Sodern recent development in the design and verification of passive polarization scramblers for space applications

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I. INTRODUCTION

For an imaging spectrometer, the changing polarization properties of the incoming beam is a performance limiting factor. Indeed, while it is impossible to know in advance what the polarization state of the beam will be, the efficiency of the grating varies with it. As a consequence, imaging spectrometers must strive to achieve low polarization sensitivity \[1\]. At the same time the accuracy performances required for imaging spectrometers instruments continue to increase. For example, the atmospheric concentrations are approximately 400ppm for CO$_2$ and 1800ppb for CH$_4$ \[2, 3\] and missions currently studied require an accuracy of respectively 0.5ppm and 5ppb \[4\], a relative accuracy of the order of 0.1%. Thus, the radiometric error and the spectral oscillation in the instrument due to the absence of knowledge of the polarization state shall at least be of the same order.

An obvious solution is to use gratings which have very low polarization sensitivity. Gratings with polarization sensitivities of the order of 1% have been demonstrated, and it is possible to design gratings with even lower variation of the efficiency with respect to the polarization state. However, achieving such low polarization sensitivities may require a trade-off at the expense of higher grating efficiencies. For large bandwidths it is also exceedingly difficult or impossible to achieve such low polarization sensitivities. Therefore, the need arises for a device that would strongly depolarize the incoming beam, to be included before the most polarizing optical elements.

Depolarization is typically achieved by using a so-called polarization scrambler in a dual Babinet arrangement. Dual Babinet polarization scramblers have the advantage of using crystalline materials. These materials are intrinsically transparent over extended wavelength ranges, from the UV to the IR, a requirement for today’s imaging spectrometers. However, because a Dual Babinet polarization scrambler creates a multiple spot pattern, depolarization is achieved at the expense of the imaging quality. Depolarization is in essence limited by the acceptable image quality degradation.

Here, we first introduce the formalism necessary to describe the scrambler’s effect. We then turn to the challenges entailed by the design of scramblers. The tools are required to be at the same time accurate and fast, so as to carry both search for solutions and tolerancing efficiently. Finally, the high required performances need an accurate testing bench to validate every manufactured scrambler.

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**Fig. 1**: The classic design elements of a dual Babinet scrambler: on the left, the angles of the wedges plates, on the right is the direction of the normal to the tilted face of the wedge the orientation of the crystal axes. Another parameter is the orientation of the optical axis of each wedge.
II. PHYSICAL FORMALISM AND PERFORMANCE CRITERIA

A Babinet is made of two wedges of birefringent materials, generally quartz, with the crystal axes rotated by 90° between the two wedges (see Fig.1). This has a depolarizing effect: depending on the position at which a ray goes through the two wedges, the retardation changes. With a single Babinet, this creates a mix of polarization states, but the polarizations aligned with the fast and slow axes are unaffected. If a second Babinet is added, with the crystal axis rotated 45° from the first, there is no polarization direction left unperturbed. Thus, for an extended beam at the input, one obtains a beam with a mix of polarization states at the output, no matter the incoming polarization state. When focused in a single point, it will appear to be only weakly polarized.

Generally, 4 major performances are considered for scramblers. The first is the spot split. As the beam is refracted through a dual Babinet, it forms 4 spots. Image quality is degraded as a result: placing a limit on this degradation is required. The spot split \( d \) at the image plane is directly proportional to the angles of the Babinet \( \theta \).

It is defined by:

\[
d(\theta) = 2 \cdot f \cdot \theta \cdot \Delta n,
\]

with \( f \) the focal of the instrument and \( \Delta n \) the difference between the ordinary and extraordinary refractive index. The second is the polarization dependent pointing, which creates a co-registration error. For a Dual Babinet, if the input beam is linearly polarized, depending on the polarization direction 2 spots will be illuminated, or, if the polarization state is rotated by 90 degrees all the intensity will move towards the two opposite spots, giving a different position to the image barycentre.

