Enlightening students: optics applications in the math classroom

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ABSTRACT

Since 2013, we have been introducing topics in optics/photonics in middle and high school mathematics classes. Our goals have been to illustrate applications of mathematics to students who may not otherwise find mathematics useful, to promote critical thinking and problem-solving skills, and to introduce science and technology topics not usually addressed at these grade levels. In this paper we describe our activities, programs and their outcomes.

Keywords: secondary school, middle school, inquiry, STEM, outreach, active learning, problem-based learning, Dumpster Optics

1. INTRODUCTION

In the past six years we have conducted five programs to enrich the middle and high school mathematics curriculum at St. Bernard School (Uncasville, CT, USA) through the use of examples from optics/photonics. Several of the projects were supported by outreach grants from SPIE; others were self-supported. While some were specific to a grade level or math course, others were open to participation for all students attending the school. In each case our goals were to:

1. Illustrate the practical use of math principles to answer the question “why do we have to learn this?”
2. Introduce the science and technology of optics and photonics to groups of students who were unfamiliar with them.
3. Develop critical thinking and problem solving skills in the context of math applications.

1.1 Organizations involved

St. Bernard School (SBS) is a Catholic co-educational school located in Uncasville, CT co-sponsored by the Diocese of Norwich, CT and the Xavierian Brothers. SBS serves 350 students in grades 6-12 (approximately 12 to 18 years of age). Author M. Donnelly has been a mathematics instructor and Head of the Mathematics Department at SBS since 2012. As of the 2018-2019 school year SBS has the most advanced mathematics program in Eastern Connecticut, offering courses including Multivariable Calculus, Differential Equations, and Mathematics Research. The department also offers co-curricular courses such as Philosophical Problems in Space and Time and Astrophysics. For the past several years SBS has had a thriving international student program allowing students from around the world to attend school in Connecticut. International students are primarily from China (from areas near Beijing and Shanghai) but students from Brazil and several European countries have attended as well. Approximately 15% of the student body is international.

Our first project (2013-2014) was conducted in part at Three Rivers Community College (TRCC), a two-year state-supported college in Norwich, CT, a few miles from SBS. Author J. Donnelly (who retired from teaching in summer, 2014) was program coordinator for Connecticut’s only two year program in Laser and Fiber Optic Technology. TRCC’s SPIE and OSA student chapters were enthusiastic and talented students ranging in age from 18 to 55 who designed and conducted outreach projects for hundreds of local students each year, primarily in 5th grade (Jr. Laser Camp) and high school (Laser Camp) but also for college students and the local community.

1.2 Project design

Although the projects were all quite different in scope, there were several similar features. Each began with an introduction to optics and photonics and their importance in students’ lives. Three of the four projects included pre- and post-surveys to assess changes in attitudes toward mathematics and problem solving. All projects included hands-on inquiry-based activities or required students to engage in critical thinking and problem solving. Many of the activities were developed by the TRCC’s OSA and SPIE student chapters or during the course of the SPIE-supported Dumpster Optics project In this paper, we will summarize the key points of each project including what worked well and what did not. Some of the lessons can be found at www.lasertechonline.org .

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2. ILLUMINATING MATH WITH OPTICS (2013-2104)

2.1 Activities

The first collaboration was a year-long project to introduce 45 students in the three SBS pre-calculus classes to applications of optics/photonics that used the mathematical concepts they were studying.[1] Pre-calculus is confusing to many students because unlike the algebra or geometry courses that precede it, there is no one overarching theme. Instead, it is a collection of seemingly unrelated topics that were not addressed in previous courses but that will be needed for the study of calculus. In many cases it seems to students that the topics they are studying have no real applications because those don't become apparent until calculus. Because much of pre-calculus involves the transcendental functions (trigonometry, exponentials) we chose as our logo "Transcendental enlightenment" since it was our goal to shed light on applications of these (and other) topics. (See Figure 1) The logo appeared on tee shirts for students and volunteers and on written materials.

Figure 1 – Logo for Illuminating Math with Optics Transcendental Enlightenment project

The project began with a field trip for SBS students to TRCC to meet college students and work in the college optics laboratories. Three hour long activities were planned, including:

1. Fiber optic attenuation (logarithms). Instructed by TRCC students, the high school visitors measured loss in plastic optical fiber and learned how logarithms make the task easier.
2. Absorption in neutral density filters (exponents). Working in the optics lab, students measured the absorption coefficient by measuring transmittance thought a set of one to five filters, graphing the results to determine the value of *k.* Since the filters were marked with optical density, not absorption coefficient, students were introduced to an application of the base *e* to base 10 conversion process that they had studied in math class.
3. The third workshop took place in the manufacturing lab where students saw math applications in laser cutting and engraving, 3-D printing, CNC machining and a programmable robot arm in a model manufacturing cell. This activity was led by a member of TRCC's Manufacturing Engineering Technology program.

