

Material approaches to thermal management in advanced fiber lasers and amplifiers

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ABSTRACT

For as long as light and matter have partnered, impurities have played a role in optical system performance. This remains generally true for photonic heat engines and especially the case for optical refrigeration. Building on the history of light and glass, including the materials development of low loss telecom fibers, this paper briefly discusses the sources of heat generation in materials and all-material means for their reduction. Particularly attention will be paid to active optical fibers and connect thermal management to parasitic optical nonlinearities, both critical to high and low power amplifier and laser systems.

Keywords: Thermal management, optical fiber, amplifiers, lasers, anti-Stokes fluorescence, transverse mode instability.

1. INTRODUCTION

Heat generation in fiber-based amplifier and laser systems is of great practical consequence. At high optical powers, a system's size, weight, and power (SWaP) consumption is considerably predicated on the heat load and its mitigation, usually through cooling. Further, heat generation in high power laser systems leads to transverse mode instability (TMI [1]), which is the primary current limitation to continued power-scaling, as well as to thermal lensing [2], excess frequency and intensity noise, and degradation of the fiber's protective plastic coatings. At low optical powers, reduced thermal noise is of benefit to ultra-stable lasers and sensors.

Thermal management in amplifier and laser systems is conventionally performed using heat exchangers and mechanical chillers, which can significantly contribute to SWaP and to vibrations. Recently, analysis has been conducted on how the glass comprising the fiber itself may help reduce, even possibly mitigate heat generation completely [3]. The purpose of this paper is to briefly discuss the sources of heat generation in active optical fibers and material-means for their reduction.

2. SOURCES OF HEAT GENERATION IN ACTIVE OPTICAL FIBERS

2.1 Introduction

In some ways, the modern maturity of (silica) optical fiber, with billions of kilometers installed terrestrially, is a blessing and a curse. The blessings are more obvious and stem from global ubiquity of data and all that it enables. The curse is that material intricacies of fibers, and its processing, have been largely forgotten. Indeed, most papers in fiber-related journals and conferences focus on networking designs and protocols as well as non-telecom applications of fibers, such as lasers and sensors. While this is a natural consequence of technology maturation and adoption, glass is not "just glass," [4] and Mother Nature always has something up her sleeve.

When it comes to sources of heat, here too, time has led to much underappreciation. This section briefly discusses the origins of heat generation in active fibers and their connection to the glass from which the fiber is made.

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2.1.1 Extrinsic

The ubiquity of fiber-based data communications is facilitated by silica's remarkably low attenuation. While this is today considered an intrinsic property of the glass, it is only so because extrinsic sources of loss, and therefore heat, have been removed. Transition metal impurities, specifically, Cu and Fe, play the most consequential roles at communication wavelengths, which had been established in the 1970s [5,6]. To remove these extrinsic absorbers, thermodynamics in the form of vapor pressure difference between the glass (e.g., SiCl_4) and impurity (e.g., Fe_2Cl_6) precursors was employed with immediate impact. A good review of this importance, but largely forgotten amongst the current fiber community, is provided in Ref. [7]. Attenuation due to OH species in the glass have also been mitigated in long-haul fibers through judicious drying protocols involving chlorine. Modern silica fibers are essentially free of extrinsic sources of loss, hence heat generation, entirely due to the materials science of the chemical vapor deposition (CVD) process. However, as discussed in Sections 2.1.2 and 2.2, the blessings of CVD for intrinsically low loss fibers results in a curse when it comes to compositional tailorability of the fiber core [8].

