Conventional Adaptive Optics System

A conventional (linear) adaptive optics system, whether it is used for imaging or whether it is used for laser beam propagation, consists of three principal subsystems: a wavefront sensor to detect the optical disturbance, an active mirror or deformable mirror to correct for the optical disturbance, and a control computer to monitor and decode the sensor information for the active mirror.

Adaptive optics imaging system.

Adaptive optics laser projection system.
Image Spread with Atmospheric Turbulence

Linear systems theory shows how an image is composed of an object convolved with the point spread function (PSF) of the imaging system. Atmospheric turbulence degrades the PSF and smears the image.

The PSF is the image of a point source of light. The imaging process experiences diffraction, and the object is convolved with the PSF. The resultant image is a blurred version of the true object.

Adding aberrations to the optical system results in a broadening of the PSF and increased blurring.

Adaptive optics can compensate for the aberrations and reduce blurring.

What is adaptive optics? Answer for the common man--Atmospheric turbulence screws up the image. Adaptive optics unscrews it.
The Principle of Phase Conjugation

All systems of **adaptive optics** generally use the principle of **phase conjugation**. An optical beam is made up of both an amplitude $A$ and a phase $\phi$ component and is described mathematically by the electric field $A\exp(-i\phi)$. Adaptive optics reverses the phase to provide compensation for the phase distortion. The reversal of the phase, being in the exponent of the electric field vector, means changing the sign of the term behind the imaginary number. This mathematical conjugation corresponds to phase conjugation of the optical field, just what is needed to compensate for a distorted phase.

While **Horace Babcock** is generally thought to be the “inventor” of adaptive optics with his paper “The possibility of compensating astronomical seeing,” [Publ. Astron. Soc. Pac. 65, 229, (1953)] his exact idea was never put into practice. It wasn’t until the technological developments in electro-optics in the late 1960s and early 1970s that made a working adaptive optics system possible.
For an uncompensated astronomical telescope the point spread function is limited by the diffraction of the optics and the atmospheric turbulence. The PSF spot has a central core width and an angular width proportional to $\lambda/D$, where $D$ is the telescope pupil diameter. A halo surrounding the core has a width with an angular size of roughly $\lambda/r_0$, where $r_0$ represents the strength of atmospheric turbulence.
**Fried’s Coherence Length**

*Fried’s coherence length* is a widely used descriptor of the level of *atmospheric turbulence* at a particular site. For a fixed wavelength $\lambda$, *astronomical seeing* is given by the angle $\lambda/r_0$. For a known structure constant profile $[C_n^2(z)$, where $z$ is the altitude] and a *flat-Earth assumption*, the coherence length is given by

$$r_0 = \left[ \frac{0.423 k^2 \sec \zeta \int_{\text{Path}} C_n^2(z) dz}{\zeta} \right]^{3/5},$$

where $k = 2\pi/\lambda$, $\zeta$ is the *zenith angle* (0 deg is straight overhead), and the integral is over the path to the ground-based telescope from the source of light.

Under turbulence, the resolution is limited by Fried’s coherence length rather than the diameter of the telescope. Since $r_0$ ranges from under 5 cm with poor seeing to more than 20 cm with good seeing, even in the best conditions, a large diameter telescope without *adaptive optics* does not provide any better resolution than a telescope with a smaller diameter.
The term **brightness** represents the brightness of an object in the heavens. As the object such as a star is observed, the amount of light (number of photons) collected by an aperture (such as the human eye) per second is **astronomical brightness**. The **visual magnitude** $m_v$ of a star is a logarithmic measure of the star’s brightness in the visible spectrum. Smaller numbers represent brighter stars; negative numbers represent even brighter stars. One expression that accounts for atmospheric absorption relates visual magnitude to brightness:

$$B_{\text{astro}} = \left(4 \times 10^6\right)^{-m_v} \cdot 10^{2.5} \text{ photons/cm}^2\text{sec}.$$

It takes about one millisecond for light to pass vertically through the Earth’s atmosphere.
Isoplanatic Angle

Light traveling from a wavefront beacon should traverse the same atmosphere as the light from the object of interest. When the angular difference between the paths results in a mean-square wavefront error of 1.0 rad\(^2\), the angular difference is called the isoplanatic angle. For a given structure constant profile \(C_n^2(z)\) where \(z\) is the altitude), and a flat-Earth assumption, the isoplanatic angle is given by

\[
\theta_0 = \left[ 2.91k^2 \sec^{6/3} \zeta \int_{\text{Path}} C_n^2(z)z^{5/3} dz \right]^{-3/5},
\]

where \(k = \frac{2\pi}{\lambda}\), \(\zeta\) is the zenith angle, and the integral is over the path from the ground-based telescope to the source of light above the surface.