The remaining three workshops took place at SBS in M. Donnelly's mathematics classroom between December 2013 and March 2014. Each focused on applications of one area of mathematics from the pre-calculus course:

1. Measuring the wavelength of light (trigonometry). Using a variety of diffraction gratings and red and green laser pointers, students used the diffraction equation to measure the lasers' wavelengths.
2. Simple lens shape analysis (matrices). After two homework assignments to introduce refraction and Snell's law, students were shown how to use the matrices for refraction and translation to predict the path of light through a simple lens. They used this knowledge to construct a system matrix for a flat acrylic semi-circular "lens" and to predict the focal length. The measured focal length compared well to prediction.
3. Telescope mirrors (conic sections). A short presentation introduced students to the law of reflection and the geometry of mirrors ranging from parabolic reflectors in flashlights to ellipsoidal solid state laser cavities to the complex mirrors of modern space telescopes. Since we did not have time or budget to build a telescope, students created edible conic sections made from cone-shaped cakes and frosting. (See Figure 2a.)

The year-long program ended with a special session on "polarized light art". Students first worked through an inquiry-based lesson on polarization from TRCC's *Introduction to Light and Lasers* course that used a set of questions and hands-on activities to introduce polarization and birefringence. They were then given supplies to create colorful art pieces with transparent tape and two polarizers. (See Figure 2b.) Students from the AP (advanced placement) calculus course joined in for this activity and their questions led to a discussion of the mathematics behind Malus' Law.

Figure 2 – a) Cut and frosted cone-shaped cake reveals a conic section figure. b) Layers of transparent tape between two polarizers create unique works of “art”

2.2 Results of pre and post surveys

We modified a survey for pre/post testing from the Student Engagement in Mathematics (SEM) developed by Kong, Wong and Lam (2003).[2] In retrospect, this was not the best choice since the survey was developed to measure cognitive, affective and emotional engagement of middle school students in Shanghai, Some of the survey statements were not particularly appropriate for U. S. high school students. The post-survey did show some positive changes, for example significant increases in response to the following statements

When I learn mathematics, I wonder how much the things I have learned can be applied to real life.

I am very interested to know how to solve new mathematics problems. Mathematics always gives me pleasure.

The most valuable outcome were the student comments which were generally all positive and appreciative.

Thanks for making math relevant. The projects were nice; math is much more fun when we can understand the applications.

Loved the projects, they helped me understand what we were doing in class. A little more time to work on them would be good.

3. DARK SKIES AT ST BERNARD (2016)

3.1 Project description

This self-supported mini project was prompted by the discovery of a box of sky quality meters (SQM) in the author’s basement. The meters had been purchased for a combined STEM/cultural project with elementary school students in four U.S. states, Canada and Romania.[3] Funded by a grant from the OSA Foundation, Dark Skies at EASTCONN provided each school with resources to learn about light pollution and astronomy and to take part in the Globe at Night project to measure the darkness of the night sky.[4] Our goals at SBS were to introduce concepts of light pollution (and the mathematics used to describe them), raise awareness of the health implications of light at night, and make personal connections with high school students in Slatina, Romania. The Romanian teacher, Elena Vladescu of Colegiul National Vocational "Nicolae Titulescu", is an enthusiastic veteran of several projects with U.S. partners including the Dark Skies at EASTCONN project and PHOTON PBL (Problem-Based Learning).

We began with an exchange of letters to introduce students in the two countries. At SBS, the project was required of all students in the Honors Pre-Calculus class but it was voluntary for the Romanian students. As a result, all of the Romanian students were girls since (according to Ms Vladescu) the boys “are not interested in extra work.” Students in both classes were instructed to write a letter to their pen pal and talk a bit about themselves – favorite food, interests outside of school and so on. They were then introduced to the problems of light at night and the Globe at Night web site through a PowerPoint presentation and the documentary film, *The City Dark*.[5] Each student was asked to take visual and/or SQM measurements of the night sky in their local neighborhood and enter them into the Globe at Night site so that we could view the maps of night sky darkness in the areas around the two schools. (See Figure 3.) Eighteen measurements were reported in southeastern Connecticut and more than 40 from in and around Slatina, Romania between January and March of 2016. Some of the Romanian students also took measurements from places they traveled during the duration of the project.