2.1.2 Intrinsic

Intrinsic sources of loss in optical fibers relate to both the glass host and the active dopant (e.g., Yb, Er, Tm). As relates to the former, absorption due to electronic transitions and charge transfer at shorter wavelengths or multiphonon vibrations at longer wavelengths are the main intrinsic heat sources. While the impacts of Rayleigh scattering are captured in attenuation (e.g., cut-back) measurements, it is not, per se, an intrinsic absorption process unless the scattered light exits the fiber and is absorbed by the polymer coating. Rayleigh scattering is the principal intrinsic "loss" in silica fibers at 1 μm (Yb-doped systems), whereas Rayleigh scattering and multiphonon absorption contribute together at 1.55 μm (Er-doped systems), and multiphonon absorption dominates at 2 μm operation (Tm-doped systems), setting aside any extrinsic absorption due to the fundamental OH vibration. As noted, the CVD processes employed to fabricate modern silica-based fibers lead to base glasses that are intrinsically pure for all intents and purposes.

As relates to the active dopant, numerous phenomena contribute to heat generation including the quantum defect, multiphonon relaxations, energy transfer, and photodarkening, to name a few. The quantum defect (QD) relates to the energetic difference between the pump and emission photons and represents the theoretical minimum driver of heat generation given an otherwise lossless fiber system (i.e., not considering inefficiencies with pump diodes, coupling, etc.). While the energies of the rare-earth dopant electronic states are fairly immune to host glass composition, the spectral lineshapes can be somewhat tailored resulting in a quantum defect from Yb, for example, that can range from about 8% to less than 1% [8]. Multiphonon relaxation results from electron-phonon coupling between the excited rare-earth dopant ion and the host glass phonons. For example, the highest energy Si-O vibration in silica glass occurs at an energy of about 1100 cm^{-1} . Accordingly, whereas 5 phonons are required to non-radiatively relax an electron across the $^4\text{I}_{13/2}$ to $^4\text{I}_{15/2}$ energy gap in Er^{3+} , which yields the 1.55 μm emission, 9 are required to bridge the $^2\text{F}_{5/2}$ to $^2\text{F}_{3/2}$ transition in Yb^{3+} , which yields the 1 μm emission, for example, making Yb more radiatively efficient. In fairness, the erbium emission at 1.55 μm is also quite efficient from a practical sense, so this illustration is more for relative considerations.

Energy transfer and photodarkening may result when rare-earth ions cluster at higher dopants concentrations. A degradation in laser performance occurs via energy transfer when the excited electron migrates from dopant to dopant in close proximity, ultimately non-radiatively relaxing when it encounters an (extrinsic) impurity or (intrinsic or extrinsic) defect or color center. Photodarkening can occur when excitation energy is cooperatively upconverted and creates color centers in the glass host, leading to added attenuation over time (or optical power). As ion clustering is fundamentally thermodynamic in origin [9], it is noted here as an intrinsic source of loss, hence heat.

Lastly, it is worth noting that optical nonlinearities, such as stimulated Brillouin scattering (SBS) and stimulated thermal Rayleigh scattering (STRS [2]), can also be considered intrinsic sources of loss as they nonlinearly convert signal energy or power to other wavelengths or propagating modes that can subsequently be absorbed or facilitate other parasitic effects, such as TMI. Such nonlinearity scattering phenomena are also fundamentally thermodynamic [10].

2.2 Thermodynamics versus Kinetics

The previous Section invoked thermodynamics as the origin for a variety of intrinsic materials-related phenomena that can directly or indirectly generate heat. Thermodynamics relates to natural end-states under (generally) equilibrium conditions and, while instructive, glass from optical fibers are made is, by definition, a kinetic state of matter, i.e., intrinsically non-equilibrium. Indeed, much of the performance of optical fibers results from an interplay between

thermodynamics and kinetics. Accordingly, understanding this balance is critical to developing ever-more-advanced systems. A fuller discussion of “Thermodynamics versus Kinetics” is beyond the scope of this paper. However, a few examples are provided for illustrative purposes.

