Transition Bandwidths

Optical transitions all have a finite bandwidth, such as a finite range of frequencies or wavelengths where the corresponding cross sections are large. Different physical phenomena influence the transition bandwidth.

The term homogeneous broadening refers to cases where all involved atoms or ions have the same spectral width and position of the considered optical transition. In many cases, the transition linewidth is determined by the finite lifetime of involved energy levels. In the case of Stark level manifolds in solid-state media, the relevant lifetime can be that of a single sublevel of a manifold, which can be rather short if the interaction with crystal lattice phonons leads to fast transitions between different sublevels. This effect increases the linewidth of transitions in laser crystals by orders of magnitude above the linewidth that would be calculated from the lifetime of the whole Stark level manifolds.

Additional inhomogeneous broadening means that different atoms or ions differ in the spectral positions or widths of their optical transitions, so that the overall cross-section spectrum, which is a kind of average over many contributions, becomes broader. Inhomogeneous broadening occurs, for example, when laser-active ions can occupy different lattice positions in a laser crystal (e.g., in a disordered crystal), and similar effects occur in glasses. In a gas laser, atoms move with different velocities, so that inhomogeneous broadening results from the Doppler effect.

A certain transition is often called (in)homogeneously broadened when (in)homogeneous broadening is dominant. The type of broadening also affects the saturation characteristics (see p. 9) and thus the performance details of many laser systems.
Calculating Laser Gain

In the simplest case, we only have ions (or atoms) in the upper laser level with a number density $N_2$. We then obtain a gain coefficient of

$$g(\lambda) = N_2 \sigma_{em}(\lambda)L,$$

where $L$ is the length of the medium. This translates into a power amplification factor of

$$G = \exp(g) = \exp(N_2 \sigma_{em}(\lambda)L)$$

for the incident light.

If there is also absorption from the lower laser level with a population density $N_1$, for example, we must take this into account and obtain

$$g(\lambda) = (N_2 \sigma_{em}(\lambda) - N_1 \sigma_{abs}(\lambda))L.$$

If the population densities are varying with the position $z$ along the beam, we have to integrate

$$g(\lambda) = \int (N_2(z)\sigma_{em}(\lambda) - N_1(z)\sigma_{abs}(\lambda)) \, dz.$$

The $z$-dependent population densities can be calculated with coupled rate equations (i.e., with differential equations for the temporal evolution). Alternatively, the steady-state population densities as calculated from the local optical intensities of pump and laser radiations can be used. These intensities themselves then depend on population densities, as those determine the gain or absorption. Numerical models can handle such issues. They are particularly complicated when transverse dimensions are also of interest.

Fortunately, simpler models can be used in many cases. For example, the transverse dimension can often be neglected so that the overall gain or absorption in a laser crystal depends only on the spatially integrated population densities, not on their detailed distribution. Also, it is often sufficient to calculate steady-state values, not considering the dynamical aspects.
Basic Principles of Lasers

Gain Saturation

Stimulated emission not only amplifies light, it also removes laser-active ions from the upper state. Even if the gain medium is constantly pumped, a high rate of stimulated emission (resulting from a large amplified intensity) will reduce the population of the upper laser level and therefore “saturate” (reduce) the laser gain.

In many cases, the **steady-state** value of the gain is of interest. Rate equations for an optically pumped gain medium, combined with a few (often valid) assumptions, lead to the simple equation

\[ g(I_p, I_L) = \frac{g_0(I_p)}{1 + I_L/I_{L,\text{sat}}}, \]

where \( I_p \) and \( I_L \) are the pump and laser intensity, respectively, \( g_0 \) is the so-called small-signal gain (achieved for \( I_L = 0 \)), and \( I_{L,\text{sat}} \) is the saturation intensity according to

\[ I_{L,\text{sat}} = \frac{\hbar \nu_L}{\sigma_{\text{em}} \tau_2}, \]

where \( \hbar \nu_L \) is the photon energy of the amplified laser light, \( \sigma_{\text{em}} \) is the emission cross section, and \( \tau_2 \) is the upper-state lifetime.

![Graph showing saturated gain vs. intensity/saturation intensity](image)
Gain Saturation (cont.)

If the intensity of the laser light equals the saturation intensity, the gain is reduced to one-half the small-signal gain. The latter depends on the pump power applied to the gain medium. This dependence may or may not be linear, depending on the circumstances. When constant pump and signal power is applied, the gain usually requires a few times the upper-state lifetime until it reaches the steady-state value as given by the formula.