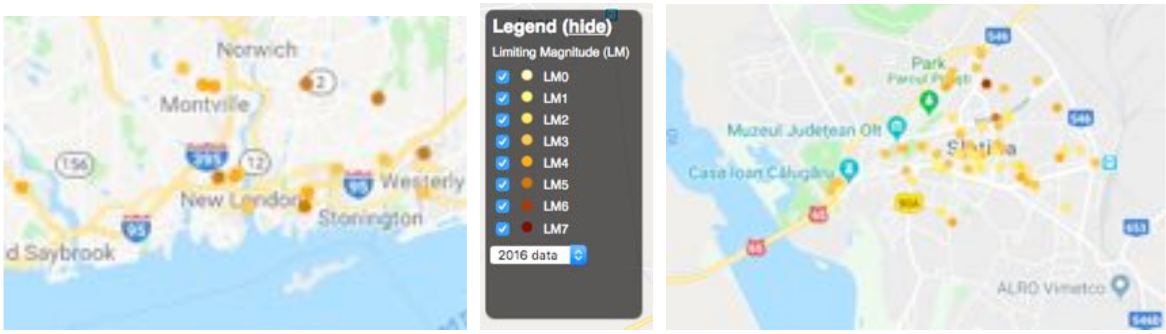

Figure 3 – Globe at Night data from SBS students (left) and Romanian students (right). The limiting magnitude legend is in the center. Not surprisingly, the night sky in Slatina (population nearly 80,000) is in general much brighter than the area around Montville-New London Connecticut (population around 27,000 and 20,000 respectively).

3.2 Results

Unfortunately, the time difference between the two schools made the planned video-conference impossible during the school day. Without face-to-face communication further work such as comparing data from the two schools was difficult. Nonetheless, students were generally interested to "meet" with students from another culture and follow them on social media, at least during the project. (We did not monitor that aspect.) Students were also quite interested in the deleterious health effects of blue light at night, and at least one student reported the project prompted him to install a dimmer app on his game system that made sleep easier at night.

4. PROBLEM BASED LEARNING IN OPTICS/PHOTONICS (2016-2017)

4.1 Teaching problem solving and critical thinking in calculus

A recurring complaint of employers of college graduates is that they know all the content taught in their courses but not how to use it to solve real-world problems. Between 2006 and 2016, the New England Board of Higher Education received three grants from the National Science Foundation's Advanced Technological Education (NSF-ATE) program to develop materials for Problem-Based Learning and to provide secondary and post-secondary educators with professional development in their use.[6] Over ten years, the PBL team developed 21 "Challenges" (case studies) to bring real-world industry problems to the classroom; many of the collaborating companies were from optics/photonics industries or research university departments. An NSF-ATE grant awarded to Springfield (MA) Technical Community College in 2018 will develop eight more Challenges in Advanced Photonics Manufacturing.

The 18 students of Honors Calculus course at SBS had good backgrounds in mathematics and science, many had completed or were currently enrolled in physics or chemistry, but mostly they were taught with traditional lecture/lab pedagogy. None had been exposed to PBL or similar methods of teaching. A previous study with 2-year college students had shown[7] that students' motivation for learning and their confidence in their problem-solving ability increased after completing three PBL Challenges. We decided to incorporate into the Honors Calculus course three PBL Challenges with topics related to calculus. Our goal was to improving problem-solving skills in this group of students by teaching them a structured approach.[8]

4.2 The PBL Challenge Whiteboards

The PBL Projects' Challenges have been described elsewhere[6,7,9] and are freely available on the web site www.pblproject.org . The Challenges are designed to be solved by teams of students using a tool called the Whiteboards (because they were originally intended to be projected onto a classroom whiteboard). The Whiteboards were developed in the first PBL Project (PHOTON PBL, 2006-2009) with the assistance of engineers from a variety of photonics companies in southern New England who were asked, "How does your company solve problems?" It is a variation of the science or engineering problem solving cycle with the addition of thought-provoking questions that students must answer before proceeding. Many students who just want to "get an answer" find this attention to detail frustrating at first, but soon learn the value of carefully following the process. It is not uncommon to have students say they use the Whiteboards in other courses when they have a complex project to complete.