2.2.1 Low reduction in modern long-haul telecom fibers

The performance of an optical fiber is dependent on both the waveguide design and the materials from which it is made. But, as noted above, glass is not “just glass” and the process by which the glass is made, specifically its time / temperature history, may also be deterministic. Here, the on-going effort to reduce attenuation in long-haul fibers, even by small fractions of a dB/km, is a good exemplar of this thermodynamic / kinetic balance. As represented well in Ref. [11], the development of “ultra-low loss” fibers (attenuation below 0.15 dB/km) was predicated on reducing the Rayleigh scattering of the pure silica glass core. This was done through thoughtfully controlling the cooling rate of the fiber to permit greater relaxation of the glass network, thus reducing the distribution of ring structures. Materially, this reduces the fictive temperature of the glass, which lessens both the compositional and density contributions to Rayleigh scattering [10]. While Rayleigh scattering is not normally a source of heat in fibers, nonetheless, that the attenuation of the pure silica core depend on cooling rate is proof-positive of kinetics at play.

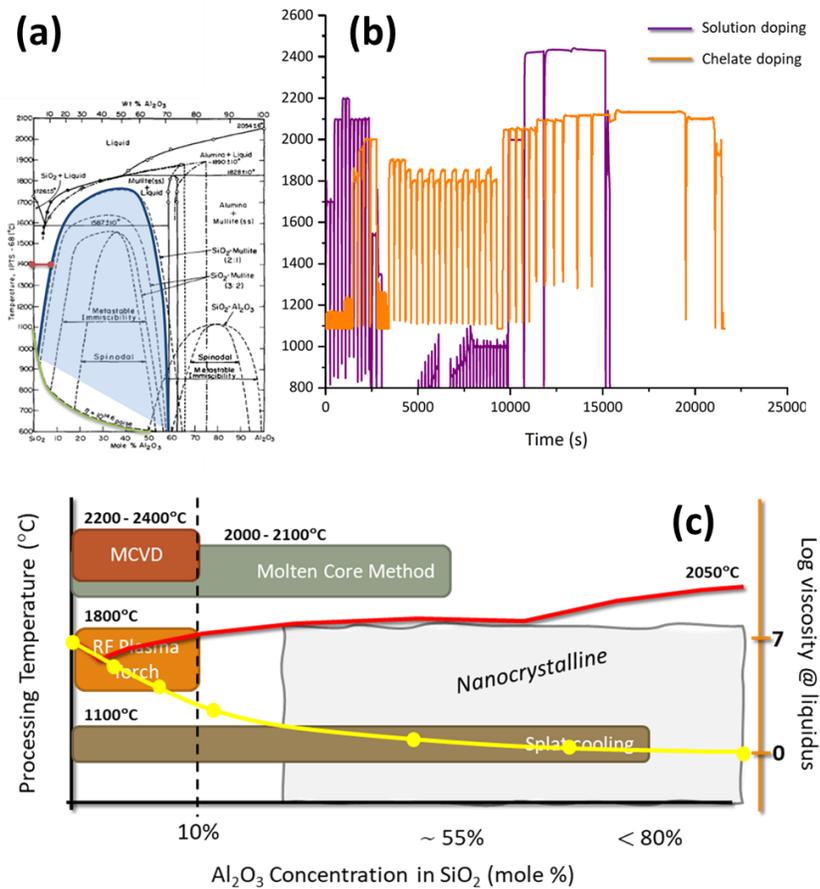


Figure 1. Thermodynamic and kinetics effects in the SiO₂-Al₂O₃ system. (a) The SiO₂-Al₂O₃ equilibrium phase diagram with colored overlay of the high-silica content immiscibility region where phase separation can occur. Reproduced with annotations with permission of John Wiley & Sons and the American Ceramic Society (2022). (b) Representative time-temperature profiles for MCVD with solution doping (“solution doping”) compared to that for chelate doping (“chelate doping”). The temperature ordinate is aligned with that from (b) so one can better understand how the processes relate to the thermodynamic phases and phase behaviors. (c) A relative comparison of processing temperature as a function of permissible Al₂O₃ concentrations for various glass processes, including the liquidus temperature (red curve) and log viscosity of the melt (yellow curve with dotted data points). Underlying data from Refs. [12-16].