The other 2 major performances are related to polarization. The formalism of Stokes vectors and Müller matrices is used because it is the best suited framework to describe a beam made of a mix of polarization states. Every optical element can be associated to its 4x4 real matrix called Müller. To evaluate these performances, one considers the idealized set up of Fig.2 and looks at the behaviour of the transmission \( T \) of the output beam. If \( M \) is the Müller matrix of the scrambler under study, we have:

\[
T = \frac{1}{2} \cdot S(\beta) \cdot M \cdot S(\alpha), \quad \text{with} \quad S(\alpha) = \begin{bmatrix} 1 \\ \cos 2\alpha \\ \sin 2\alpha \\ 0 \end{bmatrix}
\]  

One can then define the Polarization Sensitivity (PS) of the scrambler and the Relative Spectral Radiometric Accuracy (RSRA). All these performances can be calculated from the Müller matrices elements related to linear polarization.

The polarization sensitivity measures the residual polarization of the beam for a fixed wavelength. This translates into an uncertainty on the actual transmission of the instrument, as the polarization state of the incoming beam is a priori unknown in orbit. It is defined by:

\[
PS(\lambda) = \frac{\max_{\beta} T(\lambda, \alpha, \beta) - \min_{\alpha} T(\lambda, \alpha, \beta)}{\max_{\alpha} T(\lambda, \alpha, \beta) + \min_{\alpha} T(\lambda, \alpha, \beta)}
\]

with \( \lambda \) the wavelength, \( \alpha \) the orientation of input linear polarization and \( \beta \) the orientation of the linear polarization along which the transmitted beam is analyzed.

The RSRA is the variation of transmission over intervals of wavelengths \( \Delta \lambda \): it is an indicator of how much the radiometric accuracy of the instrument changes over the wavelength interval because of the polarization of the input beam. Here we define it by:

\[
RSRA(\lambda, \Delta \lambda) = \frac{\max_{\lambda, \beta} T(\lambda, \alpha, \beta) - \min_{\lambda, \beta} T(\lambda, \alpha, \beta)}{\langle T(\lambda, \alpha, \beta) \rangle_{\lambda, \Delta \lambda}}
\]
Fig. 2: Idealized setup to calculate performances of a scrambler. The fully polarized input beam has an arbitrary angle $\alpha$, the output beam goes through a perfect polarizer which has an arbitrary angle $\beta$.

III. SCRAMBLER DESIGN

A. Model and computation software

For a circular pupil and a perfectly collimated beam at normal incidence, McGuire & al. [5] have shown that formulas are available to compute the elements of the Müller matrix at any wavelength. They show that the envelope curve of the polarization sensitivity is decreasing for increasing wedge angles. The relation between the wedge angles and the RSRA is much less obvious. The greater wedge angle isn’t necessary the one for with the RSRA is minimized. Because of this non obvious relationship, it is necessary to search for the best solution, as it is not enough to select an angle to the maximum allowed by the spot split.

The formulas derived by McGuire & al. are also of limited use to the designer. Actual optical designs have often extended fields of view, the pupil may have an arbitrary shape (e.g. rectangular) and the beam at the position allocated to the scrambler may not be perfectly collimated. The designer must therefore turn to ray tracing to compute the actual performances of the scrambler.

Large numbers of Müller matrices must be computed. Searching for solutions requires trying a large number of design parameters. Tolerancing is typically achieved by Monte-Carlo simulations where the performances of about 1000 scramblers with random parameters must be calculated. Calculating the RSRA also requires a fine wavelength sampling of the order of 0.1 to 0.2 nm: $\Delta \lambda$ values of 1 to 3nm are typical to constrain fast variations with respect to the wavelength. One can see that computing millions of Müller matrices is necessary to design a scrambler.

Sampling of the pupil is also increasingly important as the required maximum PS and RSRA decrease: precision of the numerical simulation must increase accordingly. Today, tracing 256 rays across the pupil is required to obtain a suitable accuracy.

It is thus obvious that computation speed is of prime importance to design a scrambler. Various computing software can be used to calculate Müller matrices, be it dedicated optical software (Code V, ZEMAX,...) or general purpose computation tools (Matlab...). A benchmarking of these available computing tools shows that computing a Müller matrix requires a time of the order of 10 seconds in the best case (see Table 1). For an efficient design of the scrambler, it is necessary to reach computing times of much less than 1 second for a signal Müller matrix. Therefore, the development of a dedicated computing tool in C/C++ was undertaken. The speed boost is very large, as a factor about 1000 is gained.

Table 1: Müller matrix computation times.