The Whiteboards begin with *Problem Analysis,* requiring student teams to clearly state the problem in their own words and to reflect on the requirements of a successful solution, what is known, what needs to be learned and what, if any, assumptions need to be made. Since the problem is presented in the context of, say, an industry team meeting or a customer request this is the most difficult and most important part of the process. Next students create a plan and a timeline for learning what they need to know, using the *Independent Research* Whiteboard. Once research is complete, teams reconvene to *Brainstorm* solutions. Finally, they engage in *Solution Testing* to determine if their solution has met all the requirements they laid out in the first Whiteboard. This is a recursive process; if all requirements are not met, they must return to the problem analysis phase to reexamine their original work. (See Figure 4.)

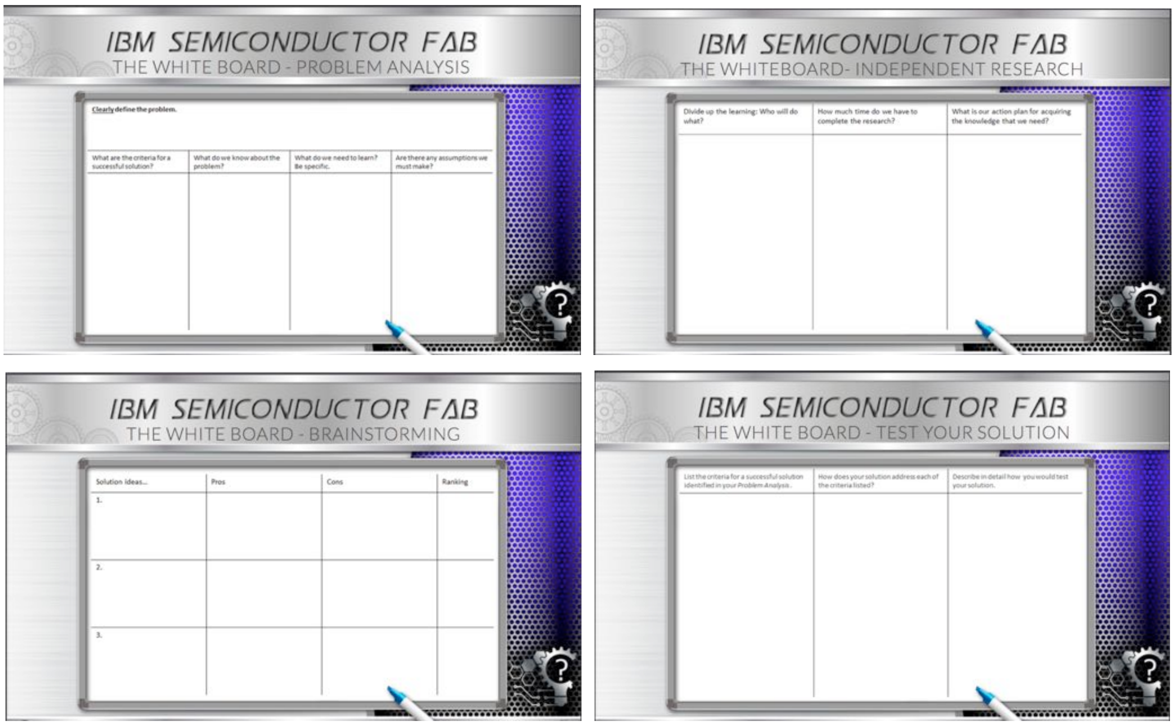

Figure 4 – The Whiteboards: (top) Problem Analysis, Independent Research (bottom) Brainstorming, Test Your Solution

4.3 PBL and Calculus

The first Challenge took place over a 3-day span in Fall 2016 at the beginning of the school year. The problem was to determine if a pilot could have suffered permanent eye damage after being targeted by a laser pointer as he was landing his plane. Although some details were given (e.g., distance from the laser to the plane) students had to make many assumptions as well as learn about laser output divergence and maximum permissible exposure. Most student teams came to a well-reasoned solution, and they were assessed mainly on the quality and completeness of their Whiteboards. Several said they enjoyed using trigonometry knowledge in this type of real-world problem.

The next Challenge was a bit more complex, requiring students to compare the luminous output of fluorescent and incandescent light bulbs. This required acquiring much new knowledge in a short period of time, including radiant and luminous units and how light bulb power is measured. The calculus connection involved converting from watts to lumens and then estimating the area under the power curves for each bulb. Students were given a week to complete the task using whatever geometric means they preferred. About half of the teams were able to complete the computations correctly. Their final reports showed that they were missing some basic skills from previous courses in geometry and unit conversions.