2.2.2 Compositional limits in CVD optical fibers: The aluminosilicate system

A second important consideration as relates to optical fiber generally, and heat generation specifically (though, perhaps, not obviously), is the compositional limit of the core glass. As noted, the properties of glass depend on (1) its thermal history and (2) its composition. However, these two are themselves casually related. The range of glass-forming compositions can be tailored by how the glass is fabricated and processed, i.e., its time / temperature history. This is represented in Figure 1. Figure 1(a) shows the equilibrium (i.e., thermodynamic) phase diagram for the (crystalline) $\text{SiO}_2\text{-Al}_2\text{O}_3$ system. There is a well-defined liquidus (melting point) for each composition. Shaded in blue is one of two immiscibility regions where system will phase separate into silica-rich and alumina-rich regions. This is critical because it defines the maximum concentration of Al_2O_3 that can be added to SiO_2 and still form a homogeneous material. Figure 1(b) provides a representation of the time / temperature profile for two CVD processes (modified chemical vapor deposition, MCVD) where the doping is performed either using a solution (“solution doping”) or metal-organic chelate vapors (“chelate doping”). Clearly observed are differences in both the time and the temperature of the various process stages from the start of the preform (time = 0) to the finished collapsed rod (time where data ends). The temperature ordinate in Figure 1(b) is scaled to align with the temperature ordinate of Figure 1(a) for the reader to see how the process temperature overlay against the melting temperatures and immiscibilities. Lastly, Figure 1(c) approximately presents the maximum Al_2O_3 concentration that can be added to SiO_2 for several different processes, their maximum process temperature, as well as melting points and viscosities. Similar limits exist for other dopants into silica [8]. In truth, it is best to consider each composition a fundamentally different glass since all the thermal and physiochemical properties are different. Further, it is worth remembering that glass does not per se have a melting temperature since, above the glass transition temperature, the viscosity changes continuously with temperature. Here, too, the fact that the maximum concentration of alumina prior to phase separation is dependent on processing conditions speaks to the kinetic qualities of glass.

This is relevant to heat generation since clustering of the active rare-earth dopant facilitates energy transfer, concentration quenching, and photodarkening. While there are always glass forming limits, understanding the kinetic aspects of the material system and method of manufacturing offer tools to enhance performance.

3. MATERIAL APPROACHES TO THERMAL MANAGEMENT

The previous Sections discussed sources of heat generation as well as the thermodynamic and kinetic balance of making optical fiber glasses. This Section brings together the previous ones to discuss several novel materials-related approaches to thermal management.

3.1.1 Low quantum defect fibers

The CVD process for manufacturing silica-based optical fibers is compositionally limited by virtue of (i) the availability of suitable vapor-phase precursors and (ii) the time / temperature profile, which restricts the allowable concentration of volatile glass components (e.g., F, B_2O_3 , P_2O_5 , and GeO_2) and can promote phase separation in immiscible material systems. That said, there are other fiber fabrication processes that afford greater compositional flexibility while also being sufficiently scalable at specialty fiber levels, albeit with higher optical losses. One such process is the molten core method (MCM), which is thoroughly reviewed in Ref. [17]. Of note to this Section, the MCM permits considerably higher doping into silica, including, for example upwards of 50 mole percent for Al_2O_3 and 4-5 mole percent fluorine, versus about 10 and 2 percent, respectively, for CVD.

Two approaches to reduced quantum defect (Yb-doped) fibers have employed the MCM. The first relies on a highly modified composition, specifically fluorine doping, to permit a significant reduction in the QD through tailoring of the Yb spectroscopy. Quantum defects below 1% have been realized in this way [18]. The second, while also benefitting from the “intrinsically low” QD in similar MCM silicate fibers, further drives low QD performance by using two wavelengths (Stokes and anti-Stokes pumping) in a pulsed configuration to create “excitation balancing” [19]. In this case, the anti-Stokes pumping is done at a wavelength greater than the lasing wavelength, ultimately contributing a negative QD to the system via conversion of phonons into light energy. Such as system offers the possibility that net zero heat is generated during the stimulated emission process, benefiting lasers as relates to both TMI and thermal lensing.