<table>
<thead>
<tr>
<th></th>
<th>Matlab</th>
<th>Code V</th>
<th>C/C++</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 wavelength</td>
<td>~165s</td>
<td>~15s</td>
<td>~20ms</td>
</tr>
<tr>
<td>Spectrometer band</td>
<td>~90h</td>
<td>~8h</td>
<td>~40s</td>
</tr>
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With such a computation speed, it is possible to explore exhaustively a solution space and run Monte-Carlo simulations with more than a thousand random trials.
IV. SCRAMBLER VERIFICATION TEST BENCH

While customers are reassured by simulations, they require that performances be validated by appropriate testing means. A test bench for scramblers has been built to this end. A schematic of the bench is shown on Fig. 3.

A. Test bench description

In order to build the bench a simple architecture has been retained. The principle of operation is based on synchronous detection of a reference path and of a measurement path. On Fig. 3, one can see that the source injects light into a monochromator \( M_0 \), which selects the appropriate wavelength. The beamsplitter \( BS \) separates the reference path from the measurement path. On the measurement path, thanks to an optical system, the right aperture and beam footprint for the scrambler’s illumination are generated. The scrambler \( SCR \) is placed between two rotating polarizers \( P1 \) and \( P2 \). Photodiodes \( PD1 \) and \( PD2 \) are linked to Lock-in Amplifiers and are used to perform synchronous detection at the rotation frequency of the chopper \( C \). At the output of the LIA, one gets two voltages \( V_R \) and \( V_M \) for the reference and measurement paths, respectively. The useful measured signal is \( S = \frac{V_M}{V_R} \).

In order to compute the performances of the scrambler, it is necessary to calibrate the bench to remove the contribution of the bench to the variation of \( S \) during the measurements. Two calibrations are necessary. The first aims at removing the effect of the partial polarization of the beam. Because the monochromator includes a grating, the polarization state of the beam varies with the wavelength selected as the polarization sensitivity of the gratings varies. It is therefore necessary to make a blank run by measuring \( S \) with just a rotating polarizer on the measurement path. The second calibration aims at removing the effect of the varying transmission of the optical train with the wavelength. A blank run is made with all the elements in the optical train except the scrambler.

Not all effects can be calibrated, however. Some contributors to the variations of \( S \) must be entirely removed. The bench is built to measure relative effects of the order of 0.1%, therefore even small effects have to be removed. As a result, the beam must be appropriately limited so that the same extent is measured on the reference and measurement paths. With the small spectral bandwidth at the output of the monochromator, effects caused by the partial coherence of the beam must also be avoided. Random noise is impossible to avoid, but SNR values of the order of \( 10^4 \) must be obtained in order to observe relative effects of 0.1%.

To obtain the values of \( PS \) and \( RSRA \), a statistical treatment of the measurement is necessary. Thanks to the calibration, an estimator \( \hat{T} \) of the transmission is obtained. The first idea one can have to compute \( PS \) and \( RSRA \) is to plug the values of \( \hat{T} \) into (1) and (2). But because they are defined with the help of the \( max \) and \( min \) operators, they have detrimental statistical properties. Indeed, they introduce a bias into the result: the expected value of the maximum of observed values is always greater than the true value of the maximum of the underlying physical property. If the distribution of the noise on the measurement is unbounded, such as the Gaussian distribution, the bias may tend to infinity when the number of measurement points tends to infinity. Even if the growth of the bias is extremely slow, increasing the number of points past a certain number will actually increase the error.
For PS, this effect is relatively limited, because the number of measurement points is relatively low. For low values of PS of the order of 0.1%, the bias is nonetheless significant. For RSRA, though, this effect is major. RSRA values to be measured are typically below 0.5%. The number of points necessary to compute it accurately is high. Typically, one would have 4 analyzer positions, 10 input polarizer positions, a sampling of the order of 0.1nm and a wavelength range of 3nm: all in all, typically 1000 measurement points are necessary. In other words $\max(\hat{\mathbf{T}}) - \min(\hat{\mathbf{T}})$ is commonly above 6 standard errors even if the true value is 0: the RSRA can be very quickly drowned in the noise. This effect is shown on Fig.4 where simulated measurements are shown: with an SNR of 2000, the results are rendered meaningless by the statistical noise.

On Fig.4, the theoretical curve (in blue) extends from 305nm to 500nm, while the simulated measurements extend from 340nm onwards. The top curve is for an SNR of 2000. The curve simulated with a SNR of 10000 is much closer to the theoretical curve, yet it is still noisy and the bias is still clearly visible. The horizontal lines represent the analytical calculation of the minimum value that can be returned on average, a "lower bound" of the algorithm. As it is illustrated, the simulated curves for both SNR fit correctly the lower bound estimation. For an SNR of 2000, it is close to 0.6%, much higher than the theoretical performance.