The final Challenge was the most complex and a real test of how well students could use the problem solving process. IPG Photonics (Oxford, MA) provided a problem on laser cleaning of aluminum in the context of a customer request. Data was provided on the laser, two types of delivery fiber, the beam scanner and customer requirements. The information was somewhat simplified compared to its presentation in a college laser technician course, but it still presented many unknowns to high school students who had never seen a high power laser or were aware of laser surface cleaning. During the three days teams worked on the project we noticed improvement in teamwork strategies and use of the Whiteboards but again, they were bogged down in what should have been fairly straightforward calculations with units.

4.4 Results

We administered the Motivated Strategies for Learning Questionnaire (MSLQ) before presenting the first Challenge and after the last Challenge. This is a widely used, validated survey for assessing college students' motivation and strategies for learning. The same instrument was used in the earlier community college study, but we saw little significant change with high school students. We also asked a teacher not involved in the project to work with a focus group of five students at the end of the project to determine their attitudes toward PBL. Students agreed that completing Challenges in topic areas where they had little or no previous knowledge increased their confidence by giving them specific tools for approaching problems. Several also commented that they found the optics/photonics applications interesting as well.

5. SEEING THE LIGHT: ENRICHING MIDDLE SCHOOL MATH (2017-2018)

After completing the PBL project with Honors Calculus students we agreed that students needed to experience math applications and open ended problem solving at a much younger age. This year-long project was funded by an outreach grant from SPIE and included 77 students in five classes in grades 6-9. Over the course of the year, students engaged in optics lab activities and used math to analyze data and solve problems. At the conclusion of the year all students at SBS had the opportunity to participate in a photo contest celebrating the International Day of Light.

5.1 Activities

With so many classes at different levels it was impossible to match the optics topic to current classroom topics but we tried to be sure that whatever math involved would be at least familiar to all of the students. The lessons were planned to take 1–3 days but some required more time especially with younger students who needed extra help with calculations. The lessons are listed below; they have been detailed elsewhere.[10]

1. Fractions, Lenses and Telescopes (September 2017). Using a meter stick optical bench students explored images formed by a single lens and calculated focal length of the lens using the Thin Lens Equation. The telescope portion was adapted from Hands-On Optics *Terrific Telescopes*[11]. Computations with fractions proved problematic for some younger students, and only the older, more experienced students were able to complete the telescope resolution activity. (See Figure 5.)

Figure 5 – Assisting students with measurements of object and image distance with the meter stick optical bench.

2. Structured Problem Solving. (October 2017) We decided to try a modified PBL activity to introduce students to structured problem solving. The sixth and seventh graders completed a PBL Challenge involving neighborhood efforts to mitigate storm water and the older students did the laser safety Challenge. As is often the case with older students, the lesson revealed the need to assist students with effective solutions for teamwork.

3. Similar Triangles–Light and Shadows and Pinhole Images. (November 2017) This lesson was based on the Dumpster Optics lesson, *Light and Shadows*.[12] Students used LED finger lights and a triangle hole mask and respond to questions after trying each activity. The activities led to understanding the origin of pinhole images.

We intended to follow up with an experiment to predict the size of an image of a light bulb filament, but with warm weather and energetic students we decided to go outdoors and use the same similar triangle calculations to measure the height of a tall tree. While students were computing tree height in the classroom, we showed an oatmeal box pinhole camera and took a photo of the classroom. (See Figure 6.)

4.

Figure 6 – (Left) Pinhole photo taken with a cylindrical oatmeal box camera, 30-minute classroom exposure. (Right) Inverted image.

5. Inverse Square Law and Spreading Light. (March, 2018) After a pause due to holidays, vacations and snowy weather, we resumed with a lesson using a flashlight bulb, square hole mask and grid to illustrate the inverse square law. Most students were able to quickly measure the number of grid squares illuminated as a function of distance from the bulb, but some groups had trouble with alignment. (See Figure 7.) For many students this was their first experience with a science experiment that required careful measurement. All students quickly noted the quadratic relationship, which they graphed the following day. The youngest students needed considerable assistance in making a graph with data. The older students were introduced to using the curve fit function of Graphical Analysis software (www.vernier.com). After the lesson was complete, the application was used to begin an introduction to graphs of functions.