The graph illustrates the isoplanatic angle versus wavelength for the Hufnagel-Valley H-V model and the Stragic Laser Communication SLC model of turbulence.
**Zernike Polynomials**

Optical phase can be represented by a 2D surface over the aperture. The deviation from flat (or some other reference surface) is the wavefront error sensed by the wavefront sensor. A very useful infinite-series representation of the wavefront is the Zernike polynomial series. Radial (index \( n \)) and azimuthal (index \( m \)) polynomials are preceded by Zernike coefficients \( A_{nm} \) and \( B_{nm} \) that completely describe the wavefront up to the order specified by the largest \( n \) or \( m \). The series is written

\[
\Phi(r, \theta) = A_{00} + \frac{1}{\sqrt{2}} \sum_{n=2}^{\infty} A_{n0} \mathcal{Y}^0_n \left( \frac{r}{R} \right) + \sum_{n=1}^{\infty} \sum_{m=1}^{n} (A_{nm} \cos \theta + B_{nm} \sin \theta) \mathcal{Y}^m_n \left( \frac{r}{R} \right),
\]

where the azimuthal polynomials are sines and cosines of multiple angles and the radial polynomial is

\[
\mathcal{Y}^m_n \left( \frac{r}{R} \right) = \sum_{s=0}^{\frac{n-m}{2}} (-1)^s \frac{(n-s)!}{s! \left( \frac{n+m}{2} - s \right)! \left( \frac{n-m}{2} - s \right)!} \left( \frac{r}{R} \right)^{n-2s}.\]

The series is especially useful in adaptive optics because the polynomials are orthogonal over a circle of radius \( R \), common to many optical system geometries. For \( R \) normalized to unity, the first few radial terms are given here.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( n = 0 )</th>
<th>( n = 1 )</th>
<th>( n = 2 )</th>
<th>( n = 3 )</th>
<th>( n = 4 )</th>
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<tbody>
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<td>1</td>
<td>2r^2-1</td>
<td>6r^4 - 6r^2 + 1</td>
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<tr>
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<td>( r )</td>
<td>3r^3-2r</td>
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</tr>
<tr>
<td>( m = 2 )</td>
<td>( r^2 )</td>
<td>4r^4 - 3r^2</td>
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<td>( r^3 )</td>
<td></td>
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<tr>
<td>( m = 4 )</td>
<td>( r^4 )</td>
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</table>
Zernike Polynomials (cont’d)

A few terms are graphed to visualize their relationship to third-order optical aberrations.

Reflecting telescopes of the Cassegrain design have a central obscuration, which requires an extremely large number of Zernike coefficients—more than can be adequately described. A set of annular Zernike polynomials may be obtained from Gram-Schmidt orthogonalization, and this series is generally used for optical systems with central obscurations.
Atmospheric Turbulence Models

One of the most widely used models for the atmospheric turbulence structure constant as a function of altitude is the H-V model:

\[ C_n^2(h) = 5.94 \times 10^{-23} h^{10} \left( \frac{W}{27} \right)^2 \exp(-h) + 2.7 \times 10^{-16} \exp(-2h/3) + A \exp(-10h), \]

where \( h \) is the altitude in kilometers, and \( C_n^2 \) is in units of \( \text{m}^{-2/3} \). The parameters \( A \) and \( W \) are adjustable for local conditions. For the most common **H-V 5/7 model** (leading to \( r_0 = 5 \text{ cm} \) and \( \theta_0 = 7 \mu\text{rad} \)), the structure constant at the surface \( A \) is \( 1.7 \times 10^{-14} \), and the wind velocity aloft \( W \) is 21.

For conditions other than the 5/7 model, one can calculate \( A \) and \( W \) from

\[ A = 1.29 \times 10^{-12} r_0^{-5/3} \lambda^2 - 1.61 \times 10^{-13} \theta_0^{-5/3} \lambda^2 - 3.89 \times 10^{-15}, \]

\[ W = 27(75 \theta_0^{-5/3} \lambda^2 - 0.14)^{1/2}, \]

where the **coherence length** \( r_0 \) is in centimeters and the **isoplanatic angle** \( \theta_0 \) is in microradians.

Other models are layered, such as the **SLC-Night model**:

<table>
<thead>
<tr>
<th>Altitude (above ground)</th>
<th>( C_n^2 )</th>
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</thead>
<tbody>
<tr>
<td>( h \leq 18.5 \text{m} )</td>
<td>( 8.40 \times 10^{-15} )</td>
</tr>
<tr>
<td>( 18.5 &lt; h \leq 110 \text{m} )</td>
<td>( 2.87 \times 10^{-12} h^{-2} )</td>
</tr>
<tr>
<td>( 110 &lt; h \leq 1500 \text{m} )</td>
<td>( 2.5 \times 10^{-16} )</td>
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<tr>
<td>( 1500 &lt; h \leq 7200 \text{m} )</td>
<td>( 8.87 \times 10^{-7} h^{-3} )</td>
</tr>
<tr>
<td>( 7200 &lt; h \leq 20,000 \text{m} )</td>
<td>( 2.00 \times 10^{-16} h^{-0.5} )</td>
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