A better idea is try to fit the data by the statistical model provided by the Müller matrix physical model. Thanks to the periodic nature of the transmission with respect to the polarizer orientations (2), it is possible to use Fourier transforms to get least-squares estimators of the Müller matrix elements. Not all elements can be estimated however, because of the use of linear polarizers, but PS and RSRA are not impacted by the behavior of circular polarization states. A further refinement can be applied: the Müller matrix is a slow-varying function of the wavelength. With the fine sampling adopted, it is possible to make a local fit with respect to the wavelength. From the Müller matrices estimators, it is possible to calculate the performances by using the same algorithms as for the theoretical performances.

Contrarily to the plug-in estimator, these estimators have the advantage that the statistical noise decreases by a factor $\sqrt{N}$ with the number N of measurement points. Instead of using the extreme points of the measurements, all the information is used, which results into a much more accurate estimate, as shown on Fig.5.

![Simulated measurements of the RSRA of a scrambler by using directly Equation (3) compared to the theoretical curve](image-url)
Fig. 5: Simulated measurements of the RSRA of a scrambler by fitting the Müller matrix model compared to the theoretical curve. This time to add to the difficulty, a variation in transmission and a varying diattenuation were assumed. The 2 curves follow each other closely, despite the SNR of only 1000.

Thanks to the improvement of the signal processing and the uniformity of the beam the major errors come from misalignment, in particular angular misalignment and f/# uncertainties in case of converging measurement or from the scrambler itself.

B. Application on an actual scrambler measurement

Actual results are obtained on the Sentinel 4 Scrambler, a Dual Babinet scrambler in converging beam. The scrambler has crystal axes at $\alpha_1 = +45\,\text{deg.}$, $\alpha_2 = +135\,\text{deg.}$, $\alpha_3 = 0\,\text{deg.}$ and $\alpha_4 = +90\,\text{deg.}$ The Pupil is circular and all wedges are made of quartz.

The measurement presented on fig.6 was performed between 340nm and 460nm. PS00 (resp. PS45) is defined as the polarization sensitivity when the analyzer orientation is at $\beta = 0$ or $90\,^\circ$ (resp. $45\,^\circ$ or $135\,^\circ$), RSRA00 (resp. RSRA45) is the relative spectral radiometric accuracy over the spectral window of $\Delta \lambda = 3\,\text{nm}$ when the analyzer orientation is at $\beta = 0$ or $90\,^\circ$ (resp. $45\,^\circ$ or $135\,^\circ$). The dash lines is the measurement, and the solid lines is the best fit simulation based on actual scrambler geometry measurement (wedge angles, central thicknesses and refractive index) and some adjustable parameters link to the scrambler manufacturing and test bench uncertainties. The two parameters, that are used in this case, are the angular misalignment between the incident beam and the scrambler and the orientation of wedges optical axis. From this analysis it is possible to integrate an “as manufactured” geometry of the scrambler into dedicated optical software to estimate the performance of a spectrometer.

The main impact of the orientation of wedges optical axis are the increase of the minimal value of PS00 (non-zero values near 380nm and 450nm) and high frequencies oscillation on RSRA00 and RSRA45, present in both simulation and measurement. The second parameter, the incident beam angular misalignment, induced a spectral shift of the PS00 and PS45 curves.

This last effect is verified experimentally by comparing the expected behavior of the scrambler performance (solid lines in Fig.7) and the measurement (dash line in Fig. 7) when we rotated the scrambler with respect to the incident beam. As it is shown on the figure below, the experimental performance of the scrambler is well modelizing by the Sodern software.
Fig. 6: Real measurement results with the bench (black dash lines) and simulation of the As-Built design (red lines) with retro-engineering.

Fig. 7: Comparison of the impact of the scrambler angular misalignment between simulation and measurement.

V. CONCLUSION

Sodern now has extensive means to design and optimize high performance scramblers. The high accuracy of the measurement bench allows the measurement of this high level of performance. Thanks to the significant heritage of the manufacturing for space scrambling windows, Sodern masters the entire development cycle of polarization scrambling windows from design to validation tests.

ACKNOWLEDGEMENTS

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REFERENCES