Another important case is that of a short laser pulse with the fluence (energy per unit area) $F_p$. After amplification of that pulse, the gain is reduced from an initial value $g_0$ to

$$g_{\text{end}} = g_0 \exp\left(-\frac{F_p}{F_{\text{sat}}}\right)$$

with the saturation fluence

$$F_{\text{sat}} = \frac{hv_L}{\sigma_{\text{em}}}.$$

The upper-state lifetime does not occur here because the pulse duration is assumed to be much shorter than the upper-state lifetime, so that spontaneous emission within that time is not relevant.

Inhomogeneous saturation (see the following page) leads to a more complicated situation.
**Homogeneous vs. Inhomogeneous Saturation**

If all laser-active ions (or atoms) have the same emission spectra, and absorption (e.g., in a quasi-three-level medium, see p. 51) can be neglected, gain saturation will simply reduce the magnitude of the gain but not affect its spectral shape. This **homogeneous gain saturation** occurs, for example, in laser crystals where all laser-active ions occupy equivalent positions of the crystal lattice.

**Inhomogeneous saturation** can occur when different ions have different emission spectra (e.g., as a result of different lattice positions in a solid medium) or different particle velocities in a gas. In that case, a narrow-band laser beam may primarily saturate those ions (or atoms) that emit most strongly at the given wavelength. As a result, the balance of different contributions to the gain, and thus the gain spectrum, is modified. The figure below illustrates this effect for a narrow-band laser beam at 1064 nm: The gain is most strongly reduced for wavelengths around that of the saturating beam.

![Graph illustrating gain saturation](image)

Obviously, saturation effects are significantly more complicated to calculate in the case of inhomogeneous saturation. They can have a significant impact on the operation of wavelength-tunable and single-frequency lasers.
Spatial Hole Burning

In many lasers, particularly those with linear resonators, the gain medium is subject to counterpropagating laser beams, which form a so-called “standing wave” via interference, leading to strong, location-dependent laser intensity. This has essentially two effects:

- Gain saturation is stronger at locations with higher laser intensities, leading to a spatial pattern of excitation density. This effect is called spatial hole burning.

- The excitation density in regions with higher optical intensity is more important for the resulting laser gain.

The figure shows how the intensity of a single-frequency laser beam saturates the laser gain (black curve). The reduction of gain for that beam itself is stronger than that for a second beam with a lower intensity and a slightly different wavelength: the antinodes of the latter beam do not fully overlap with the strongly saturated regions of the gain medium. Effectively, we have a kind of inhomogeneous saturation (see p. 9).

The possible consequences on laser operation are manifold. Single-frequency operation becomes more difficult to achieve because the lasing mode experiences more gain saturation than any weak competing modes. The laser efficiency can be reduced if the excitation in the field nodes (dark regions) can not be utilized. Spatial hole burning also has consequences for wavelength tuning and for ultrashort pulse generation with mode locking.
Threshold and Slope Efficiency

Laser operation requires an optical gain that at least compensates for the optical losses in the laser resonator. For a pump power that is too low, this is not achieved; the laser is said to be below threshold. It then emits only some amplified fluorescence, which is normally much weaker than the regular laser emission for pumping above the threshold pump power.

Above the laser threshold, the output power often increases linearly with the pump power, assuming optical pumping. The slope of the curve for output versus input power is called slope efficiency. The figure shows a case with a threshold power of 2.5 W and a slope efficiency of 50%.

Both threshold pump power and slope efficiency of an optically pumped laser can be defined with respect to either incident or absorbed pump power.

A lower output coupler transmission leads to lower threshold pump power, but this can also reduce the slope efficiency because a larger fraction of the circulating power may be lost via parasitic losses. The value of the output coupler transmission for optimum output power depends on the pump power. Most lasers are designed so that the pump power is several times that of the threshold pump power.

Low threshold pump powers can be achieved by keeping the resonator losses small, by using an efficient gain medium with a large product of emission cross section and upper-state lifetime ($\sigma \tau$ product), by using a resonator with a small mode area, and by pumping only the region of the resonator mode within the gain medium.
Threshold and Slope Efficiency (cont.)

For example, the threshold pump power of a simple, optically pumped ring laser (see p. 55) can be calculated, assuming a four-level gain medium (e.g., a Nd:YAG crystal) with length $L$, a top-hat mode profile with area $A$ in the crystal, and total round-trip resonator losses $l$. At threshold, we have $g = l$, thus

$$N_2 \sigma_{em} L = l,$$

with the average density $N_2$ of excited laser-active ions in the crystal (within the mode volume), and the emission cross section $\sigma_{em}$ at the laser wavelength. This degree of excitation causes the emission of fluorescence with the total power of

$$P_\text{fl} = N_2 A L \frac{\hbar \nu L}{\tau_2},$$

where $\hbar \nu L$ is the laser photon energy, and $\tau_2$ is the upper-state lifetime. To compensate for that loss of energy, we need an absorbed pump power of

$$P_{\text{p,abs}} = N_2 A L \frac{\hbar \nu_p}{\tau_2}$$

at threshold and a somewhat higher incident pump power for incomplete pump absorption. (Note also that some of the pump light may be absorbed outside the mode volume, where it is useless.) Using the first equation to substitute $N_2$, we obtain

$$P_{\text{p,abs}} = \frac{A l \hbar \nu_p}{\sigma_{em} \tau_2},$$

which shows that for low-threshold pump power a resonator with a small mode area and low round-trip losses is needed in addition to a gain medium with a high $\sigma \cdot \tau$ product.