3.1.2 Radiation balanced amplifiers and lasers

The excitation balanced scheme just discussed, facilitated by the material (spectroscopically) low quantum defect, is one of two such multiwavelength approaches to thermal management. The second approach is radiation balancing, which is enabled by anti-Stokes fluorescence (ASF) cooling. Radiation balancing has a long and rich history, which is described in Ref. [20]. For the purpose of this brief review, in a pseudo-two-level system as an example, excitation at a wavelength greater than (energy less than) the mean fluorescence wavelength facilitates thermalization of the electron (Boltzmann) distribution in the excited state through the absorption of phonons. Subsequent radiative relaxation from this excited state results in the fluorescent photon essentially carrying the thermalized energy out of the system. This potentially results in net cooling if impurities, defects, or other nonradiative centers do not interfere. Even if cooling and net radiation balancing is not achieved, reduced heating is also of great benefit to overall thermal management.

As specifically relates to fibers, the ability to cool via ASF relies on fiber design and several material factors [21]. Necessarily, and materially, the fiber must be low loss and, ideally, exhibit a high critical quenching concentration and low mean fluorescence wavelength. Courtesy of CVD processes, relative to more exotic systems such as fluoride glass fibers [22], silica-based fibers win the low loss criterion, but lose on the critical concentration side. However, with sufficient reduction in extrinsic contaminants, particularly OH, as well as glass modifiers to enhance dopant solubility, such as Al_2O_3 , ASF cooling has been recently observed in silica fiber [23-25]. This represented a significant advancement since silica fibers are extremely mature, indeed commodity, and any useful phenomena that can be achieved in silica is considerably more practical in comparison to exotic glasses and fibers. With these initial successes, namely laser cooling of silica fibers in ambient conditions, i.e., room temperature and pressure, radiation-balanced fiber amplifiers and lasers have subsequently been reported [26,27]. The ideal radiation balanced system refers to an athermal active system whereby internal cooling via, for example ASF cooling, off-sets any heating such as via the positive QD.

