_2(t)$ and $\tilde{s}_2(t)$. It is clear that the result from the array averaging technique is closer to the original signal than the recovered signal from the STFD-based method.

The synthesized interference signal, or its t-f signature, can be used for interference mitigation in DS/SS communications. It is important to note that the DS/SS signal appears as noise in the t-f plane, and its contribution can be ignored, specifically for low power signals. Phase matching becomes important if the synthesized interfer-
ence is to be subtracted from the incoming data. The interference phase or magnitude do not, however, present a problem if the projection techniques are applied.

CONCLUSION

This paper discussed recent techniques for nonstationary interference mitigation in broadband communication platforms. Many of the techniques employed for stationary and quasi-stationary signals can be extended to rapidly time-varying interferers. Different classes of interferers require different methods of excisions. The paper focused on FM interferers or those that have clear time-frequency signatures using quadratic distributions. The effectiveness of an excision technique can be enhanced using multi-antenna receivers. The paper considered two key developments in this area, namely, subspace projection and signal synthesis from the time-frequency domain.

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Fig. 9. WVD of synthesized $s_2(t)$ using array-averaging

Fig. 10. WVD of synthesized $s_2(t)$ using STFD.

REFERENCES

Fig. 11. (top) Real part of original $s_2(t)$; (middle) Real part of the STFD-recovered $\hat{s}_2(t)$; (bottom) Real part of the array averaged $\hat{s}_2(t)$.