In simple cases, the slope efficiency can be calculated as a product of various factors, such as the pump absorption efficiency (for a slope efficiency with respect to incident pump power), the quantum efficiency, the ratio of pump and laser wavelength, and the output coupling efficiency (the fraction of generated photons that is extracted via the output coupler rather than lost via parasitic losses).
Various issues are important for achieving a high power conversion efficiency, particularly for optically pumped solid-state lasers:

- The gain medium should be long enough and sufficiently doped for efficient absorption of pump light.

- For (quasi-)three-level gain media (see p. 3 and p. 51), the length must also not be too long, even if this somewhat compromises the pump absorption. Otherwise, reabsorption effects become too strong.

- The threshold pump power should be kept low by minimizing the resonator losses. Also, the resonator losses should be dominated by the output coupler transmission (high output coupling efficiency), so that most of the generated photons are transferred into the output beam rather than lost through parasitic losses. The latter should be minimized, for example, by using high-quality mirrors (with high reflectivity) and materials with low scattering and absorption losses, and by minimizing the number of optical surfaces.

- The resonator should have modes with suitable effective areas, and pumping should only occur within the volume of the resonator mode(s).

- The gain medium should not exhibit detrimental processes, such as parasitic absorption (including excited-state absorption) or quenching of the upper-state population via unwanted cross-relaxation or upconversion. It should also have a small quantum defect (i.e., a high ratio of laser and pump radiation photon energies).

This shows that optimizing the power efficiency requires a balance of many factors, sometimes involving subtle trade-offs.
Amplified Spontaneous Emission

Even for pump powers below the laser threshold, a laser emits some light, usually at a low power level. This light originates from spontaneous emission in the gain medium and is further amplified. While this amplified spontaneous emission (ASE) can be very weak in lasers with small resonator losses, it often limits the achievable gain in a laser amplifier because it extracts a significant amount of power as soon as the gain reaches a level of several tens of decibels. The figure below shows this effect for a fiber amplifier, where ASE saturates the gain for pump powers above ≈0.6 W.

While fluorescence goes in all spatial directions, ASE can be strongly directional for gain media with a large aspect ratio (e.g., fibers or long laser rods), providing a longer path with amplification for some spatial direction.

Amplified spontaneous emission is not always unwanted. It can be used in cases where a broad and smooth optical spectrum, combined with significant output power and high spatial coherence, is required (e.g., for optical coherence tomography, gyroscopes, and some fiber-optic sensors). Such ASE sources (also called superluminescent sources) are essentially lasers (often laser diodes) where the optical feedback from resonator mirrors has been removed.
Characteristics of Laser Light

Laser light has a number of special properties:

- Lasers normally emit light in the form of laser beams. Their high degree of **spatial coherence** (the phase relationship over the whole beam profile) allows for a small beam divergence of a moderately sized beam (a high directionality), and also makes it possible to focus laser radiation on very small spots.

- In many cases, laser light also exhibits a high degree of **temporal coherence** (see p. 16), which means a long **coherence length** and a narrow optical spectrum (quasi-monochromatic emission). Some carefully stabilized lasers can emit monochromatic light with an extremely well-defined optical frequency. In extreme cases, the optical bandwidth is less than $10^{-15}$ times the optical mean frequency.

- Most lasers emit infrared (invisible) light, while only a few can emit visible or even ultraviolet light. However, methods of nonlinear frequency conversion can be used to generate other wavelengths (see p. 103-108).

- In most cases, laser radiation is **linearly polarized**; for example, the electric field oscillates only in one direction, which is perpendicular to the laser beam.

An additional interesting feature is that some lasers can emit light in the form of short pulses, with pulse durations of nanoseconds, picoseconds, or even down to a few femtoseconds. Even moderate pulse energies (e.g., a few millijoules) lead to enormous peak powers when combined with ultrashort pulse durations.

Some of these characteristics, in particular the possible high optical intensities, can make laser light quite hazardous (see p. 114). Diligent laser safety measures are then required, particularly for avoiding eye injuries.