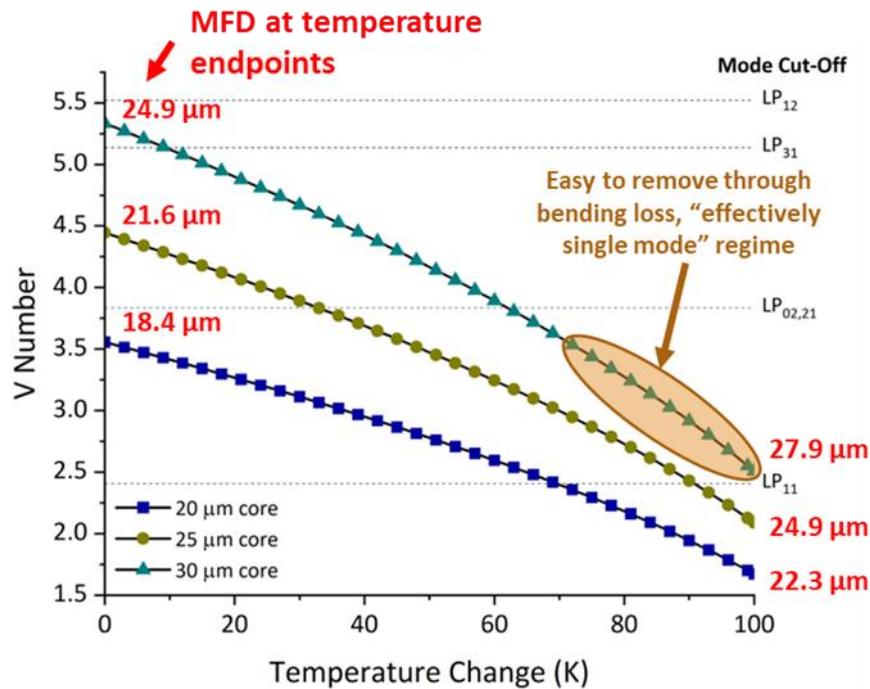


Figure 2. Change in normalized frequency (V Number) as a function of temperature for a representative intrinsically low thermos-optic (dn/dT) coefficient fiber at an optical wavelength of 1065 nm. In this example, as the fiber lases and self-heats, the index difference between core and cladding is reduced, leading to a smaller numerical aperture (NA), hence a reduced number of modes. Further, as the NA decreases with increasing temperature, the mode field diameter increases, which is of benefit to suppress intensity-related nonlinearities.

3.1.3 Low thermo-optical glasses

Discussed previously herein have been materials approaches to reduce heat generation in active fiber systems. Whether achieved completely (e.g., radiation balancing) or partially (e.g., excitation balancing or intrinsically low QD), as stated, even just a reduction in heat load has important implications for SWaP, noise, and parasitic nonlinearities.

Another materials-related approach may best be described as “can’t beat them, join them.” The exemplar here are intrinsically low thermo-optic coefficient fibers. The thermo-optic coefficient (TOC) is defined as the differential change in refractive index, n , with temperature, T , i.e., dn/dT , and is compositionally dependent. Several dopants that are compatible with CVD fabrication are known to reduce the TOC when added into silica, namely B_2O_3 and P_2O_5 (and fluorine to a lesser extent). As a matter of fact, B_2O_3 and P_2O_5 exhibit negative-valued TOCs whereas that for SiO_2 is positive. As a result, at sufficiently high B_2O_3 and P_2O_5 doping levels, the TOC can be considerably less than that for the silica cladding and, in a more extreme case, even achieve a value of zero. In such cases, an interesting and potentially ground-breaking effect is permissible: a thermally self-single-moding fiber.

In an amplifier or laser made using such a fiber, as gain generates heat, the index difference between core and cladding decreases with increasing temperature. Put another way, with increasing temperature, the refractive index of the silica cladding increases. At the same time, the refractive index of the core also increases, but to a lesser extent (possibly not at all in a zero TOC core) relative to the higher TOC cladding. This leads to a numerical aperture (NA) that decreases with increasing gain, hence temperature. This effect is passive in the sense that it is entirely controlled by the material comprising the fiber and scales according to the heat generated through gain. Figure 2 provides a representation of this phenomena for a fiber with core TOC of $2.0 \times 10^{-6} K^{-1}$.

As noted, such intrinsically low thermo-optic fibers do not, per se, change the thermal management considerations of the system, but they do offer a materials approach to lessen the detrimental consequences of heat. As a case in point, TMI is a thermally driven nonlinearity that limits power-scaling [2], with gain saturation being the critical consideration [28,29]. From a materials perspective, the TOC is central to this effect. Thus, such intrinsically low thermo-optic fibers permit considerable power-scaling advantages over conventional large mode area fibers as they reduce the impact of the heat generated. Similar arguments may be made in the case of thermal lensing.

4. CONCLUSIONS

Glass is not just glass [4] and its compositional flexibility offers numerous direct approaches of benefit to reduced heat generation and, therefore, thermal management in advanced amplifier and laser systems. Summarized herein have been a variety of examples, including intrinsically low quantum defect and thermo-optic fibers as well as excitation and radiation balancing schemes. The ability to achieve such effects is predicted on thermodynamic and kinetic considerations associated with glass stability, which critically depends on composition and on how it is made and the time / temperature history the glass experiences. This last sentence is amongst the most important, yet often least appreciated aspect of glass and the performance of optical fiber-based systems.

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