Figure 7 – Aligning the flashlight bulb, mask and grid to measure spreading light. This measurement took more care than students may have been used to.

6. Interpolation and Light Spectra (April 2018). This activity was completed with the older students only. After a presentation on spectroscopy that included watching videos of optical food sorting equipment, students were give diffraction gratings to view various light sources, including gas spectrum tubes and a variety of light bulbs. They were then shown a labeled graphic of a continuous spectrum and the hydrogen spectrum and asked to estimate the wavelength of the hydrogen alpha line. To our pleasant surprise, by repeatedly dividing the spectrum in half they were rapidly able to estimate the wavelength within a few percent. On the second day we explored the spectrum of various light sources using the Vernier Go Direct® Light and Color Sensor. The RGB output of various phone and laptop screens led to a discussion of blue light at night and the use of blue light filtering.

We decided to end the year with a photo contest to recognize the International Day of Light. Called “Light in our Life”, it replicated an international contest we helped organize during the International Year of Light in 2015.[3] All grades were invited to submit photos illustrating the theme and a short explanatory written statement. Three winners were chosen in each of two categories, middle school (grades 6-8) and high school (grades 9-12). See Figure 8.

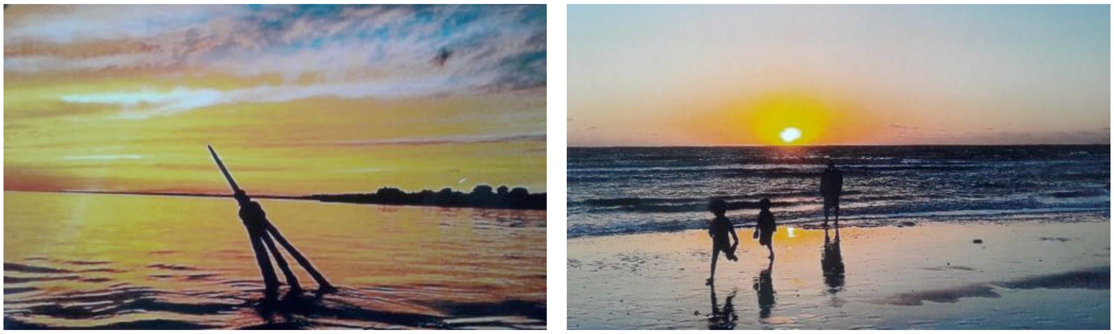

Figure 8 – Saint Bernard School’s proximity to the Connecticut shore resulted in several sunrise and sunset photos entered in the Light in our Life Photo contest.

5.2 Results

We were quite surprised at students’ initial resistance to applied math activities! Even young students are so accustomed to the usual routine of “listen to the teacher, copy the teacher, do homework, take a test” that they did not know how to react to this new way of learning. The youngest students especially were constantly asking if they were “doing things right” and wondering when they could go back to the safety of their classroom routine. Sixth graders, coming from many different sending schools, sometimes had very weak math skills that needed to be addressed as well.

We administered an informal ten-question survey before and after the program to assess attitudinal changes. We were pleased to find a significant number of students no longer felt that “Memorizing is the best way to learn math” as well as increased in persistence in problem solving and curiosity about what math is “for”. At the start of the program we asked if anyone could describe photonics, and no student could. However in the final survey most students knew it had something to do with light and about half could give a fairly good definition.

6. IDL LIGHT IN OUR LIFE PHOTO CONTEST (2019)

MathCon at Saint Bernard School is an annual celebration of mathematics, with research projects by the Mu Alpha Theta High School and Two Year College Math Honor Society on display for the entire school. In 2018 we awarded the Light in our Life prizes at MathCon, and several student visitors to the award table expressed disappointment that they were unaware a contest was taking place. For 2019, we applied for and received an SPIE IDL mini-grant to support a second Light in our Life photo contest. The grant will pay for prizes and the printing of winning photos for display. As of this writing we have spoken to seven math classes, over 100 students, introducing optics and photonics and their importance in everyday life and inviting students to enter the photo contest. Photographs are due May 1, and winners will be announced at MathCon on May 16.

7. ACKNOWLEDGEMENTS

Funding for all projects except Dark Skies and PBL was provided by outreach grants from SPIE, the international society for optics and photonics. The PBL projects of the New England Board of Higher Education were funded by grants from the Advanced Technological Education program of the National Science Foundation (NSF/ATE). Dark Skies at EASTCONN materials used in our project were purchased with a grant from the OSA Foundation.

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